Universality in the Four-Body System in an EFT Framework

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Table of Contents

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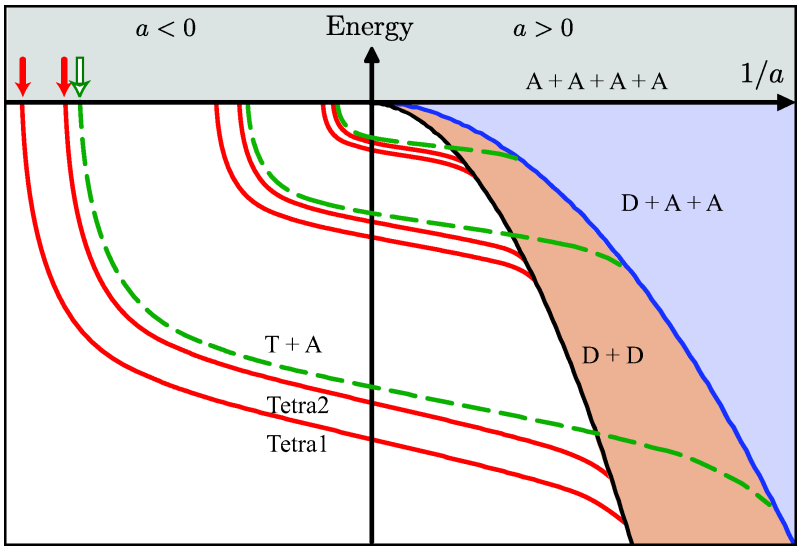
Universality

- Systems with **large scattering length** and short-range interactions behave similarly despite very different underlying physics
- Example: Efimov effect in 3-body systems (Efimov 1970/1971)



From Ferlaino *et al* (2009)

- Two universal 4-body states tied to each Efimov state (Platter/Hammer 2004-2007, von Stecher *et al* 2009, ...)

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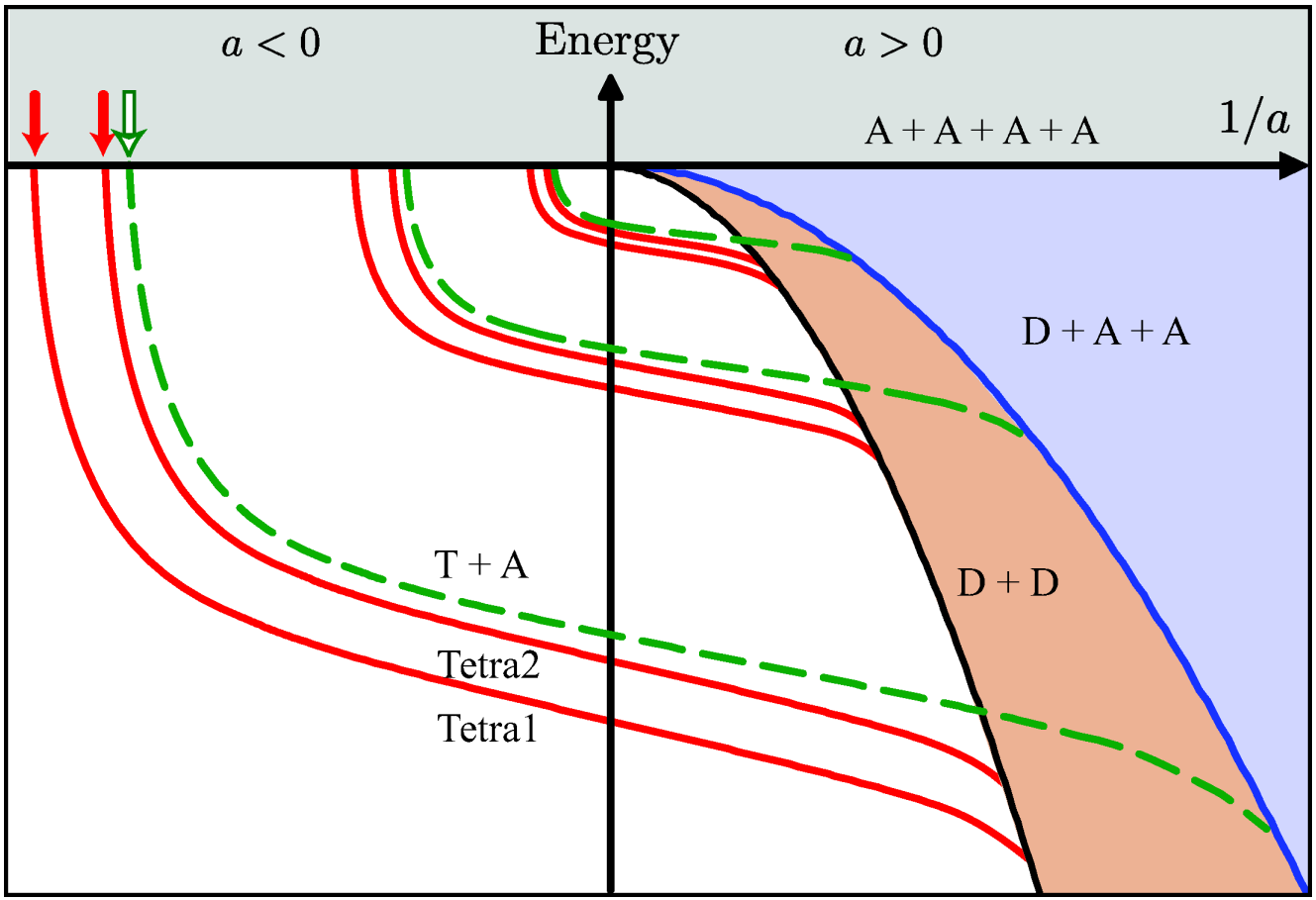
Universality



From Ferlaino *et al* (2009)

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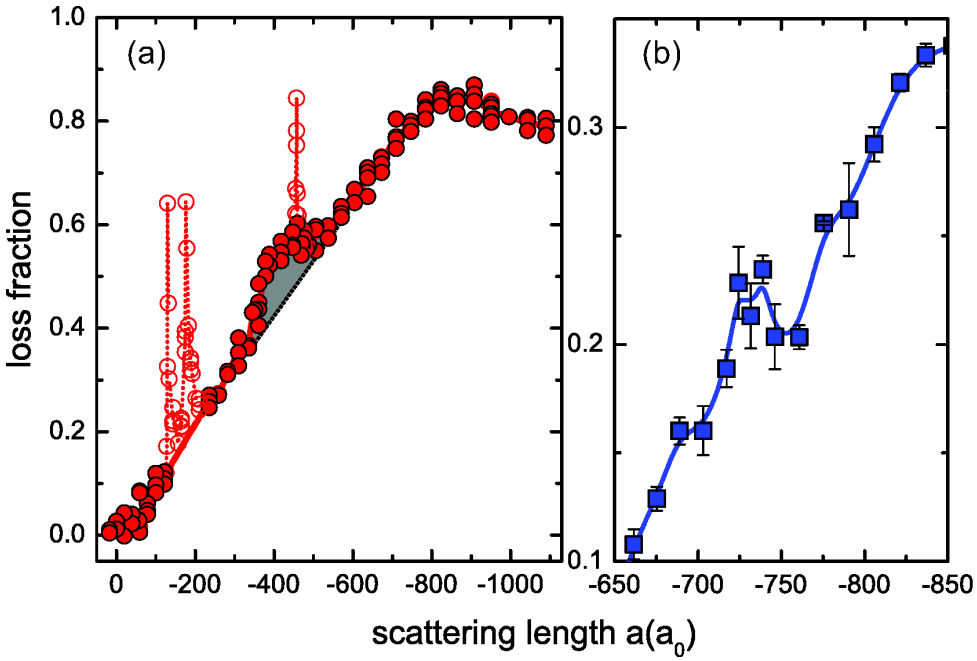
Experiments

- Experiments using Feshbach resonances confirmed theoretical predictions



From Ferlaino *et al* (2009)

- Efimov states: Kraemer *et al* (2006), ...
- Universal 4-body states: Ferlaino *et al* (2009), ...

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Theory

- Various approaches
  - Yakubovsky equations (Hammer/Platter 2004-2007, Hadizadeh *et al* 2011/2012)
  - Alt-Grassberger-Sandhas equations (Deltuva 2011-2013)
  - Functional Renormalisation Group (Ávila/Birse 2013)
  - Schrödinger equation with adiabatic hyperspherical representation for the 4-body problem (von Stecher *et al* 2009)
- Still some discussion on number of 4-body states per Efimov state
  - Most found two 4-body states
- No additional 4-body parameter found (except Hadizadeh *et al*)

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My Topic

- Hammer/Platter only took s-waves into account
- Extend their calculations to **higher partial waves**
- Test **stability of results** with included contributions from higher partial waves
- Even if 2-body interaction acts only in s-wave, higher partial wave contributions possible in 4-body system

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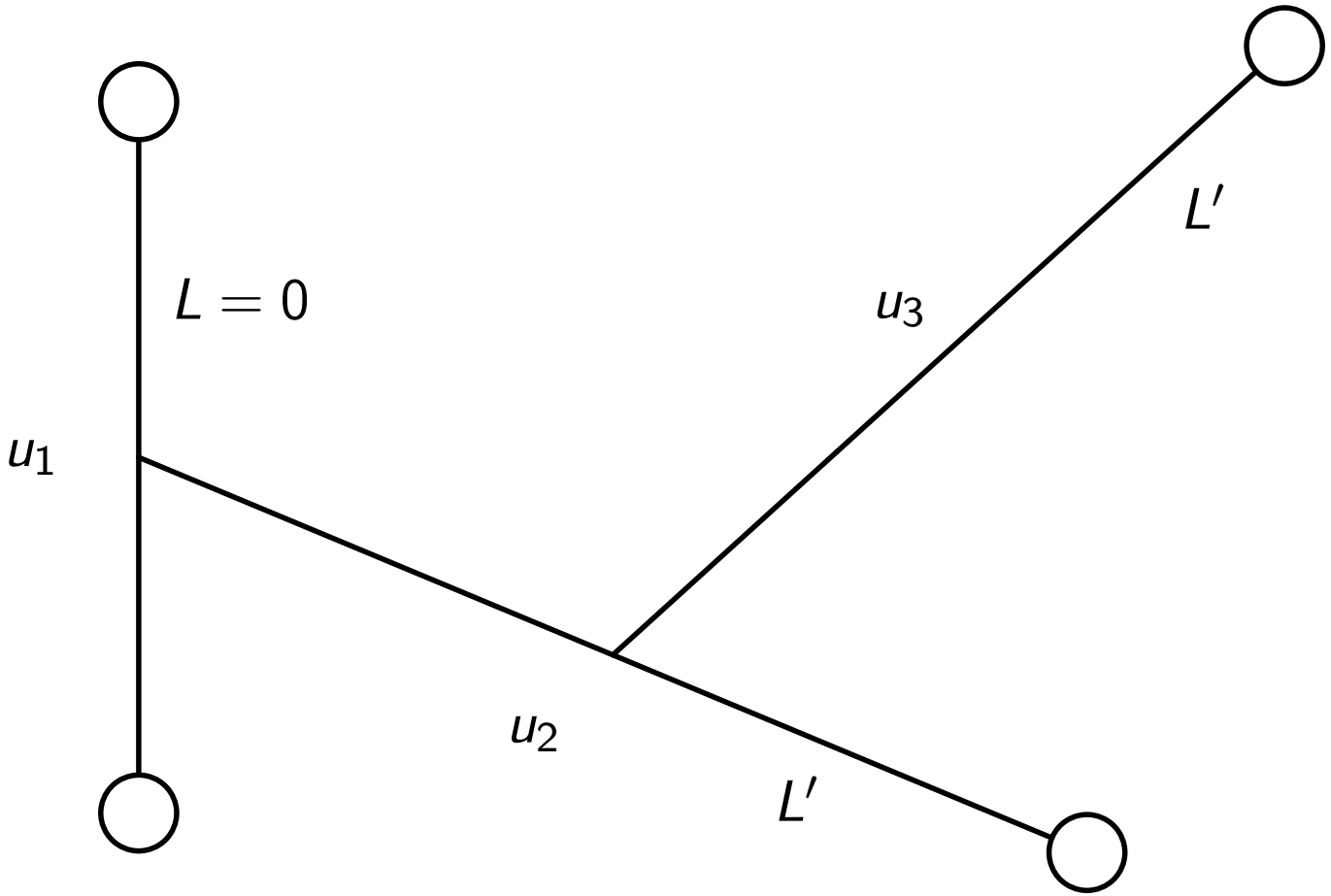
Motivation



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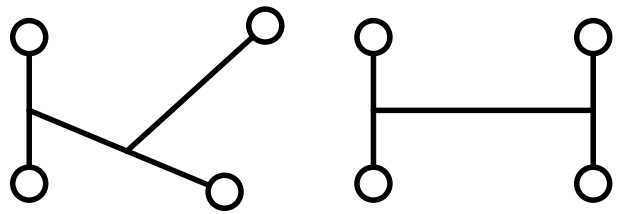
Starting point

- Work of Hammer/Platter (2004/2005)
- Effective potential from simple regulated contact interactions:

$$\langle \mathbf{p}| \, V \, |\mathbf{q}\rangle = \langle \mathbf{p}| \, g\rangle \, \lambda_2 \, \langle g| \, \mathbf{q}\rangle = \lambda_2 e^{-\frac{p^2+q^2}{\Lambda^2}}$$

- Yakubovsky equations (Yakubovsky 1967) in momentum space without 3-body force
- Two different configurations possible:



- Formulae for partial wave projections derived by Glöckle/Kamada (1992)

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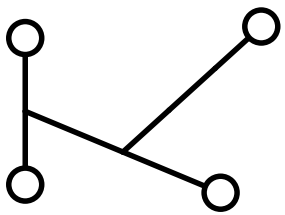
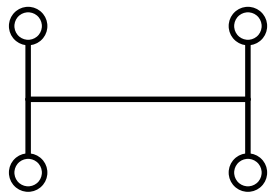
Method

- Yakubovsky equations without 3-body force:

$$\begin{aligned} \psi_A &= G_0 t_{12} P \left[(1 + P_{34}) \psi_A + \psi_B \right], \\ \psi_B &= G_0 t_{12} \tilde{P} \left[(1 + P_{34}) \psi_A + \psi_B \right]. \end{aligned}$$

- Permutation operators to get all possible particle configurations
- $\psi_A \;\hat{=}$  , $\psi_B \;\hat{=}$ 
- Higher partial waves only for ψ_A
- Rewrite coupled integral equation as $\mathbf{v} = M(E)\mathbf{v}$
- Find energy E so that some eigenvalue of M is 1

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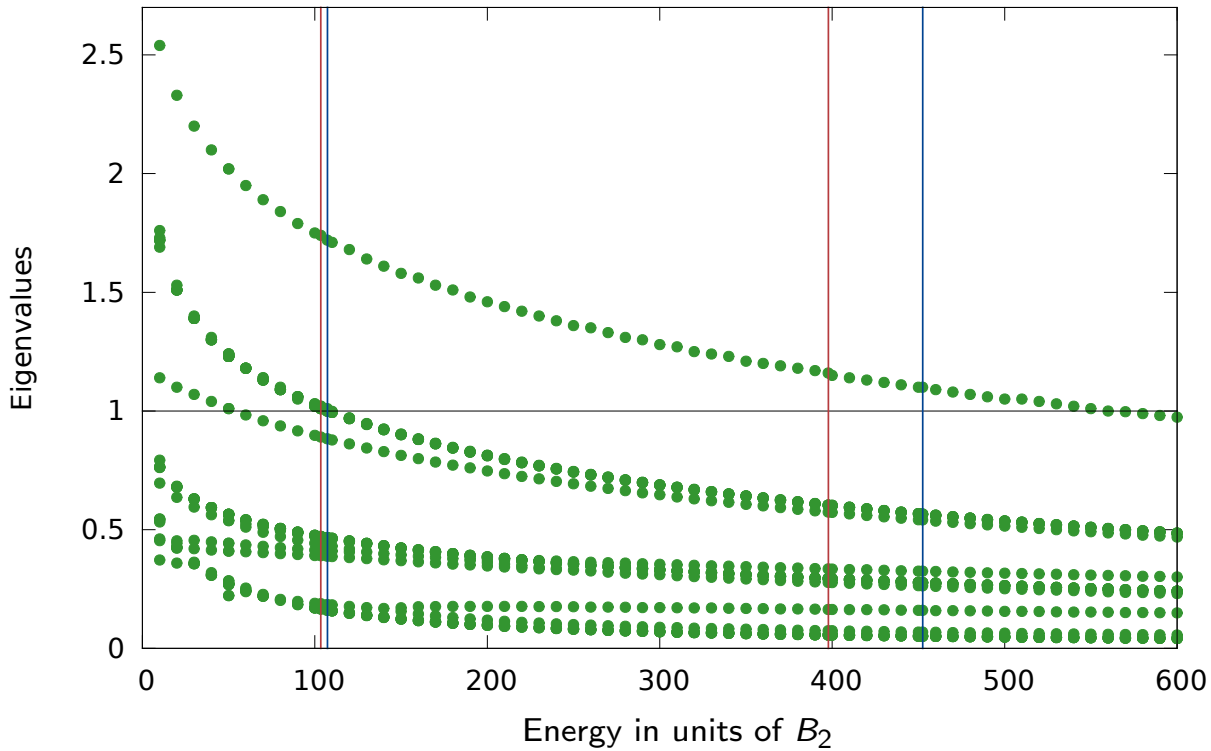
Intermediary Results

Comparison of own results for s-waves with results of Platter/Hammer (2004) and Blume/Greene (2000)



Distribution of largest eigenvalues

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Outlook

- Estimate and possibly improve numerical accuracy
- Calculate for higher partial waves
- Third bound state artefact?
- Long term:
  - Beyond $N = 4$
  - Non-universal correction: effective range

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