IMPLICIT x EXPLICIT RENORMALIZATION OF THE NUCLEAR FORCE

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OUTLINE

- Motivation
- Implicit renormalization (NN: 1S0 & 3S1)
- Explicit renormalization (Toy model)
- Implicit versus Explicit
- Final remarks

MOTIVATION

- What is the difference between

  a contact theory based on low-energy parameters

  and

  a theory with its high momentum components explicitly integrated out with a flow equation like $V_{low\ k}$ or SRG ?

IMPLICIT RENORMALIZATION

Contact theory in the continuum, regulated by a sharp cutoff

$$
\begin{aligned}
V_\Lambda(p', p) &= C_0 + C_2(p^2 + p'^2) \\
&+ C_4(p^4 + p'^4) + C'_4 p^2 p'^2 + \ldots.
\end{aligned}
$$

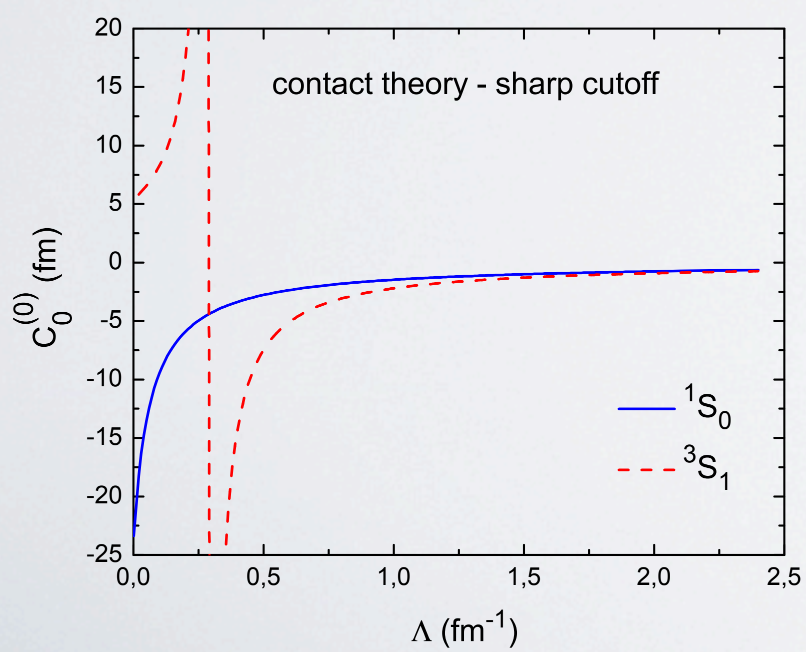
LO: $C_0 \to \alpha_0$

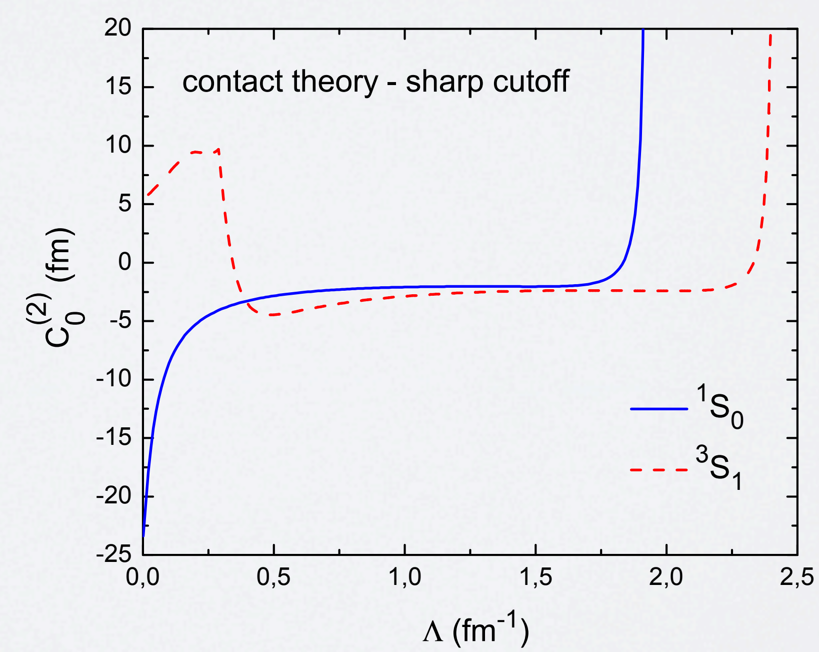
$$C_0(\Lambda) = \frac{\alpha_0}{1 - \frac{2\Lambda\alpha_0}{\pi}}$$

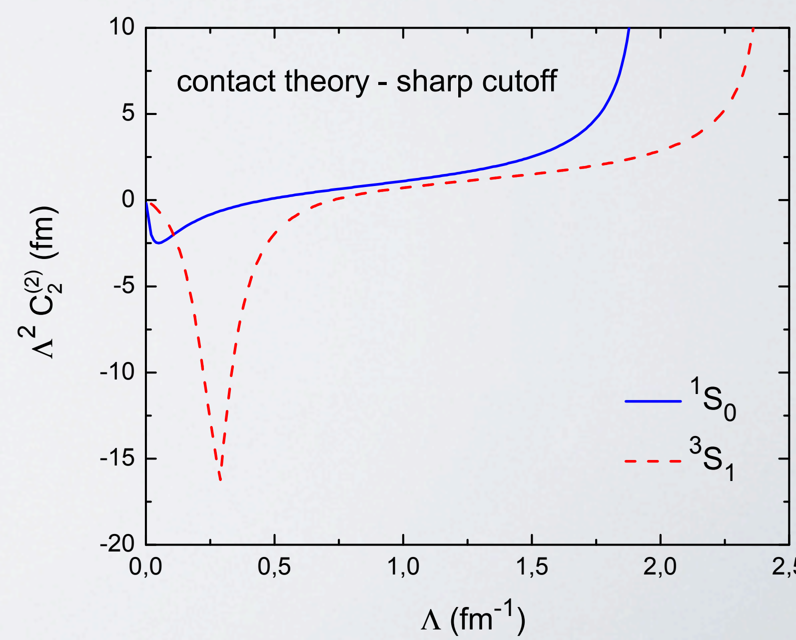
NLO: $(C_0, C_2) \to (\alpha_0, r_0)$

$$-\frac{1}{\alpha_0\Lambda} = \frac{4\left(-2c_2^2 + 90\pi^4 + 15(3c_0 + 2c_2)\pi^2\right)}{9\pi\left(c_2^2 - 10c_0\pi^2\right)}$$

$$
\begin{aligned}
r_0\Lambda &= \frac{16\left(c_2^2 + 12\pi^2 c_2 + 9\pi^4\right)}{\pi\left(c_2 + 6\pi^2\right)^2} - \frac{12c_2\left(c_2 + 12\pi^2\right)}{\left(c_2 + 6\pi^2\right)^2}\frac{1}{\alpha_0\Lambda} \\
&+ \frac{3c_2\pi\left(c_2 + 12\pi^2\right)}{\left(c_2 + 6\pi^2\right)^2}\frac{1}{\alpha_0^2\Lambda^2}
\end{aligned}
$$



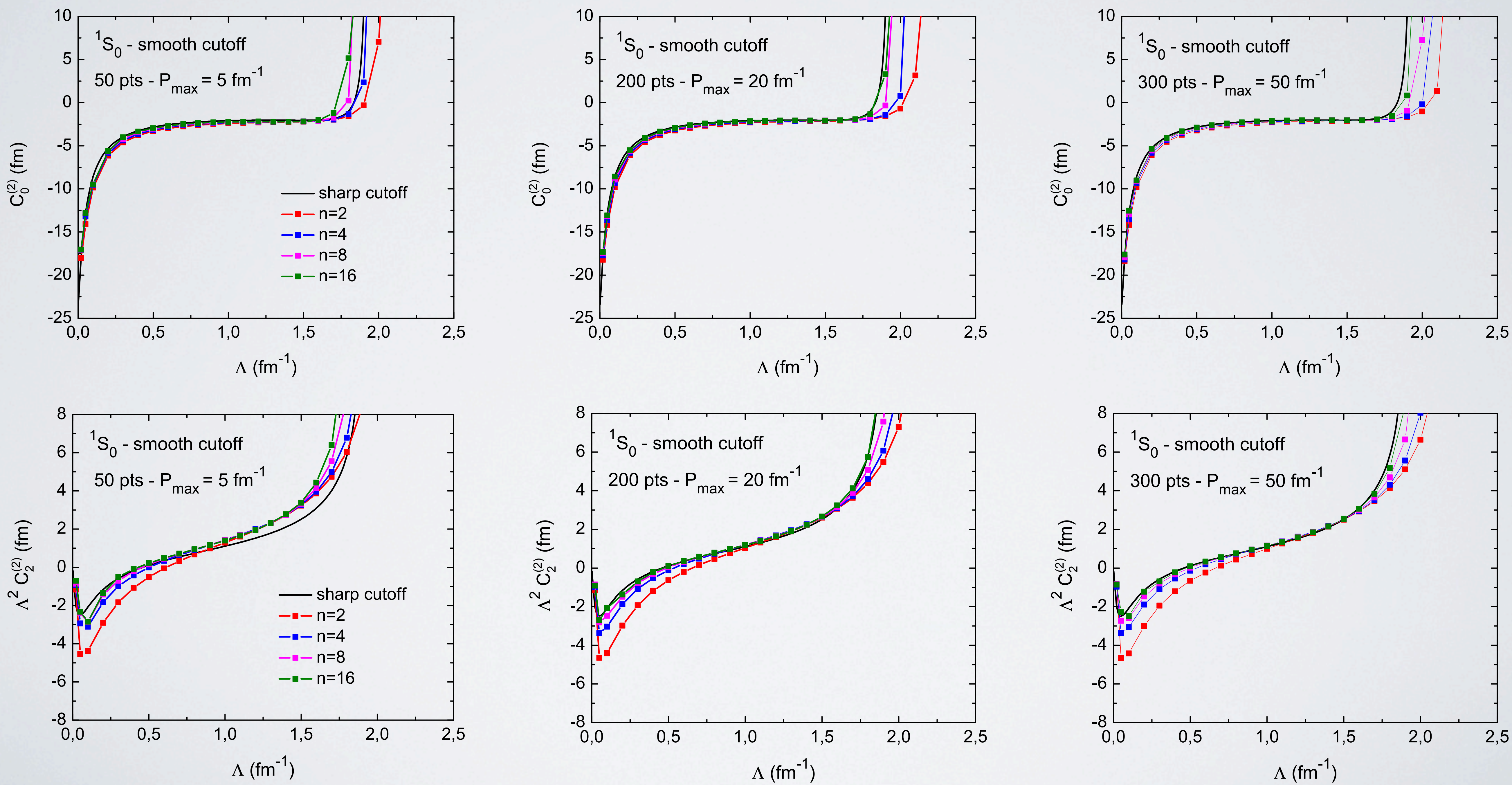




IMPLICIT RENORMALIZATION - 1S0

Contact theory on the grid, regulated by a smooth cutoff

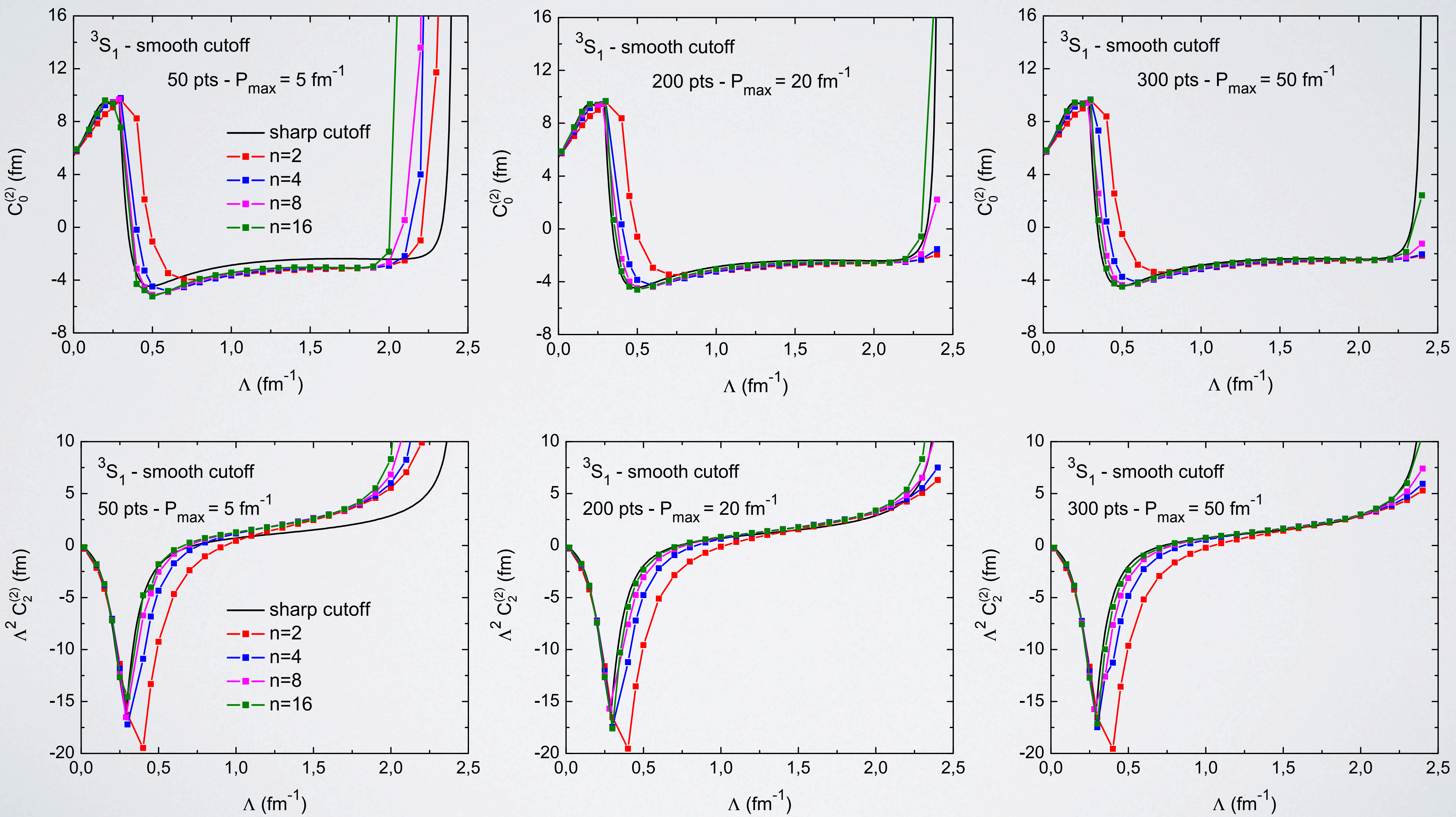
$$f_R = \exp[\mathrm{p}/\Lambda]^{2\mathrm{n}}$$



IMPLICIT RENORMALIZATION - 3S1

Contact theory on the grid, regulated by a smooth cutoff

$$f_R = \exp[\mathrm{p}/\Lambda]^{2\mathrm{n}}$$

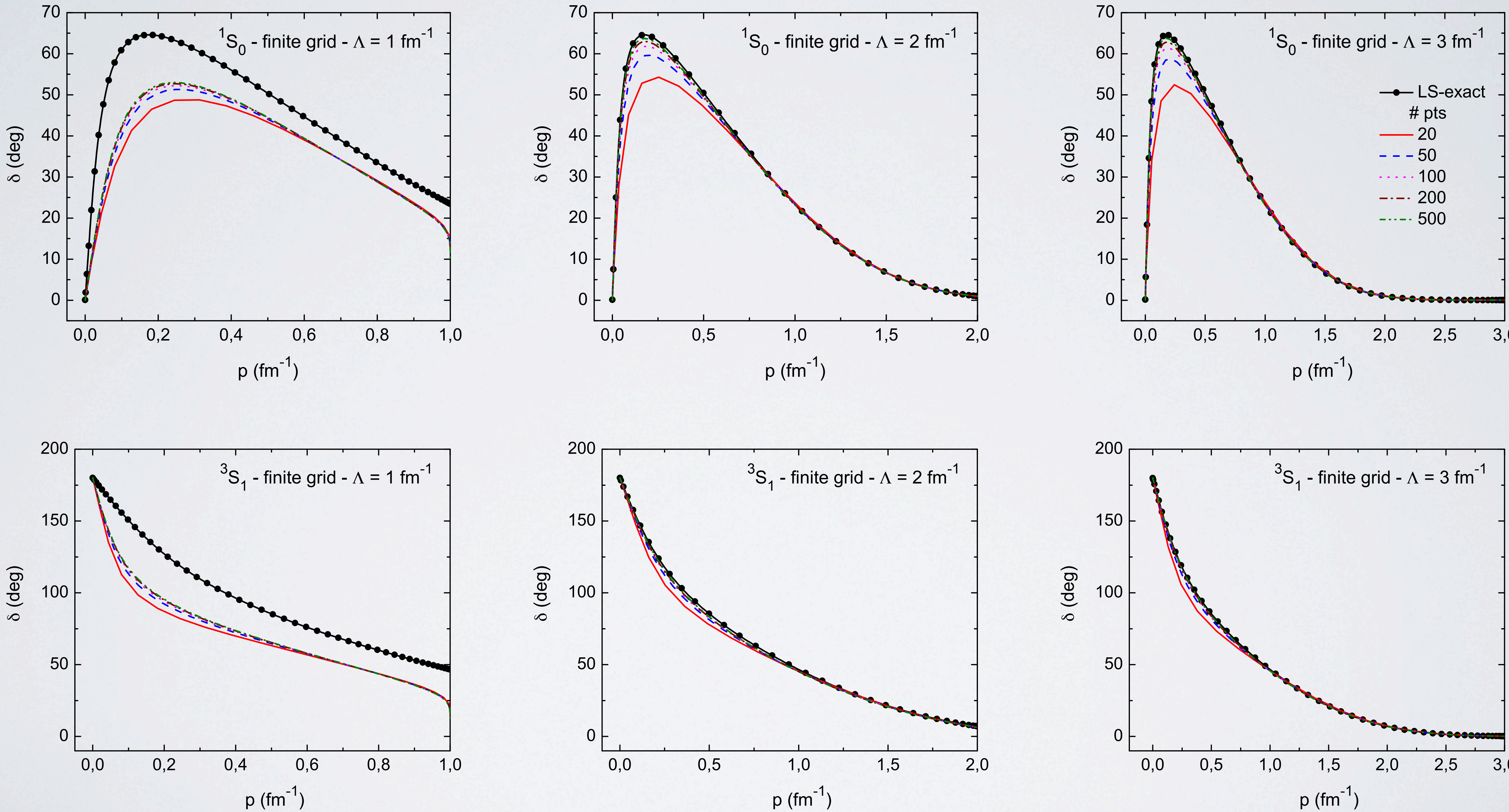


TOY MODEL - 1S0 & 3S1

$$V(p,p') = C\ e^{-(p^2+p'^2)/L^2}$$

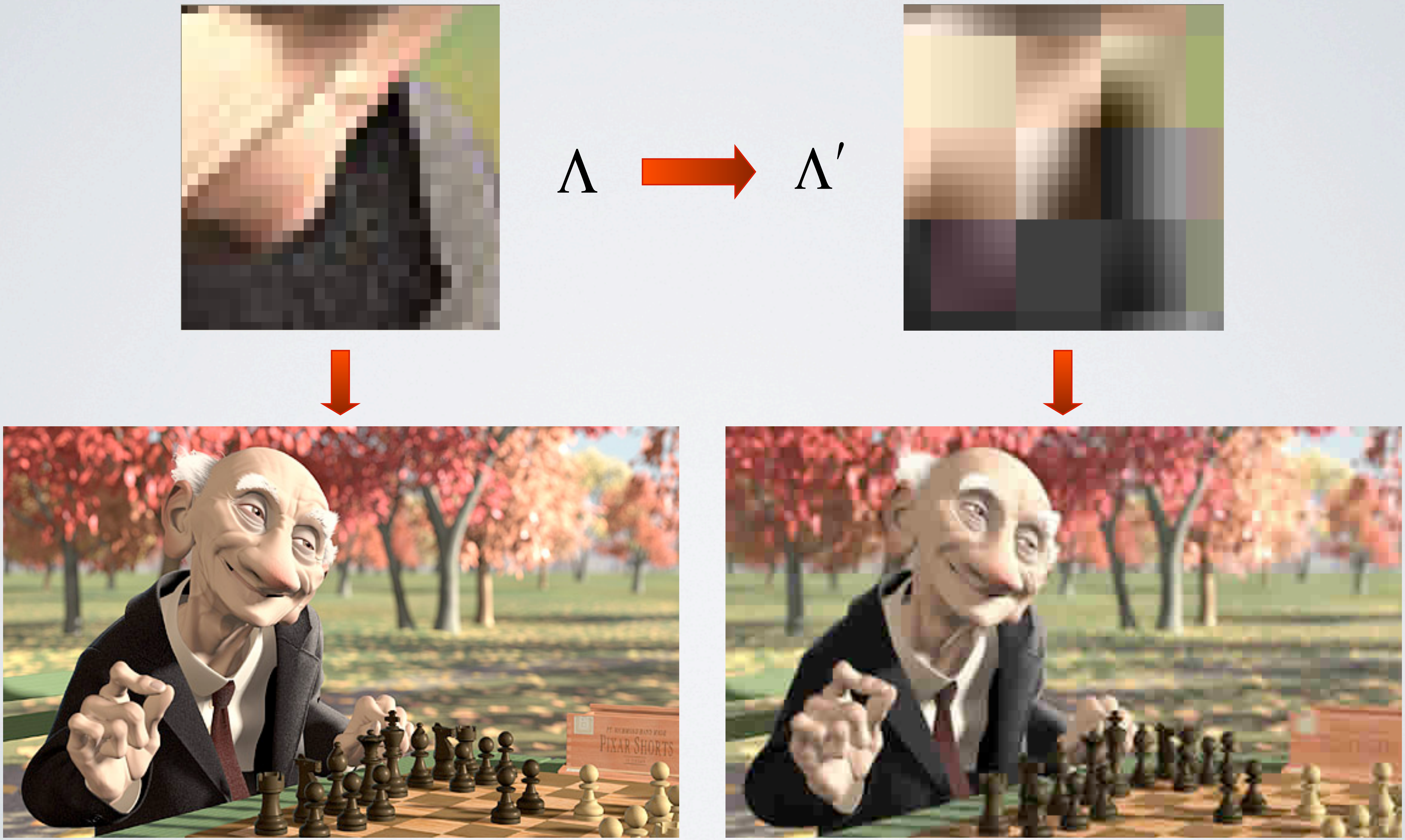
Parameter Units	α_0 (fm)	r_0 (fm)	C (fm)	L (fm^{-1})
$^1S_0$	-23.74	2.77	-1.9158	0.6913
$^3S_1$	5.42	1.75	-2.3006	0.4151

PHASE SHIFTS FROM THE TOY MODEL



Effective theory principle

Physics at low energy / large distances scales is insensitive to the details of the physics at high energy / small distances scales

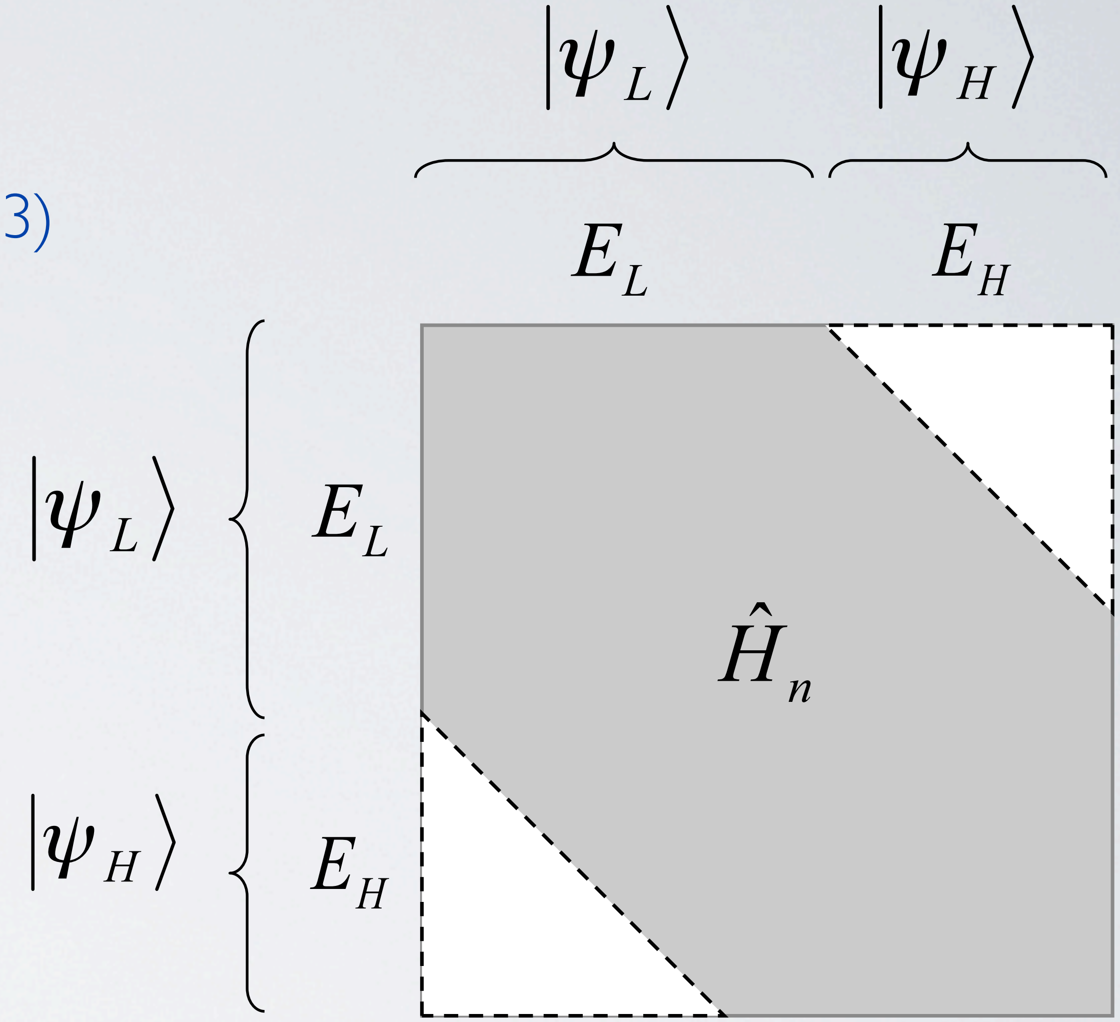


Similarity Renormalization Group

Similarity Transformation:

S. D. Glazek and K. G. Wilson, Phys. Rev. D 48, 5863 (1993)

$$H_n\left[\Lambda_n\right] = T^{(n)}_{Sim.}\left\{H_0\left[\Lambda_0\right]\right\}$$



Doesn't remove degrees of freedom

But suppresses states with large energy difference (off-diagonal elements):

$$\langle\psi_L|H|\psi_H\rangle \rightarrow \Lambda_n \leq (E_H - E_L) \leq \Lambda_0$$

Unitary transformation: doesn't affect observables $\longrightarrow$ $T^\dagger T = 1$

Similarity Renormalization Group

Wegner's formulation:

F. Wegner, Annalen der Physik (Berlin) 3, 77 (1994)

Flow equation:

$$H_s = U(s)\ H\ U^\dagger(s) = T + V_s$$

$$\frac{d}{ds} H_s = [H_s, \eta_s]$$

Flow parameter: $s = \frac{1}{\lambda^4}$ $(0 \leq s \leq \infty)$

similarity cutoff: dimension of momentum

Boundary condition: $\lim_{s \to s_0} H_s = H_{s_0}$

Transformation generator:

$\eta_s = [H_s, H_D]$ ⟶ H_D ⟶ Diagonal part of Hamiltonian

Using the free Hamiltonian:

$\eta_s = [H_s, T]$ ⟶ $\frac{d}{ds} H_s = [H_s, [H_s, T]]$

Similarity Renormalization Group

For the NN interaction

$$\eta_s = [H_s, T] \longrightarrow \frac{d}{ds} H_s = [H_s, [H_s, T]]$$

$$\frac{d}{ds} V_s(p,p') = -(p^2 - p'^2)\ V_s(p,p') + \frac{2}{\pi}\int dq\ q^2\ (p^2 + p'^2 - 2q^2)\ V_s(p,q)\ V_s(q,p')$$

S. Szpigel and R. J. Perry, in "*Quantum Field Theory, A 20th Century Profile*", ed. A.N. Mitra, (Hindustan Publishing Com., New Delhi, 2000)

S.K. Bogner, R.J. Furnstahl, and R.J. Perry, Phys. Rev. C 75, 061001(R) (2007)

S.K. Bogner, R.J. Furnstahl, R.J. Perry, and A. Schwenk, Phys. Lett. B 649, 488 (2007)

E.D. Jurgenson, P. Navratil, R.J. Furnstahl, Phys. Rev. Lett. 103 (2009) 082501

With regular or regularized potentials

SRG WITH BLOCK-DIAGONAL GENERATOR

unitary version of the V_low k

E. Anderson et al., Phys. Rev. C 77 (2008) 037001

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [G_s, H_s] \qquad G_s = H_s^{\mathrm{BD}} \equiv \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$$

Λ

$$V_s \equiv \begin{pmatrix} PV_sP & PV_sQ \\ QV_sP & QV_sQ \end{pmatrix}$$

$$P \equiv \theta(\Lambda - p); \; Q \equiv \theta(p - \Lambda)$$

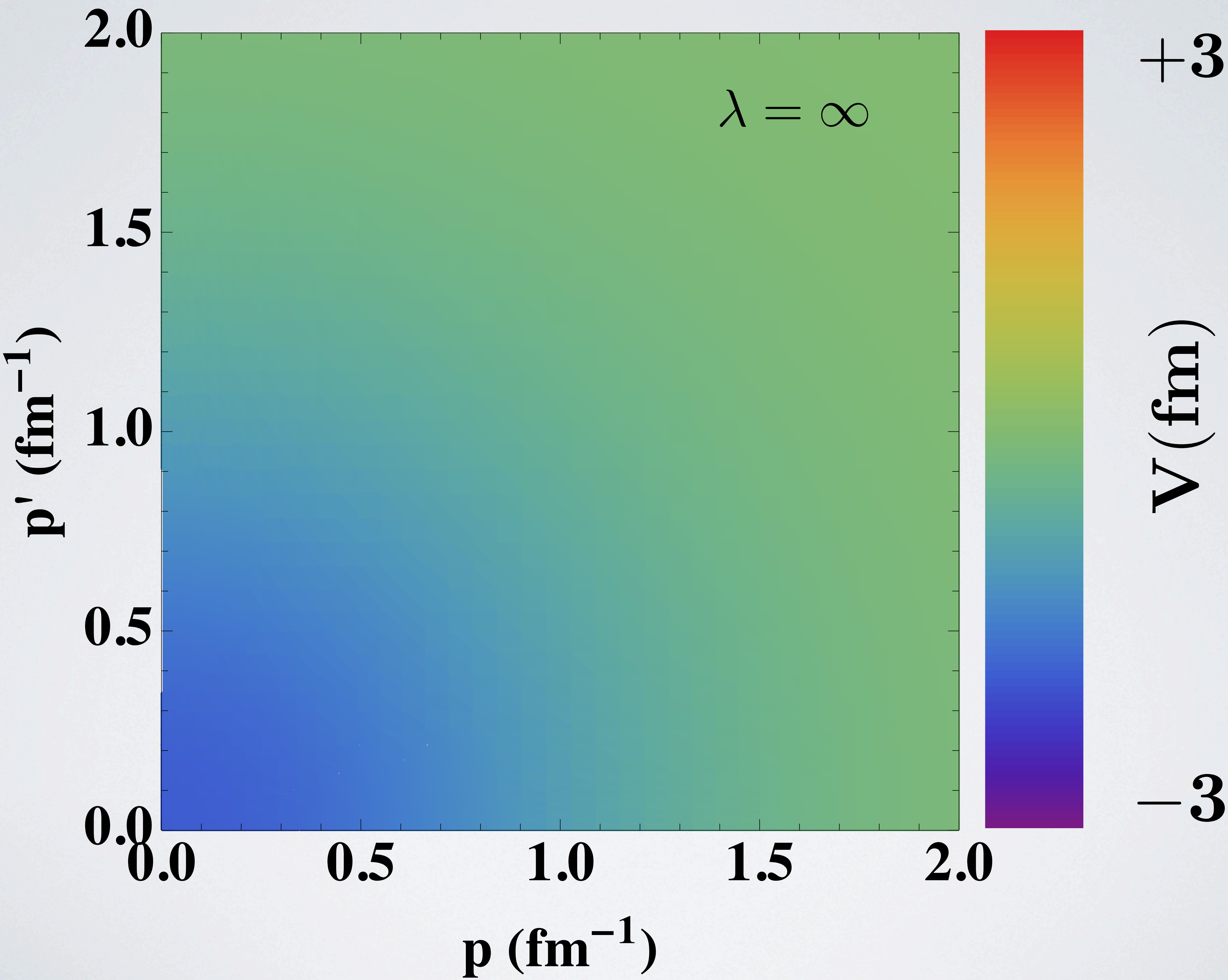
$$\frac{dV_s}{ds} = [\eta_s, H_s]$$

$$\lim_{\lambda \to 0} V_\lambda = PV_{\mathrm{lowk}}P + QV_{\mathrm{highk}}Q = \begin{pmatrix} V_{\mathrm{low\,k}} & 0 \\ 0 & V_{\mathrm{high\,k}} \end{pmatrix}$$

$$\lim_{\lambda \to 0} \delta_\lambda(p) = \delta_{\mathrm{lowk}}(p) + \delta_{\mathrm{highk}}(p)$$

Is this low-momentum interaction, obtained by the explicit integration of the high-momentum components, similar to the contact theory obtained by the implicit connection with the low-energy physics ???

EXPLICIT RENORMALIZATION - 1S0



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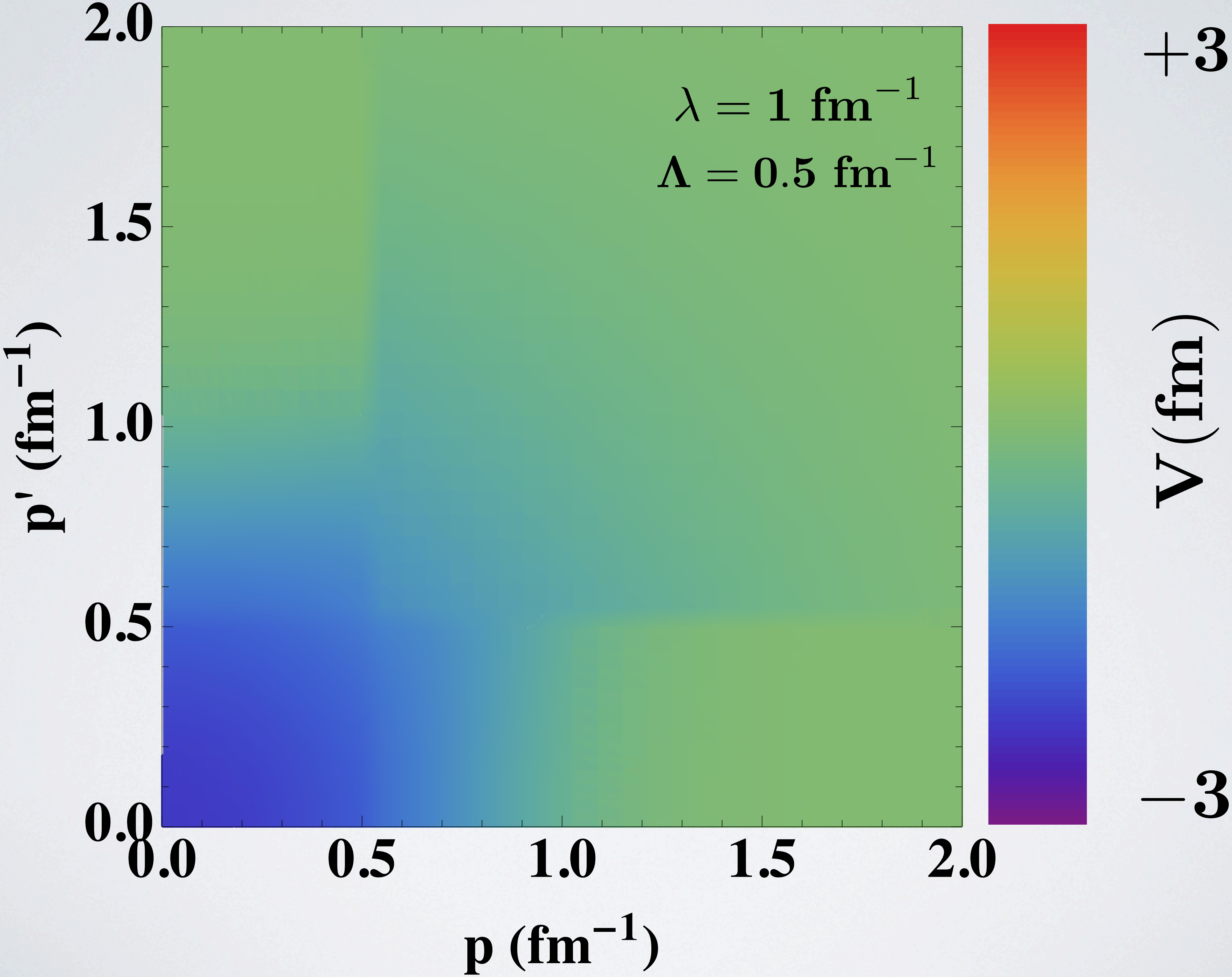

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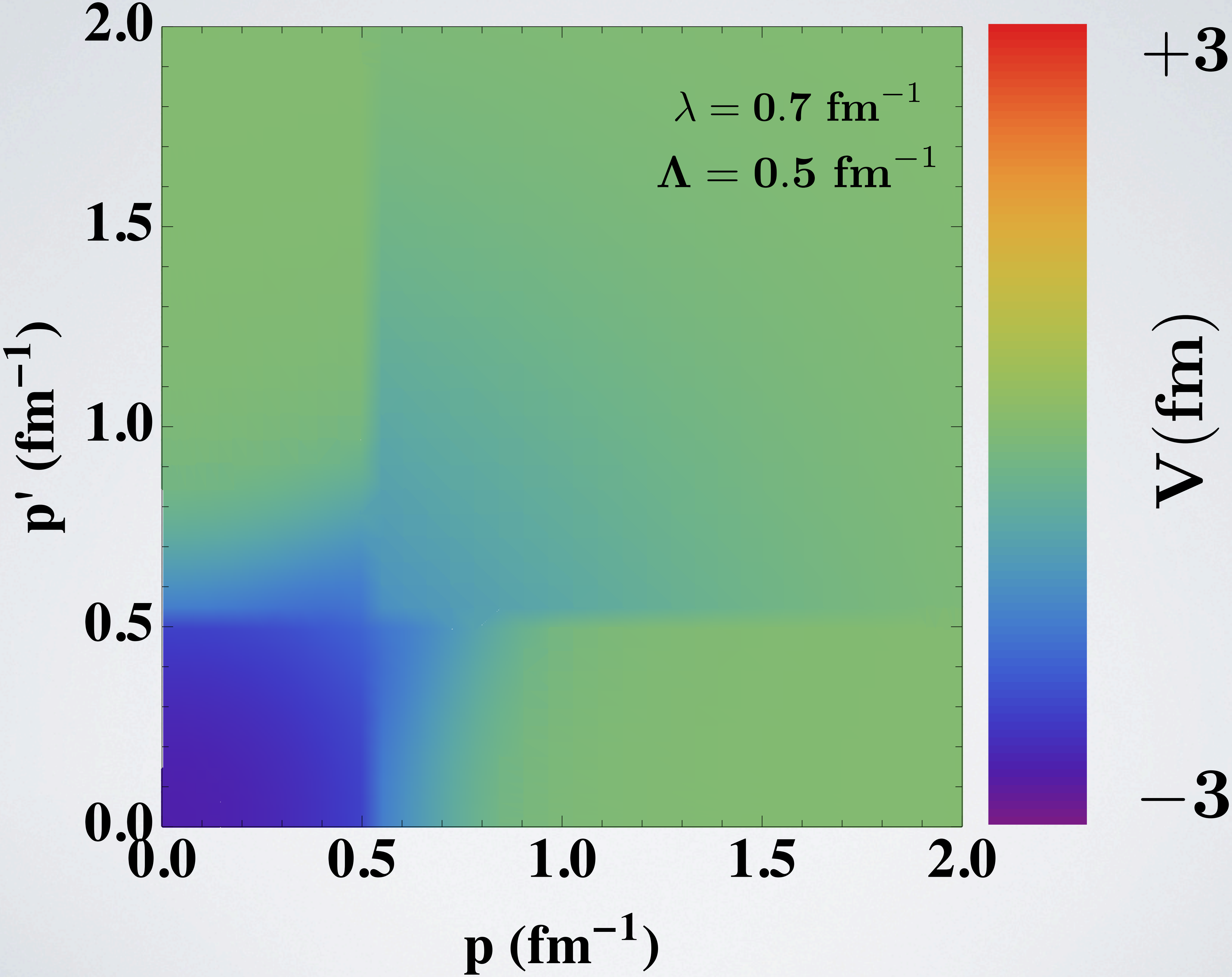

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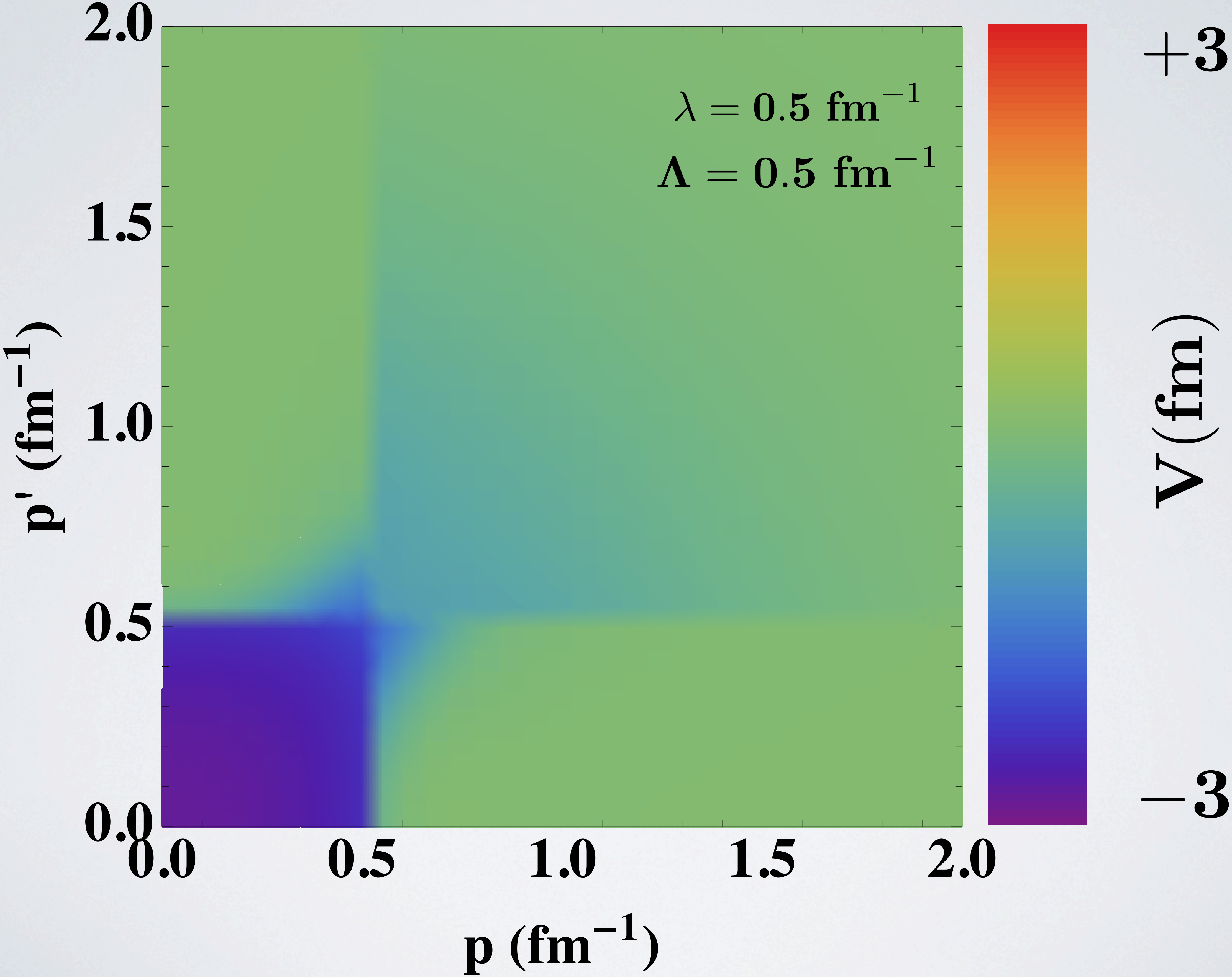

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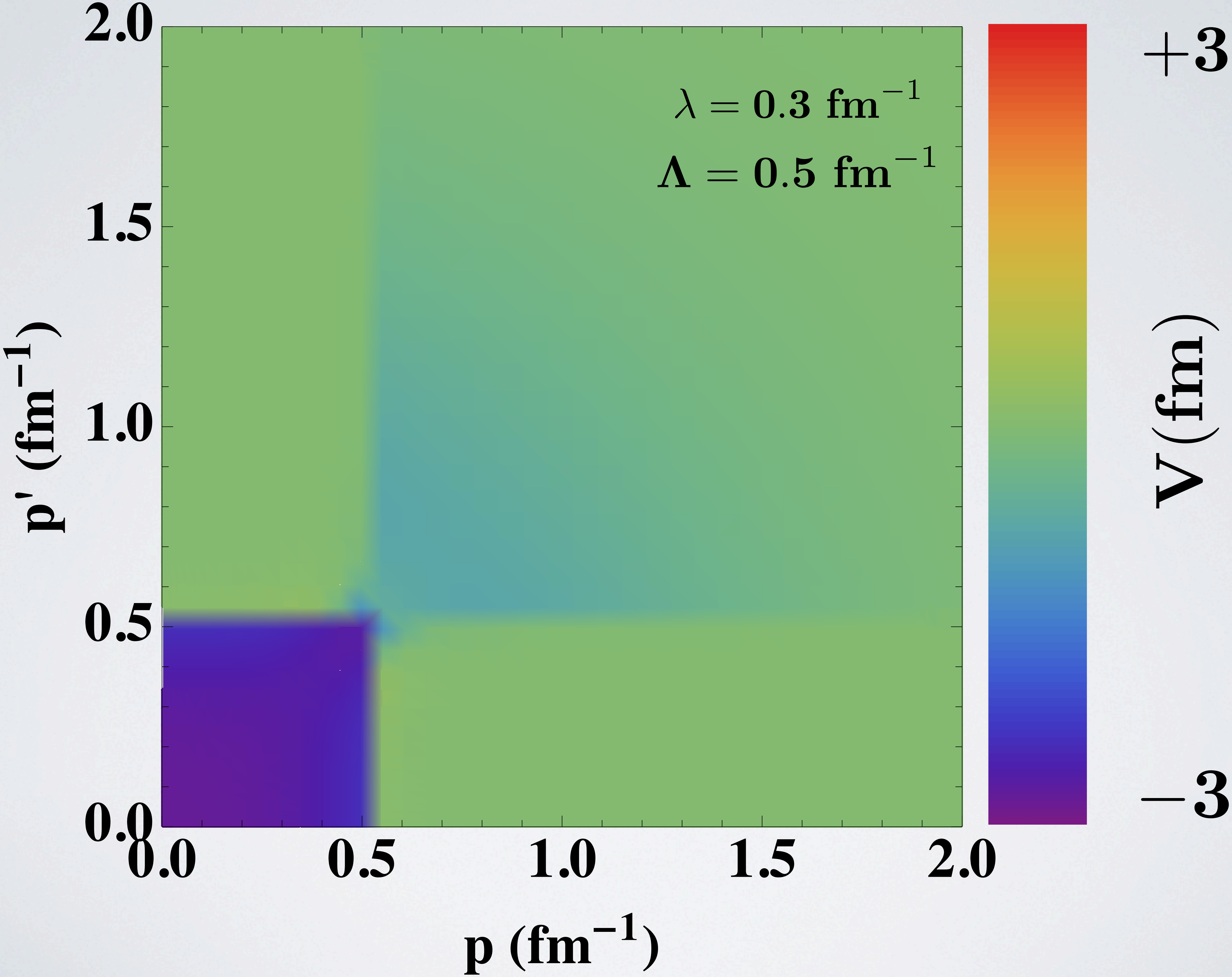

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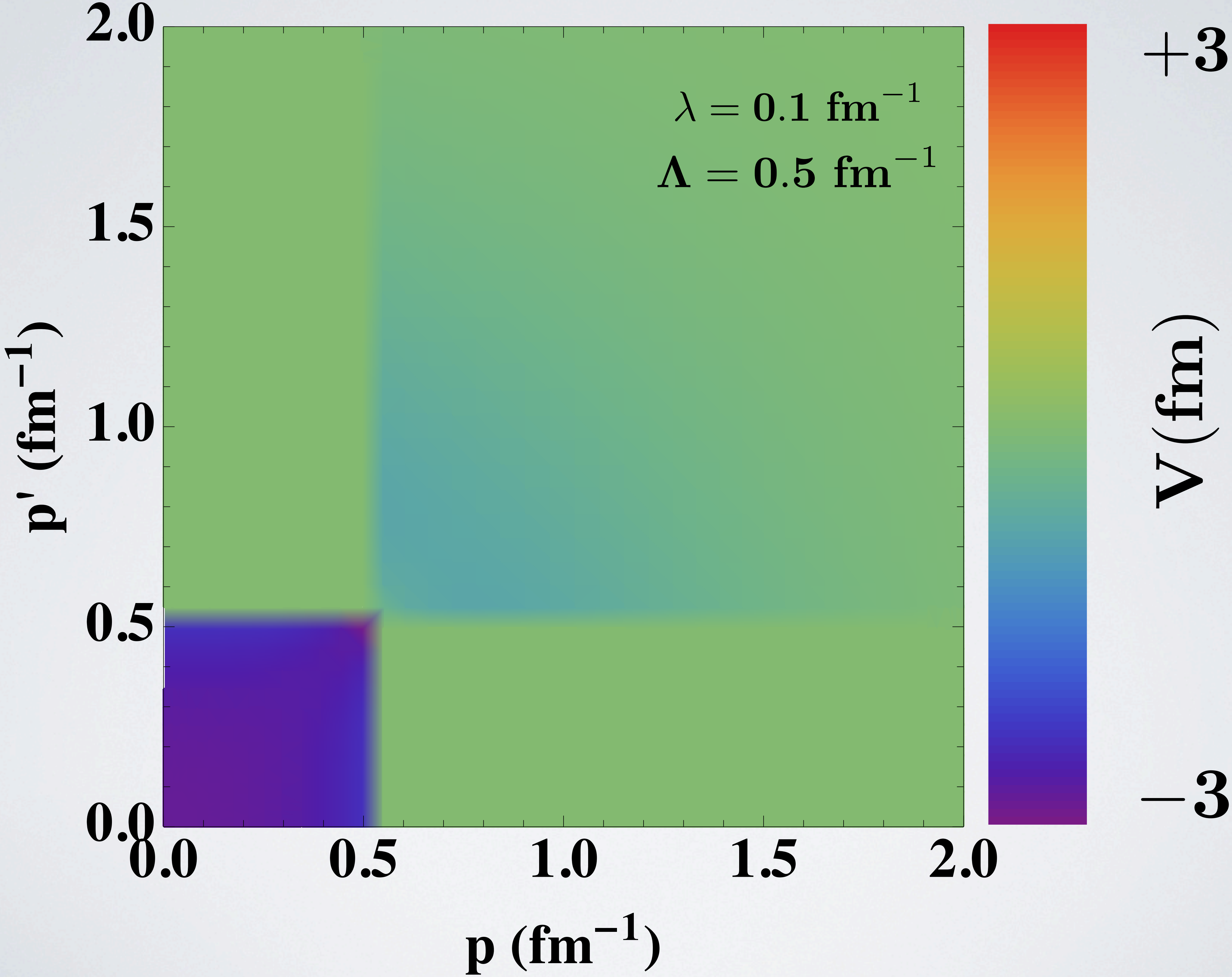
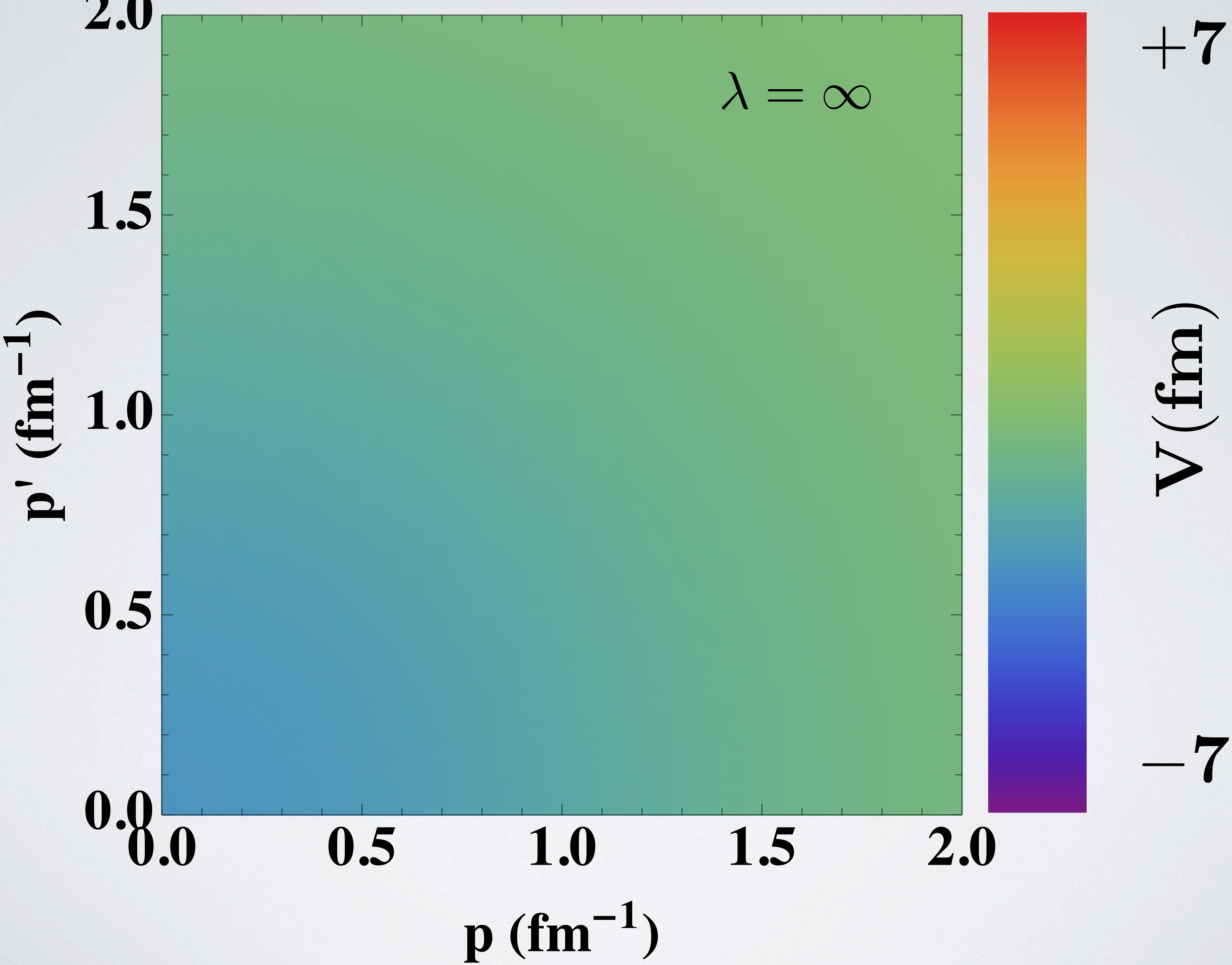


EXPLICIT RENORMALIZATION - 3S1



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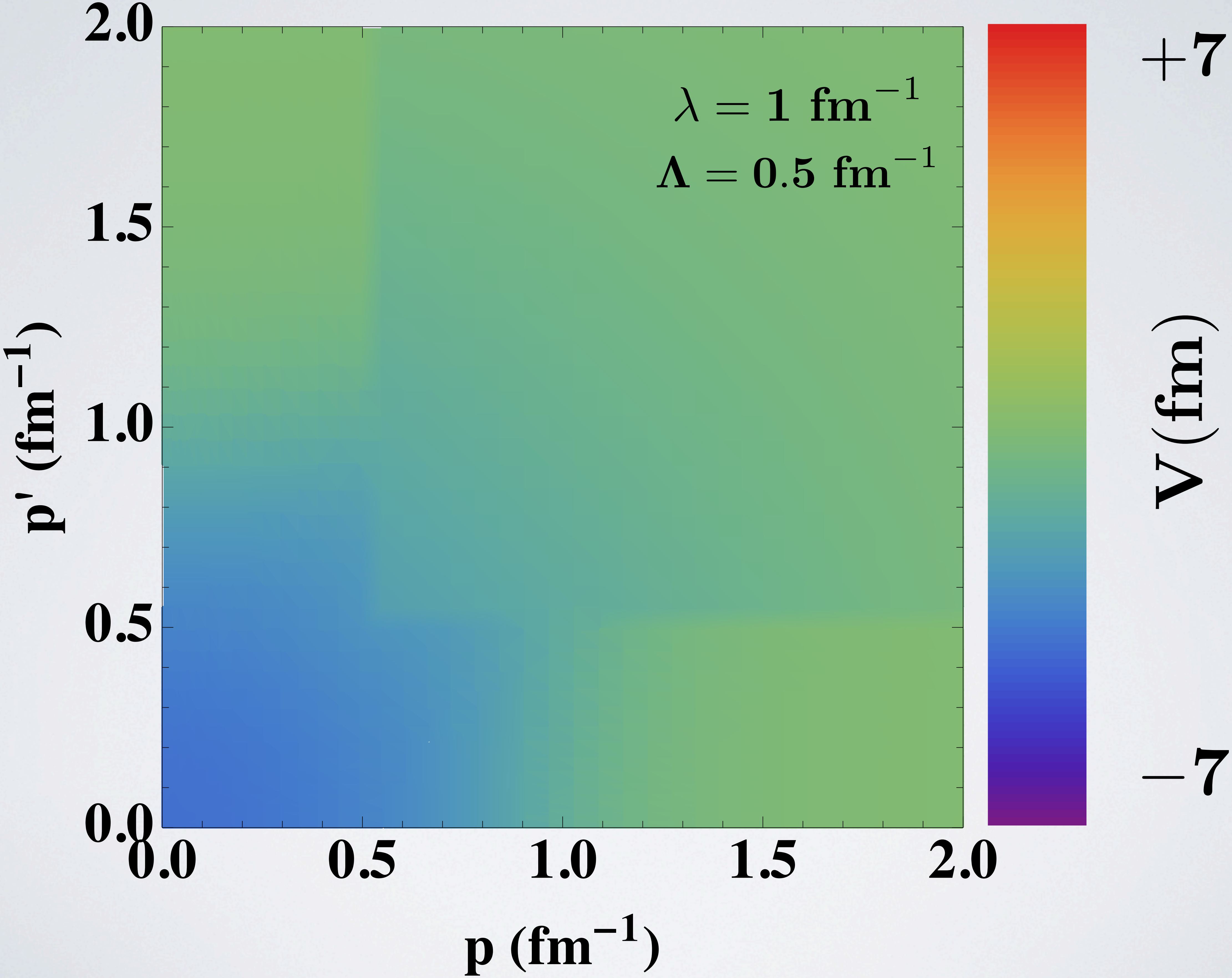

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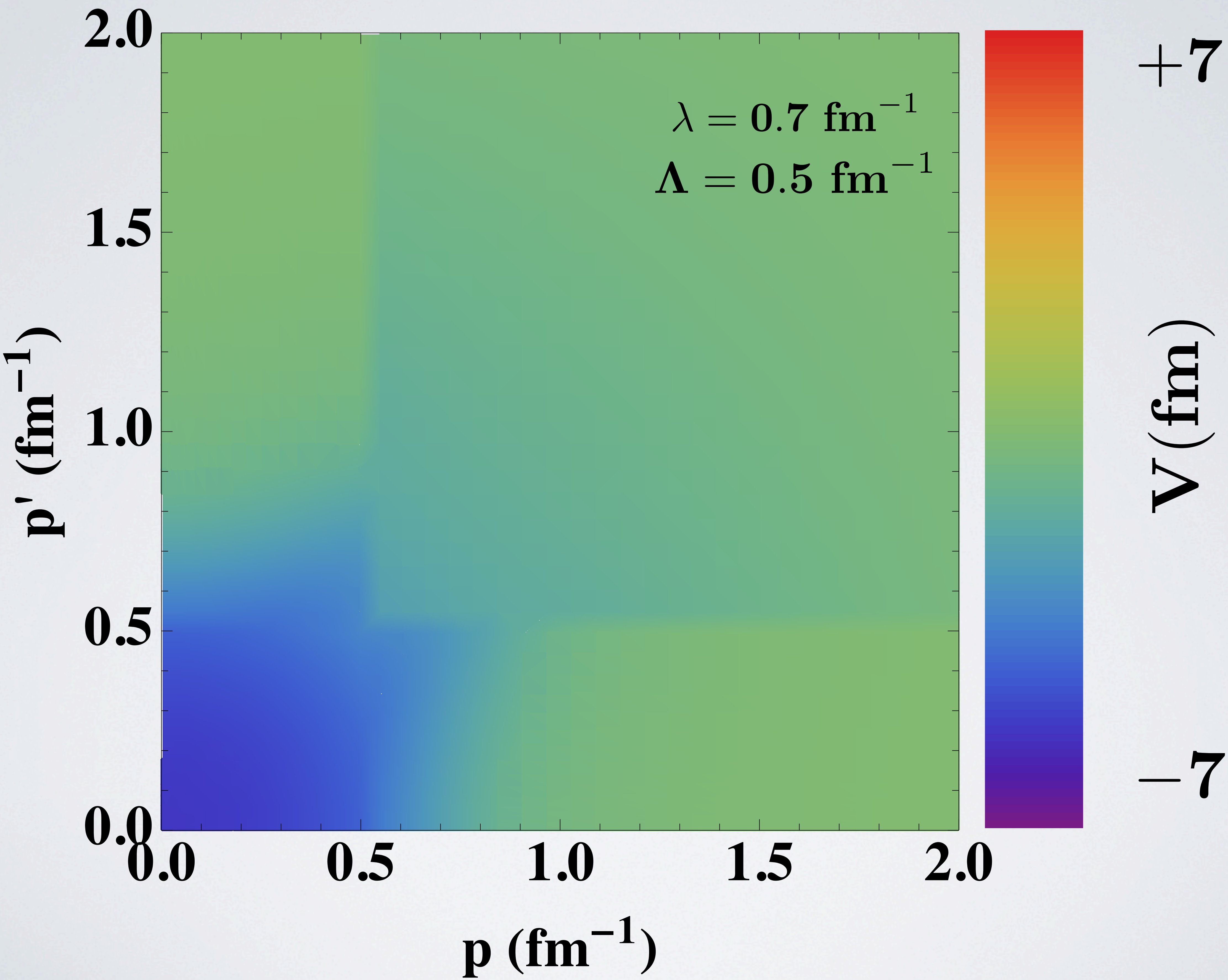

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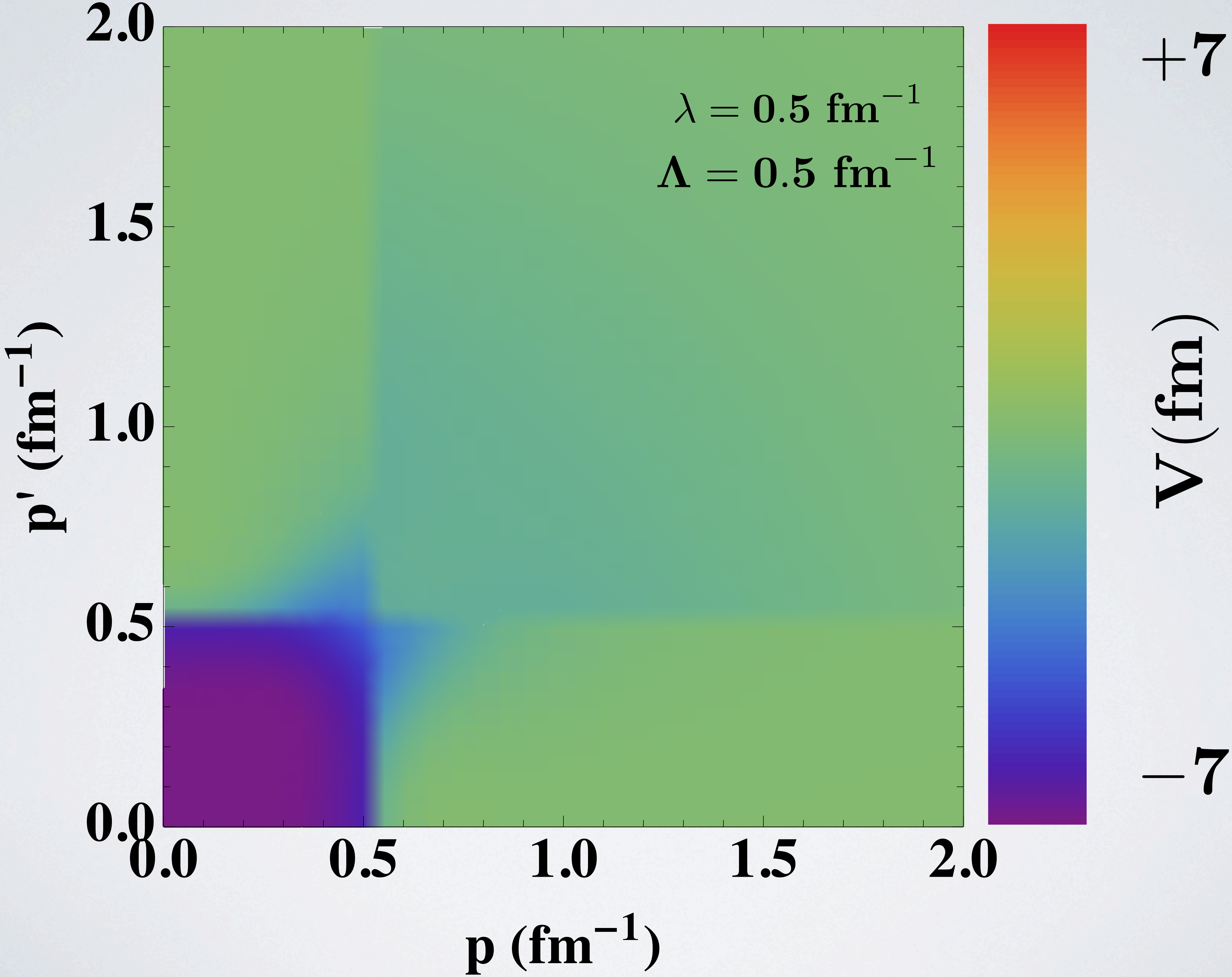
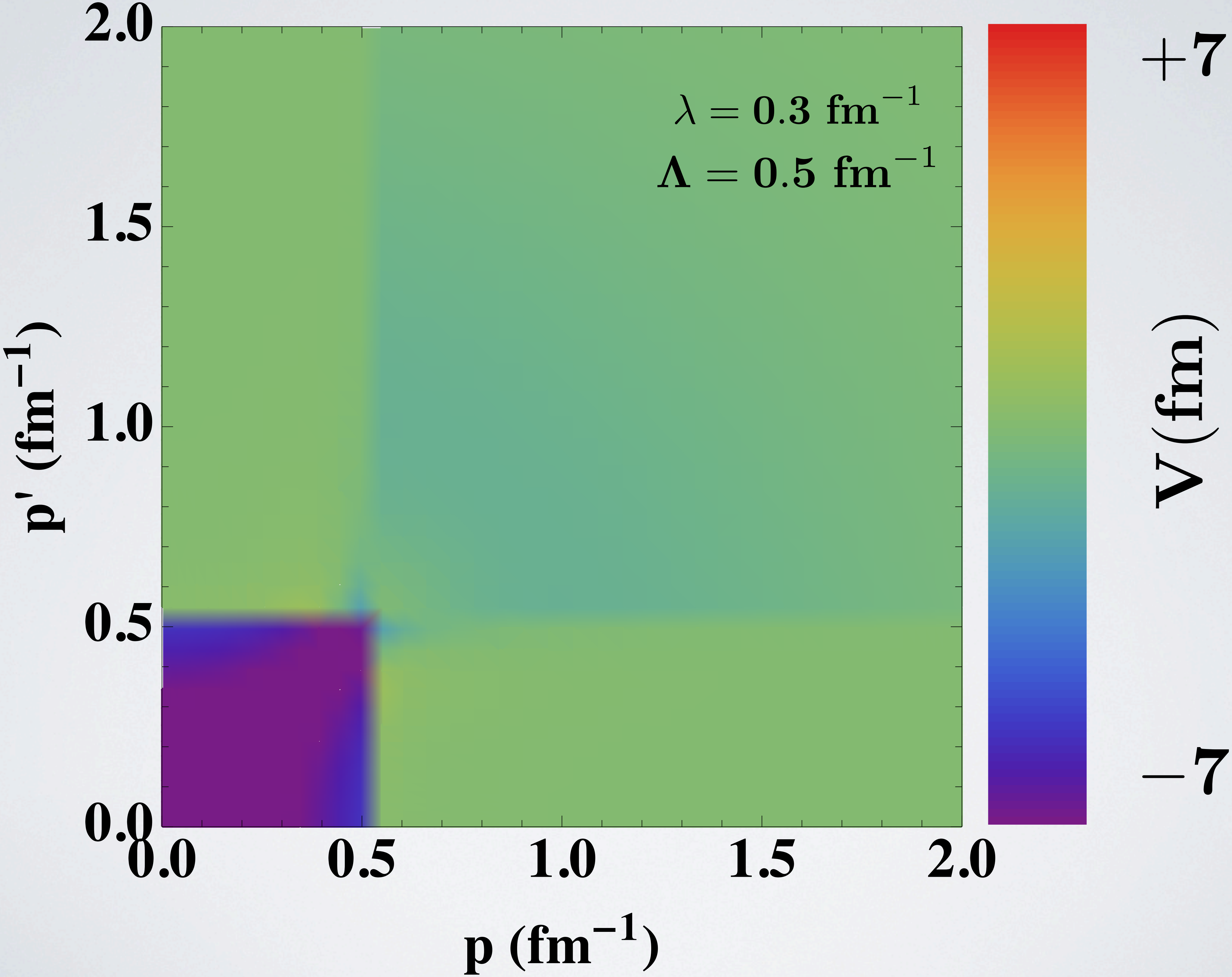


EXPLICIT RENORMALIZATION - 3S1



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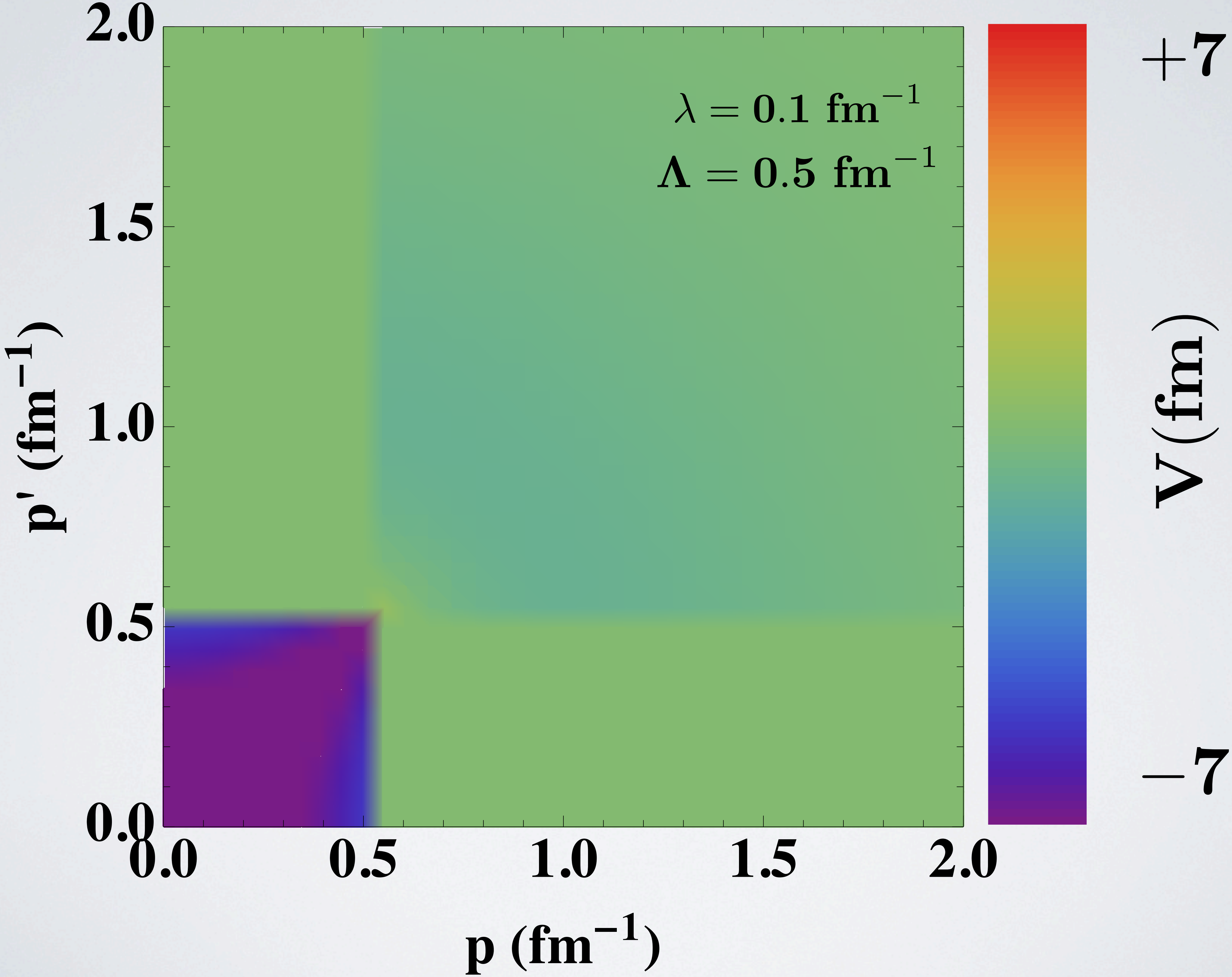
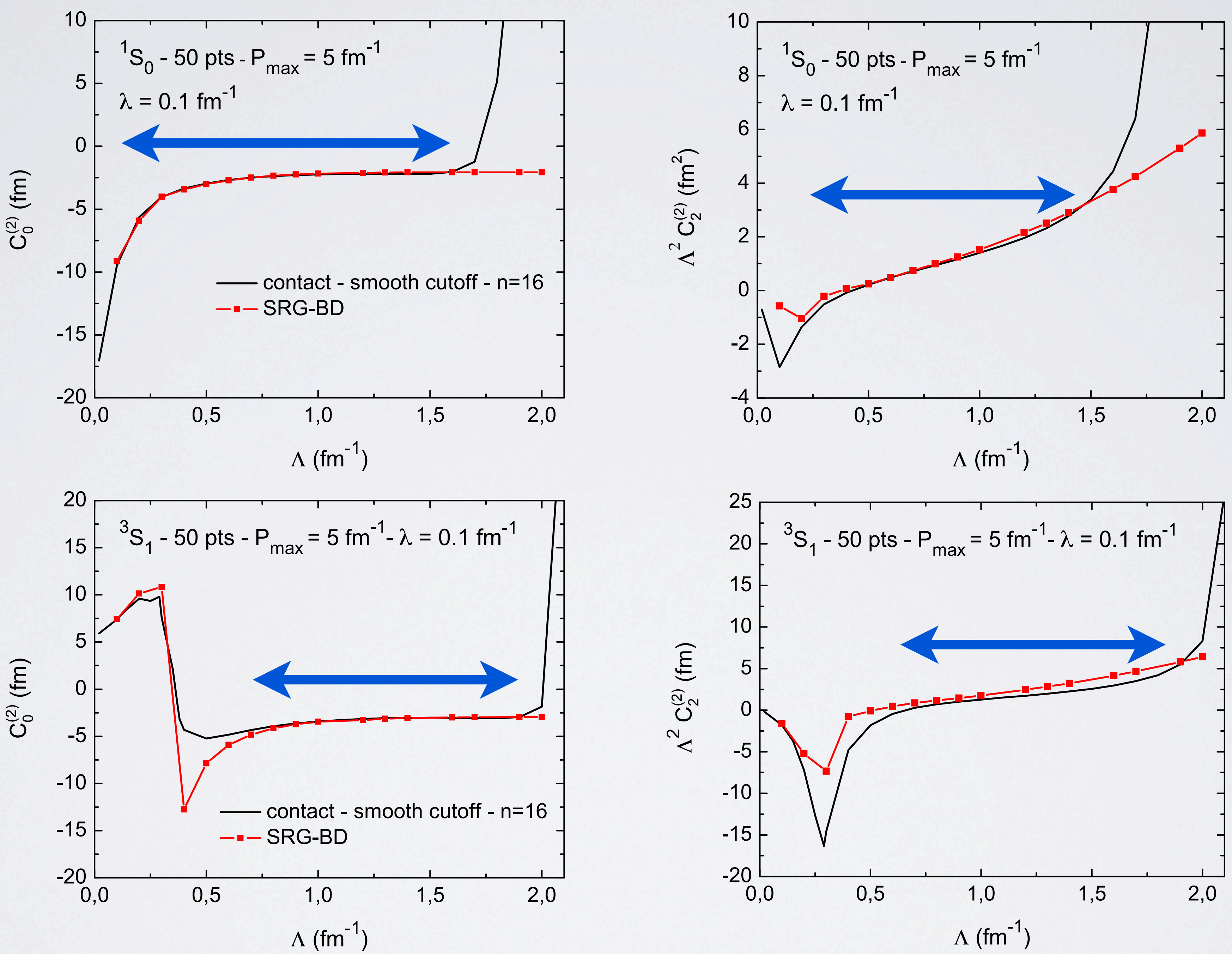


IMPLICIT x EXPLICIT - 1S0 & 3S1

$$V_{\lambda,\Lambda}(p,p') = \tilde{C}_0 + \tilde{C}_2\,(p^2 + p'^2) + \cdots$$



MATCH OVER A WIDE CUTOFF RANGE !!!

FINAL REMARKS

- The Toy model allow us to perform the SRG evolution towards the infrared region so that we have a complete separation of the V_low k and the V_high k part

- In the infrared limit, the V_low part is the same as the contact theory in a wide cutoff range

FAPESP

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

E.R. Arriola, S. Szpigel, V.S. Timóteo, arXiv: nucl-th/1307.1231