

The 22nd European Conference on Few-Body Problems in Physics  
Cracow, 9th September 2013

## **Multicomponent Strongly-Interacting Few-Fermion Systems in One Dimension**

---

**Artem G. Volosniev**

Aarhus University, Denmark

# Collaborators

Dmitri Fedorov, Aksel Jensen and Nikolaj Zinner (Aarhus University, Denmark),

Jonathan Lindgren, Jimmy Rotureau and Christian Forssén (Chalmers University of Technology, Sweden),

Manuel Valiente (Heriot-Watt University, UK).

Jonathan Lindgren et al. (2013) arXiv:1304.2992  
Artem G. Volosniev et al. (2013) arXiv:1306.4610

# Outline

- 1 System
- 2 Aims
- 3 Motivation
- 4 Approach
- 5 Results
- 6 Conclusions

# System

$N$  particles of equal mass in one spatial dimension:

- the particles are divided into classes of identical spinless fermions;
- the system is trapped by the external potential;
- the interparticle interaction is assumed to be of zero range  
 $V(x_i - x_j) = g\delta(x_i - x_j)$ .



# Aims

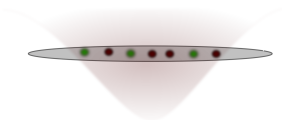
$$H = -\frac{\hbar^2}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + g \sum_{i>j} \delta(x_i - x_j)$$

The aim is to obtain analytically the eigenvalues and the corresponding eigenstates for such Hamiltonian for large repulsive interaction, i.e. in the vicinity of  $1/g \rightarrow 0$ .

# Motivation to study such systems

1. The system is experimentally realizable<sup>1</sup> and needs a thorough theoretical description.

- classes of spinless fermions:  ${}^6\text{Li}$  hyperfine states
- quasi-one-dimensional geometry: optical lattices with different aspect ratios
- interparticle interaction can be tuned using external magnetic field

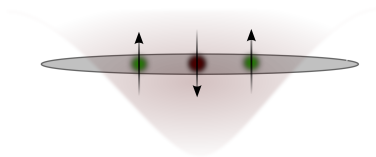


2. The analytical solution can be used as a reference point for different numerical simulations.

<sup>1</sup>See for example G. Zürn (2012) PhD thesis *Few-fermion systems in one dimension*

# Approach to obtain solutions

The approach can be shown by using the simple system of *three particles trapped in the harmonic oscillator*: two particles out of three from one class that will be referred to as *spin up* and *one particle* will be called *spin down*.

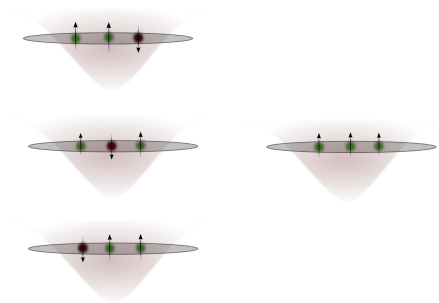


$$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_{\uparrow(1)}^2} - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_{\uparrow(2)}^2} - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_{\downarrow}^2} + g\delta(x_{\uparrow(1)} - x_{\downarrow}) + g\delta(x_{\uparrow(2)} - x_{\downarrow})$$

# Approach to obtain solutions

Note that for  $1/g = 0$

- impenetrable regime (two particles cannot exchange their position)
- fermionization:  $\Psi(x_{\uparrow(1)} < x_{\downarrow} < x_{\uparrow(2)}) \sim \Psi_{spinlessfermions}(x_{\uparrow(1)}, x_{\uparrow(2)}, x_{\downarrow})$

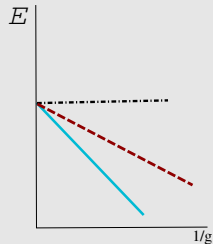


# Approach to obtain solutions

It follows

- the energy spectrum is the same as for three spinless fermions.
- the system can be found in the following independent configurations  $\uparrow\uparrow\downarrow, \uparrow\downarrow\uparrow, \downarrow\uparrow\uparrow$ , which means that the energy spectrum is three times degenerate.

- find  $E \simeq E(1/g = 0) - K/g$
- find the corresponding wave functions



# Approach to obtain solutions

$$K = \lim_{g \rightarrow \infty} g^2 \frac{\partial E}{\partial g} = \lim_{g \rightarrow \infty} \frac{g^2 \int |\Psi|^2 (\delta(x_{\uparrow(1)} - x_{\downarrow}) + \delta(x_{\uparrow(2)} - x_{\downarrow})) dx_{\uparrow(1)} dx_{\uparrow(2)} dx_{\downarrow}}{\langle \Psi | \Psi \rangle}$$

# Approach to obtain solutions

$$K = \lim_{g \rightarrow \infty} g^2 \frac{\partial E}{\partial g} = \lim_{g \rightarrow \infty} \frac{g^2 \int |\Psi|^2 (\delta(x_{\uparrow(1)} - x_{\downarrow}) + \delta(x_{\uparrow(2)} - x_{\downarrow})) dx_{\uparrow(1)} dx_{\uparrow(2)} dx_{\downarrow}}{\langle \Psi | \Psi \rangle}$$

$$\left( \frac{\partial \Psi}{\partial x_{\uparrow}} - \frac{\partial \Psi}{\partial x_{\downarrow}} \right) \Big|_{x_{\uparrow} - x_{\downarrow} = +0} - \left( \frac{\partial \Psi}{\partial x_{\uparrow}} - \frac{\partial \Psi}{\partial x_{\downarrow}} \right) \Big|_{x_{\uparrow} - x_{\downarrow} = -0} = 2g \Psi \Big|_{x_{\uparrow} = x_{\downarrow}}$$

$$K = K[\Psi(1/g = 0)]$$

# Approach to obtain solutions

$$K = \lim_{g \rightarrow \infty} g^2 \frac{\partial E}{\partial g} = \lim_{g \rightarrow \infty} \frac{g^2 \int |\Psi|^2 (\delta(x_{\uparrow(1)} - x_{\downarrow}) + \delta(x_{\uparrow(2)} - x_{\downarrow})) dx_{\uparrow(1)} dx_{\uparrow(2)} dx_{\downarrow}}{\langle \Psi | \Psi \rangle}$$

$$\left( \frac{\partial \Psi}{\partial x_{\uparrow}} - \frac{\partial \Psi}{\partial x_{\downarrow}} \right) \Big|_{x_{\uparrow} - x_{\downarrow} = +0} - \left( \frac{\partial \Psi}{\partial x_{\uparrow}} - \frac{\partial \Psi}{\partial x_{\downarrow}} \right) \Big|_{x_{\uparrow} - x_{\downarrow} = -0} = 2g \Psi \Big|_{x_{\uparrow} = x_{\downarrow}}$$

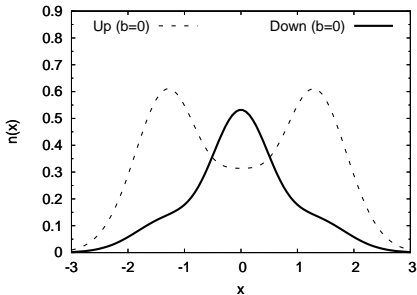
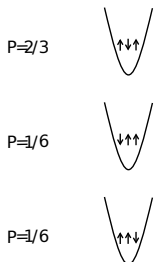
$$K = K[\Psi(1/g = 0)]$$

$$\Psi(1/g = 0) = \begin{cases} a_1 \Psi_{\text{spinlessfermions}} : \uparrow\uparrow\downarrow \\ a_2 \Psi_{\text{spinlessfermions}} : \uparrow\downarrow\uparrow \\ a_3 \Psi_{\text{spinlessfermions}} : \downarrow\uparrow\uparrow \end{cases}$$

$$K = K(a_1, a_2, a_3)$$

# Results

For the ground state:  $a_3 = a_1, a_2 = -2a_1$

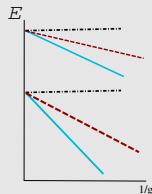


# Conclusions

For

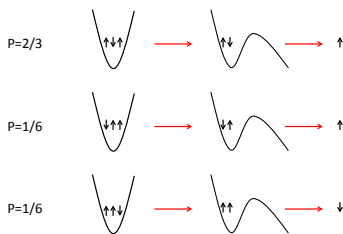
- $N$  particles of equal mass in one spatial dimension;
- the particles are divided into classes of identical spinless fermions;
- the system is trapped by the external potential;
- the interparticle interaction is assumed to be of zero range  
 $V(x_i - x_j) = g\delta(x_i - x_j)$ .

It is possible to find analytically the energy spectrum near  $1/g \rightarrow 0$  up to linear in  $1/g$  order and the corresponding wave functions at  $1/g = 0$



# Conclusions

## Experimental probe



This means that the probability to see the particle with spin down is five times smaller than the probability to detect a particle with spin up.

## A few relevant publications

### Experiment (Heidelberg)

F. Serwane PhD thesis *Deterministic preparation of a tunable few-fermion system* (2011)

G. Zürn PhD thesis *Few-fermion systems in one dimension* (2012)

Andre Wenz et al. arxiv:1307.3443

### Theory

J. Lindgren et al. (2013) arXiv:1304.2992

A. Volosniev et al. (2013) arXiv:1306.4610

S. Gharashi and D. Blume (2013) *PRL* **111**, 045302

T. Sowiński et al. (2013) arXiv:1304.8099

X. Cui and T.-L. Ho (2013) arXiv:1305.6361