Faddeev treatment of the quasi-bound and scattering states in the $\bar{K}NN - \pi\Sigma N$ system

N.V. Shevchenko
Nuclear Physics Institute, Řež, Czech Republic

K^-pp quasi-bound state

Prediction of the existence of deep and narrow K^-pp bound state (G-matrix calculation, optical potential): E_B = - 48 MeV, Γ = 61 MeV

T. Yamazaki and Y. Akaishi, Phys. Lett. B535 (2002) 70

Many theoretical calculations, different models and inputs (Faddeev, variational calculations, FCA):

$$E_B \sim -14 - 80 \text{ MeV}, \quad \Gamma \sim 40 - 110 \text{ MeV}$$

FINUDA collaboration: evidence for a deeply bound state (correlated L and *p*) with E_B = -115 MeV, Γ = 67 MeV:

M. Agnello et. al., Phys. Rev. Lett. 94 (2005) 212303

DISTO collaboration: E_B = -103 MeV, Γ = 118 MeV

T. Yamazaki et al. Phys. Rev. Lett. 104, (2010)

→ New Faddeev calculation of K^-pp quasi-bound state with phenomenological and chiral-motivated two-body input ($\bar{K}N$)

$\overline{K}N - \pi\Sigma$ is the basic interaction

How to investigate:
- fit to two-body experimental data (not enough),
- use in a few- or many-body calculation and compare with experiment.

1. Faddeev calculation of $K^- d$ scattering length with phenomenological and chiral-motivated two-body input (not directly measurable)

Experiment: measurement of kaonic deuterium *1s* level shift and width (SIDDHARTA, SIDDHARTA-2)

Calculation of the parameters, corresponding to the obtained $K^- d$ scattering length:

2. Calculation of elastic $K^- d$ scattering amplitudes with phenomenological and chiral-motivated two-body input (more information for 3.)

3. Construction of a two-body K^- - d complex potential, based on 2.

4. Use the potential for calculation of the *1s* level shift and width of kaonic deuterium (to be measured)

$\overline{K}NN - \pi\Sigma N$ system: three-body coupled-channel AGS equations (Faddeev equations in Alt-Grassberger-Sandhas form)

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}\left(1-\delta_{ij}\right)\left(G_0^{\alpha}\right)^{-1} + \sum_{k,\gamma=1}^{3}\left(1-\delta_{ik}\right)T_k^{\alpha\gamma}G_0^{\gamma}U_{kj}^{\gamma\beta}, \qquad i,j=1,2,3$$

$\pi\Sigma$ channel included directly $\Rightarrow$ particle channels:

$$\alpha=1:\left|\overline{K}_1\ N_2\ N_3\right\rangle, \quad \alpha=2:\left|\pi_1\ \Sigma_2\ N_3\right\rangle, \quad \alpha=3:\left|\pi_1\ N_2\ \Sigma_3\right\rangle$$

Input: two-body T-matrices:

T^{NN}, $T^{\Sigma N}$, and $T^{\pi N}$ are one-channel (usual) T-matrices;

$T^{KK}: \ \bar{K}N \to \bar{K}N, \quad T^{K\pi}: \ \pi\Sigma \to \bar{K}N,$

$T^{\pi K}: \ \bar{K}N \to \pi\Sigma, \quad T^{\pi\pi}: \ \pi\Sigma \to \pi\Sigma$ - elements of 2-channel $T^{\bar{K}N-\pi\Sigma}$

Quantum numbers: spin $S=0\,(K^- pp)$ or $S=1\,(K^- d$ system),

orbital momentum $L=0$, isospin $I=1/2$

Two identical nucleons - antisymmetrization $\Rightarrow$ system of 10 integral equations

Coupled-channel $\bar{K}N - \pi\Sigma$ interaction

Experimental data:

- Measured 1*s* $K^- p$ level shift and width:

$$\Delta E_{1s}^{KEK} = -323 \pm 63 \pm 11 \text{ eV}, \quad \Gamma_{1s}^{KEK} = 407 \pm 208 \pm 100 \text{ eV}$$

$$\Delta E_{1s}^{DEAR} = -193 \pm 37 \pm 6 \text{ eV}, \quad \Gamma_{1s}^{DEAR} = 249 \pm 111 \pm 30 \text{ eV}$$

$$\Delta E_{1s}^{SIDD} = -283 \pm 36 \pm 6 \text{ eV}, \quad \Gamma_{1s}^{SIDD} = 541 \pm 89 \pm 22 \text{ eV}$$

- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions,

- Threshold branching ratios γ, R_c and R_n

Phenomenological and "chiral-motivated" potentials
with isospin-breaking effects:

1. Kaonic hydrogen: direct inclusion of *Coulomb interaction*
2. Use of the *physical masses*:

$$m_{K^-}, m_{\bar{K}^0}, m_p, m_n \quad \text{instead of} \quad m_{\bar{K}}, m_N$$

Phenomenological $\bar{K}N - \pi\Sigma$ potentials with one- or two-pole structure of the Λ(1405) resonance, equally properly reproducing all experimental data, except πΛ channel

J. Révai, N.V.S., Phys. Rev. C 79 (2009) 035202,
N.V.S., Phys.Rev. C85 (2012) 034001; Nucl. Phys. A890-891 (2012) 50 (new fits)

$$V_I^{\alpha\beta}(\vec{k}^{\alpha}, \vec{k}'^{\beta}) = g_I^{\alpha}(\vec{k}^{\alpha})\, \lambda_I^{\alpha\beta}\, g_I^{\beta}(\vec{k}'^{\beta}), \quad \text{form-factors:}$$

- 1-pole Λ(1405):

$$g_{I,1pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_I^{\alpha})^2}$$

for $\alpha = K$ ($\bar{K}N$ channel) or π ($\pi\Sigma$ channel)

- 2-pole Λ(1405):

$$g_{I,1pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_I^{\alpha})^2} \qquad \text{for } \alpha = K \ (\bar{K}N \text{ channel})$$

$$g_{I,2pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_I^{\alpha})^2} + \frac{s\,(\beta_I^{\alpha})^2}{[\,(k^{\alpha})^2 + (\beta_I^{\alpha})^2\,]^2} \qquad \text{for } \alpha = \pi \ (\pi\Sigma \text{ channel}).$$

"Chiral-motivated" $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, properly reproducing all experimental data (two-pole $\Lambda(1405)$ resonance structure)

$$V_I^{\alpha\beta}(\vec{k}^{\alpha},\vec{k}'^{\beta};\sqrt{s}) = \sqrt{\frac{M_\alpha}{2\omega_\alpha E_\alpha}}\, g_I^{\alpha}(\vec{k}^{\alpha})\, \frac{C_I^{\alpha\beta}(\sqrt{s})}{(2\pi)^3 f_\alpha f_\beta}\, \sqrt{\frac{M_\beta}{2\omega_\beta E_\beta}}\, g_I^{\beta}(\vec{k}'^{\beta}),$$

Yamaguchi form-factors: $g_I^{\alpha}(k^{\alpha}) = \dfrac{(\beta_I^{\alpha})^2}{(k^{\alpha})^2 + (\beta_I^{\alpha})^2}$

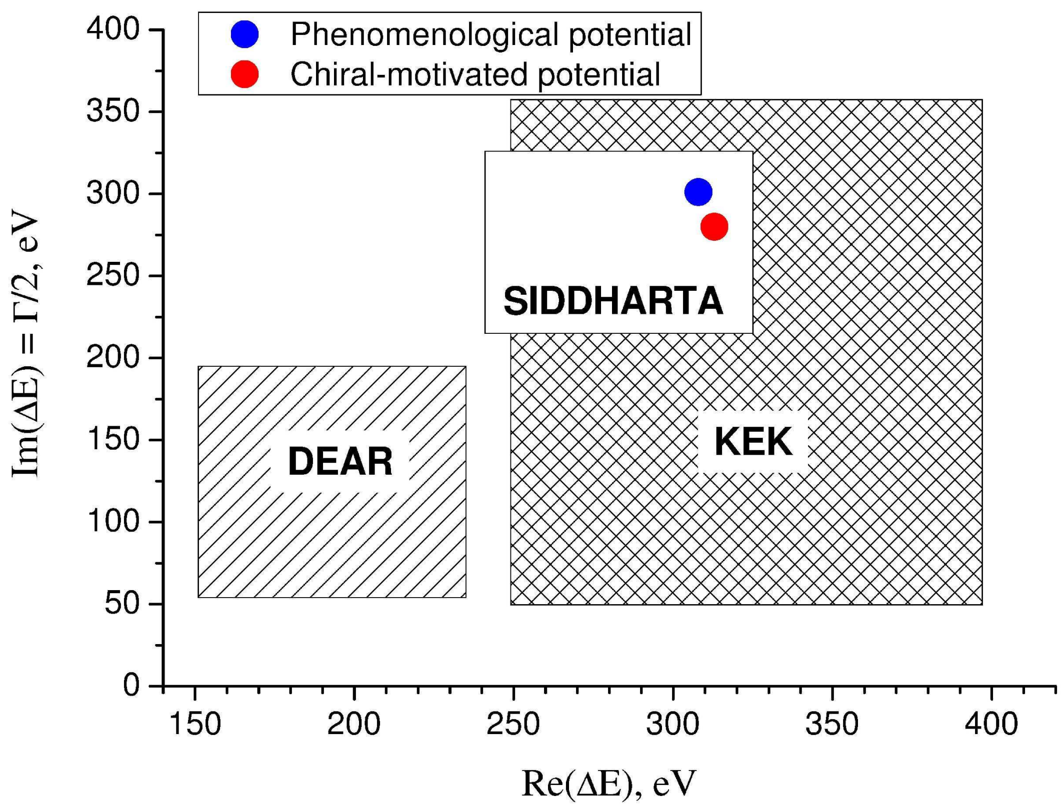
The leading-order Weinberg-Tomozawa term

$$C_I^{\alpha\beta}(\sqrt{s}) = -C^{WT}(2\sqrt{s} - M_\alpha - M_\beta)$$

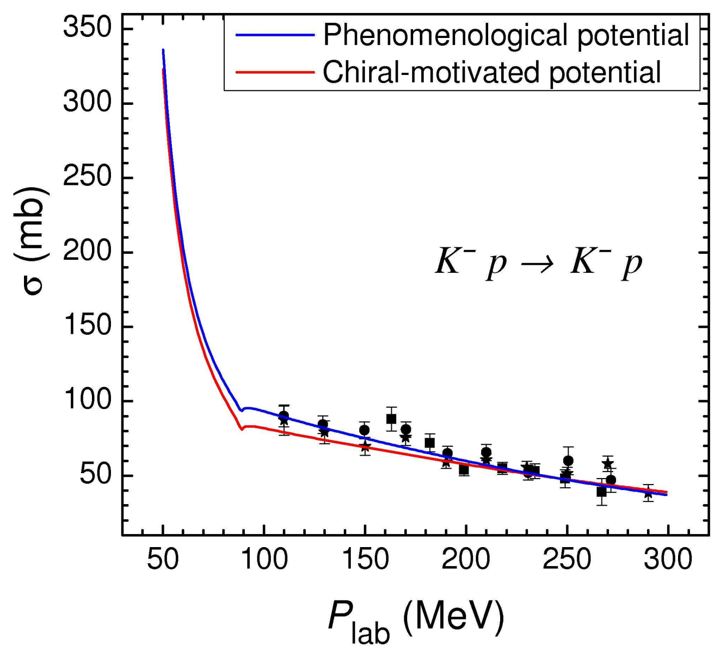
Parameters: f_K, f_π (pseudoscalar meson decay constants)

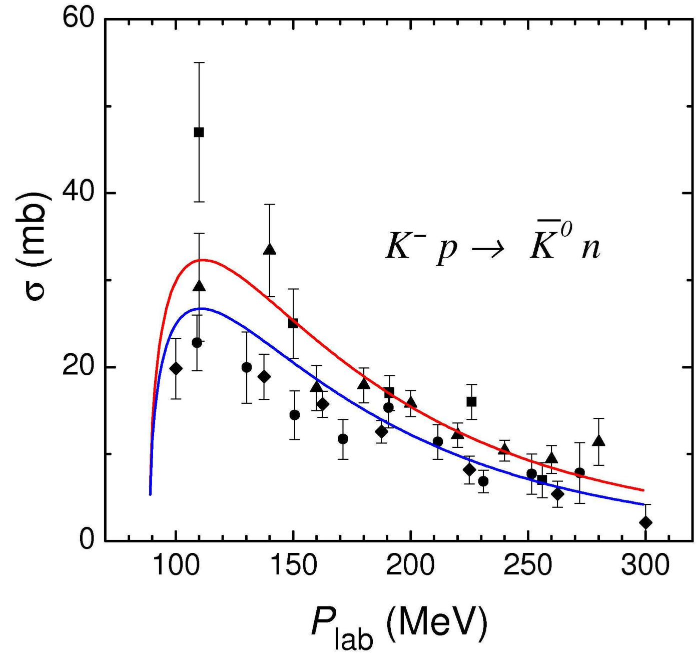
β_I^{α} (range parameters)

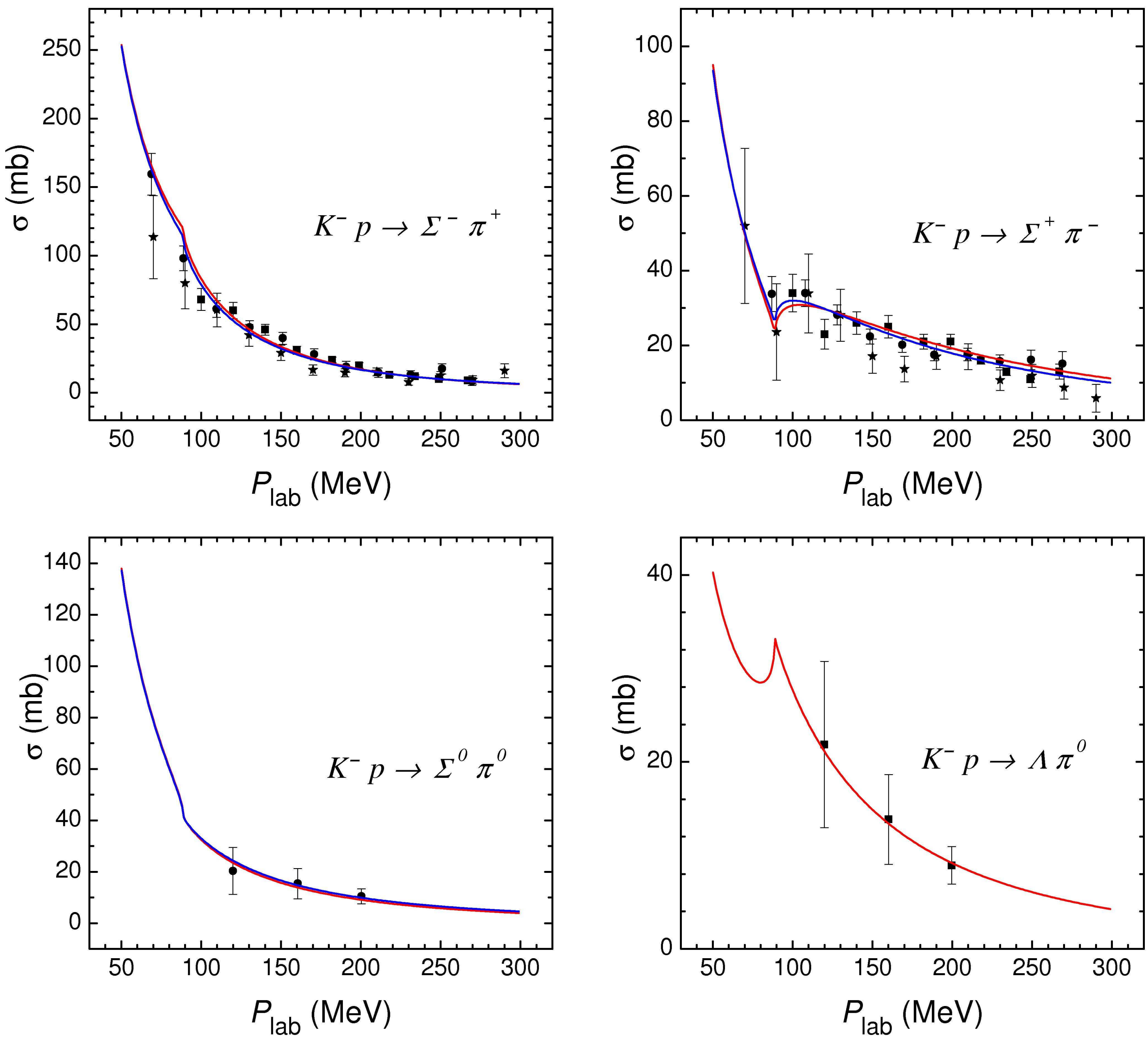
Experimental and theoretical *1s* level shifts and widths of kaonic hydrogen



Comparison with experimental data on $K^- p$ cross-sections
phenomenological and chiral-motivated potentials





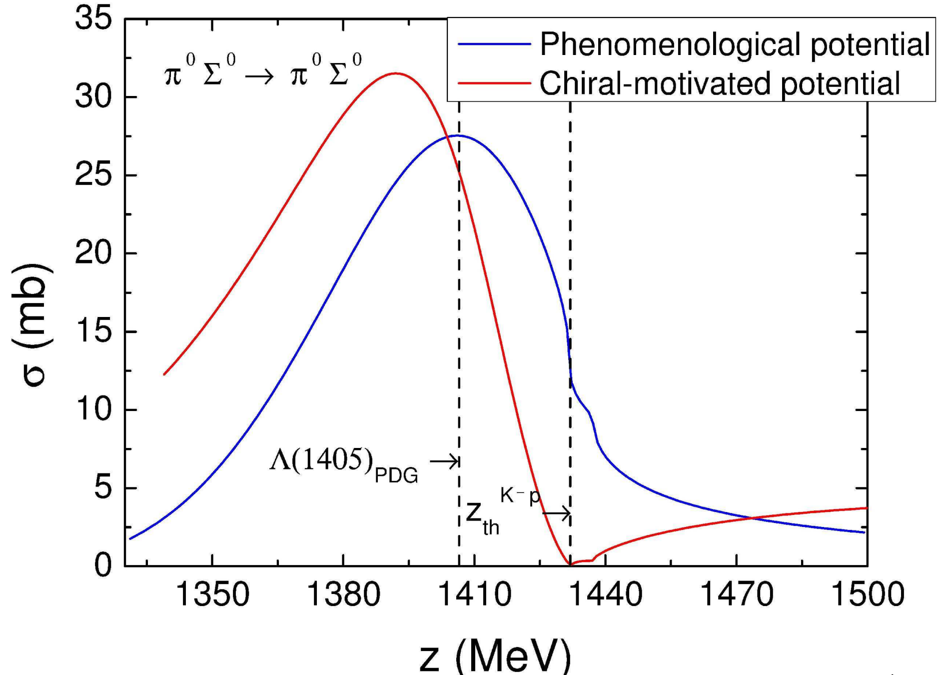


(continuation)

(Strong) pole positions

Phenomenological	Chiral-motivated
1414 – *i* 58 MeV 1386 – *i* 104 MeV	1417 – *i* 33 MeV 1406 – *i* 89 MeV

Elastic $\pi\Sigma - \pi\Sigma$ cross sections for the potentials:



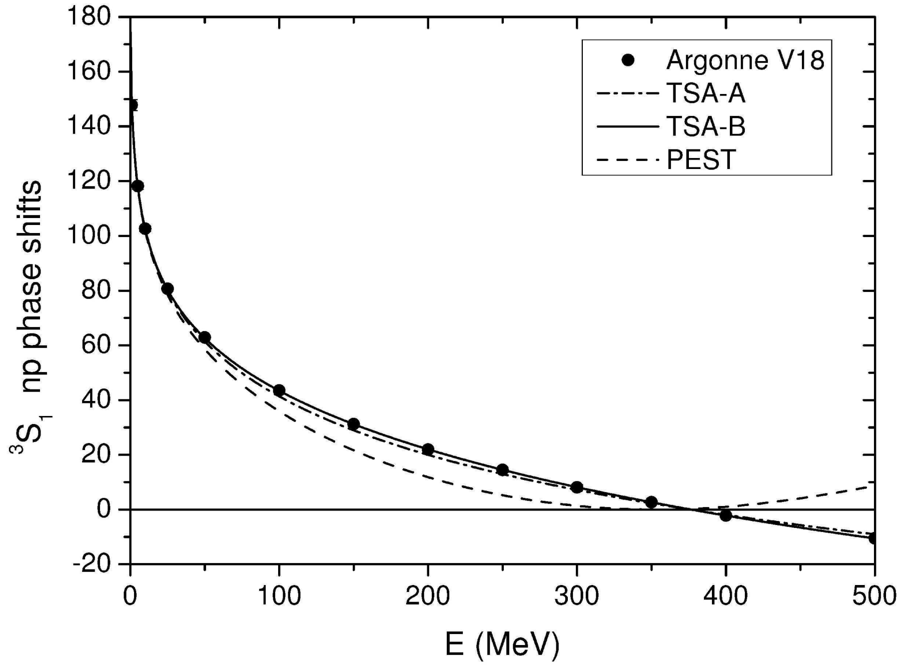
Both potentials reproduce threshold branching ratios γ, R_c, R_n $\left(\text{or } R_{\pi\Sigma} = \dfrac{R_c}{1 - R_n(1 - R_c)}\right)$

Two-term *NN* potential (TSA)

P. Doleschall, private communication, 2009

$$V_{NN} = \sum_{i=1}^{2} \left| g_i \right\rangle \lambda_i \left\langle g_i \right| \quad \rightarrow$$

$$T_{NN} = \sum_{i,j=1}^{2} \left| g_i \right\rangle \tau_{ij} \left\langle g_j \right|$$



Reproduces:

Argonne V18 NN phase shifts
(with sign change)

$$a^A(np) = -5.402\,\text{fm} \qquad a^A(pp) = 16.554\,\text{fm}$$

$$a^B(np) = -5.413\,\text{fm} \qquad a^B(pp) = 16.559\,\text{fm}$$

$$E_{deu} = -2.2246\,\text{MeV}$$

$\Sigma N(-\Lambda N)$ interaction

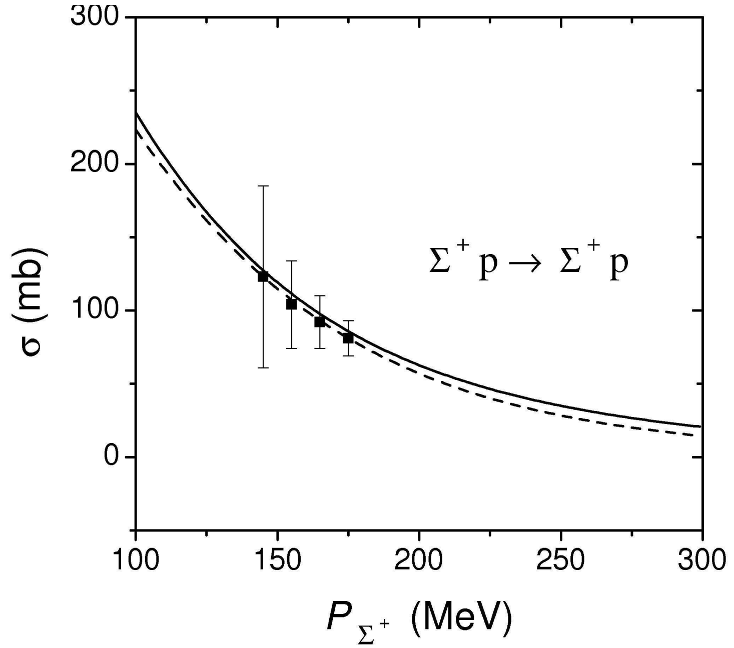
J. Révai, N.V.S., 2009

Isospin and spin-dependent $T^{\Sigma N}_{I,S}(k,k';z)$

corresponds to

$$V^{\Sigma N}_{I,S}(k,k') = \lambda^{\Sigma N}_{I,S}\; g^{\Sigma N}_{I,S}(k)\; g^{\Sigma N}_{I,S}(k')$$

with $$g^{\Sigma N}_{I,S}(k) = \frac{1}{k^2 + \left(\beta^{\Sigma N}_{I,S}\right)^2}$$

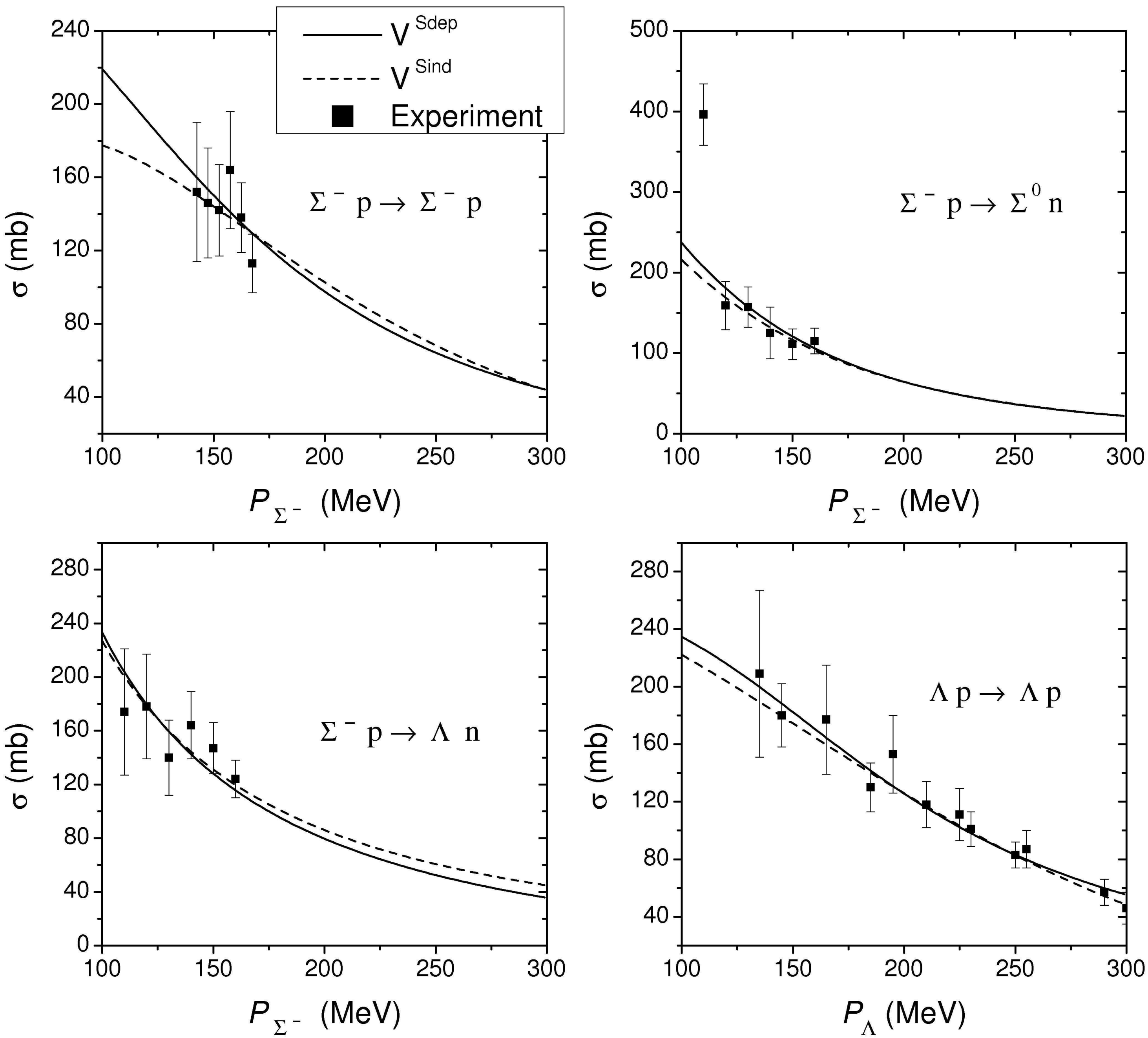


$K^- pp$ calculation: spin-singlet,
$K^- d$ calculation: spin-triplet only.

All parameters were fitted to reproduce experimental cross-sections

I=3/2
Real parameters, one-channel case

I=1/2
1. Two-channel $\Sigma N - \Lambda N$ potential, real parameters
2. One-channel (exact) optical ΣN potential, complex energy-dependent strength



V^{Sdep}
V^{Sind}
Experiment
$\Sigma^- p \to \Sigma^- p$
$\Sigma^- p \to \Sigma^0 n$
$\Sigma^- p \to \Lambda n$
$\Lambda p \to \Lambda p$
σ (mb)
P_{Σ^-} (MeV)
P_{Λ} (MeV)

The results

$K^- pp$ quasi-bound states

from coupled-channels Faddeev-type (AGS) calculations:

The pole position with phenomenological $\bar{K}N - \pi\Sigma$ potential

$$z_{pole} = -47.4 - i\,24.9 \text{ MeV}$$

much deeper!

with chiral-motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential

$$z_{pole} = -32.2 - i\,24.3 \text{ MeV}$$

$K^- d$ scattering lengths

from coupled-channels Faddeev-type (AGS) calculations:

The scattering length with phenomenological $\bar{K}N - \pi\Sigma$ potential

$$a_{Kd} = -1.51 + i\,1.25 \text{ fm}$$

with chiral-motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential

$$a_{Kd} = -1.59 + i\,1.32 \text{ fm}$$

Strong characteristics of kaonic deuterium

Step 1. Optical two-body K^{-} - d potential is constructed, it reproduces near-threshold $K^{-} d$ scattering amplitudes, obtained in the three-body calculation (scattering length and effective range are reproduced as well)

$$V^{S}_{K^{-}d}(k,k') = \lambda_{1,K^{-}d}\ g_1(k) g_1(k') + \lambda_{2,K^{-}d}\ g_2(k) g_2(k')$$

$$\text{with} \quad g_i(k) = \frac{1}{\beta^2_{i,K^{-}d} + k^2}, \quad i = 1,2$$

Step 2. Energy of the*1s* level of kaonic deuterium is calculated, Coulomb interaction was directly included into the Lippmann-Schinger equation:

$$H = H_0 + V^{S}_{K^{-}d}(k,k') + V^{Coul} \quad \Rightarrow \quad E^{S+Coul}_{1s}$$

Step 3. *1s* level shift and width caused by strong interaction are obtained:

$$\Delta E_{1s} = E^{Coul}_{1s} - \mathrm{Re}(E^{S+Coul}_{1s})$$

The results

Strong interaction shift $\Delta E_{1s}^{K^- d}$ and width $\Gamma_{1s}^{K^- d}$ of the kaonic deuterium atom $1s$ level state

Corresponding to the results obtained with phenomenological $\bar{K}N - \pi\Sigma$ potential

$$\Delta E_{1s}^{K^- d} = -797 \text{ eV} \qquad \Gamma_{1s}^{K^- d} = 1025 \text{ eV}$$

with chiral-motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential

$$\Delta E_{1s}^{K^- d} = -828 \text{ eV} \qquad \Gamma_{1s}^{K^- d} = 1055 \text{ eV}$$

Corrected Deser formula: $\Delta E_{1s}^{K^- d} - i\dfrac{\Gamma_{1s}^{K^- d}}{2} = -2\alpha^3 \mu_{K^- d}^2 a_{K^- d}[1 - 2\alpha\, \mu_{K^- d} a_{K^- d} (\ln\alpha - 1)]$

Corresponding to the results obtained with phenomenological $\bar{K}N - \pi\Sigma$ potential

$$\Delta E_{1s}^{K^- d} = -835 \text{ eV} \qquad \Gamma_{1s}^{K^- d} = 727 \text{ eV}$$

with chiral-motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential

$$\Delta E_{1s}^{K^- d} = -878 \text{ eV} \qquad \Gamma_{1s}^{K^- d} = 724 \text{ eV}$$

~ 30% error in width!

Summary

- The chiral-motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential reproducing SIDDHARTA data on kaonic hydrogen *1s* level shift and width and have two poles for the $\Lambda(1405)$ resonance was constructed and used in the three-body calculations of the $\bar{K}NN - \pi\Sigma N$ system.

- The pole position of the $K^{-}pp$ quasi-bound state obtained with phenomenological $\bar{K}N - \pi\Sigma$ potential is much deeper than that obtained with chiral-motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential.

- The corresponding to the K^{-} *d* scattering amplitudes *1s* level shift and width of kaonic deuterium were calculated using the complex K^{-} - *d* potentials.

- The chiral-motivated (energy dependent) $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential give larger shift and width of kaonic deuterium than phenomenological (energy-independent) potential.