Coupled channel approach to baryon-baryon interactions with strangeness on the lattice

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for HAL QCD collaboration

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- S=-2 Baryon-Baryon potential
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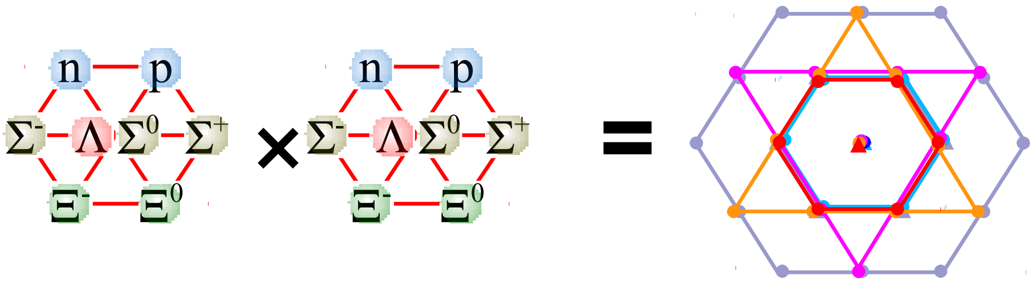
Introduction

Introduction

Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

Features of two-octet baryon system



Wide variety of BB interaction

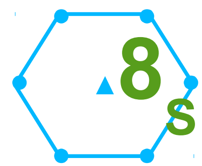
NN sector

Spin singlet

Flavor symmetric

1

H-dibaryon state is expected

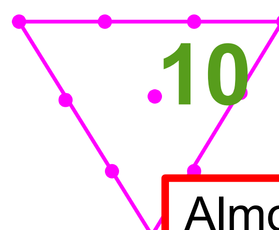


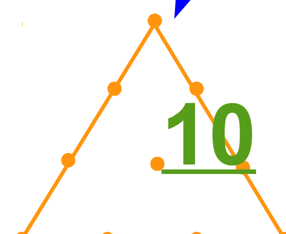
Pauli forbidden state

Short range repulsion

Spin triplet

Flavor anti-symmetric





Almost forbidden state

Strangeness brought us deeper understanding of BB interaction.

Introduction

According to quark model study

Otsuki, Tamagaki, Yasuno PTPS (1965)578
Oka, Shimizu and Yazaki NPA464 (1987)
Fujiwara, Suzuki, Nakamoto PPNP58(2007)

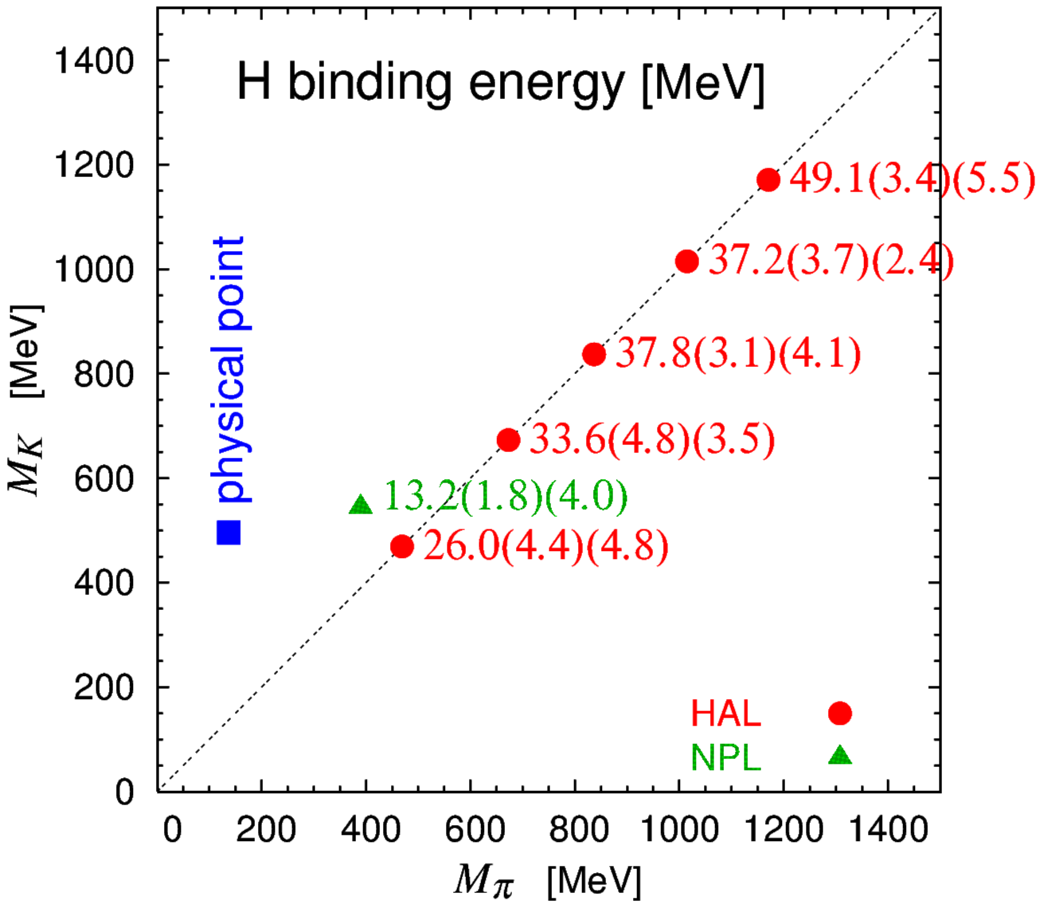
- Short range repulsion in BB interaction is result of Pauli principle and color magnetic interaction of the substructures of baryons.
- Strengths of repulsive core in YN and YY interaction are not universal but largely depend on their flavor structures.
- For the s-wave BB system, no repulsive core is predicted in flavor singlet state which is known as H-dibaryon channel.

H-dibaryon

- R.L. Jaffe predict the flavor singlet (uds)x(uds) state with J=0.
  - Strongly attractive color-magnetic interaction
  - No quark Pauli principle for flavor singlet state

Calculation directly from QCD should be important

Recent results for H-particle in Lattice QCD



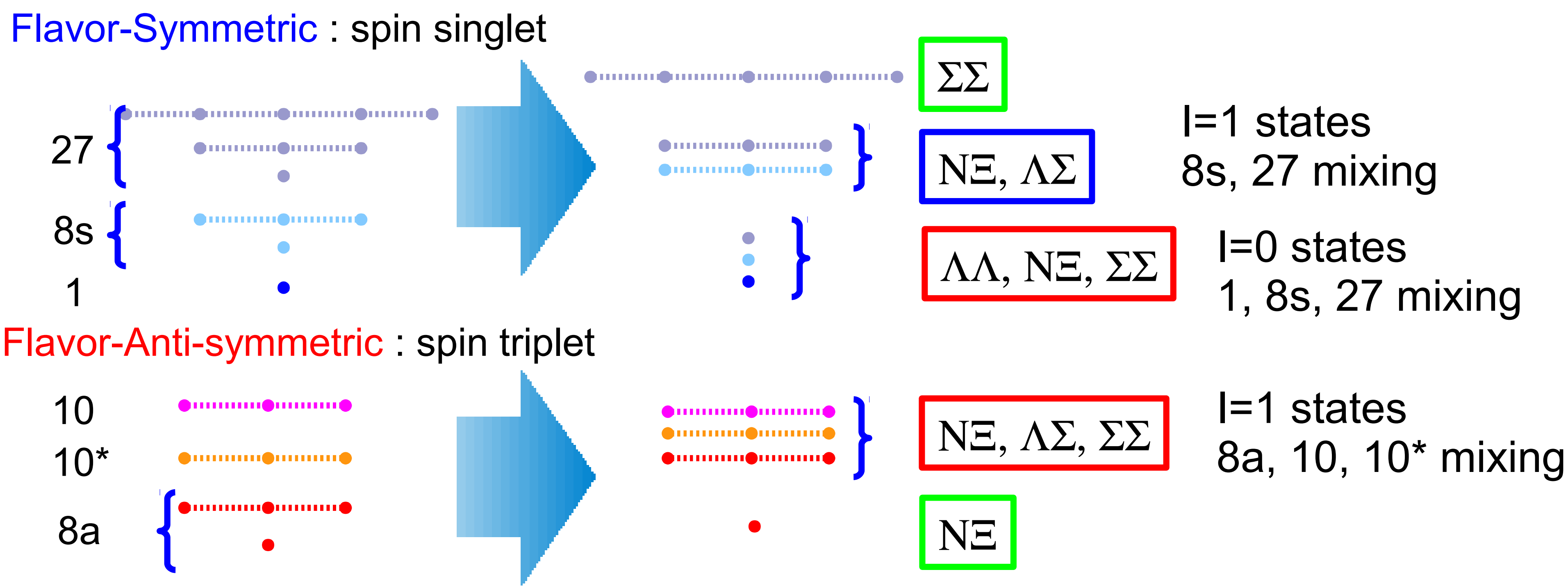
- Summary of binding energies of H-dibaryon from recent LQCD calc.
- S. R. Beane et al [NPLQCD colla.] Phys. Rev. Lett. 106, 162001 (2011), arXive: 1109.2889[hep-lat].
- The results of ours and NPLQCD look consistent
- Note that all results are still far away from physical point.
- No deeply bound H-dibaryon from experiment

What happen at physical point?

Simulation at SU(3) broken point by our method is necessary.
We have to depart from the SU(3) line.

Study of baryon-baryon interactions with S=-2

- All irreducible representations are involved in S=-2 BB system
  - Flavor singlet state in SU(3) is accessible
  - Fate of "H-dibaryon" at physical point.



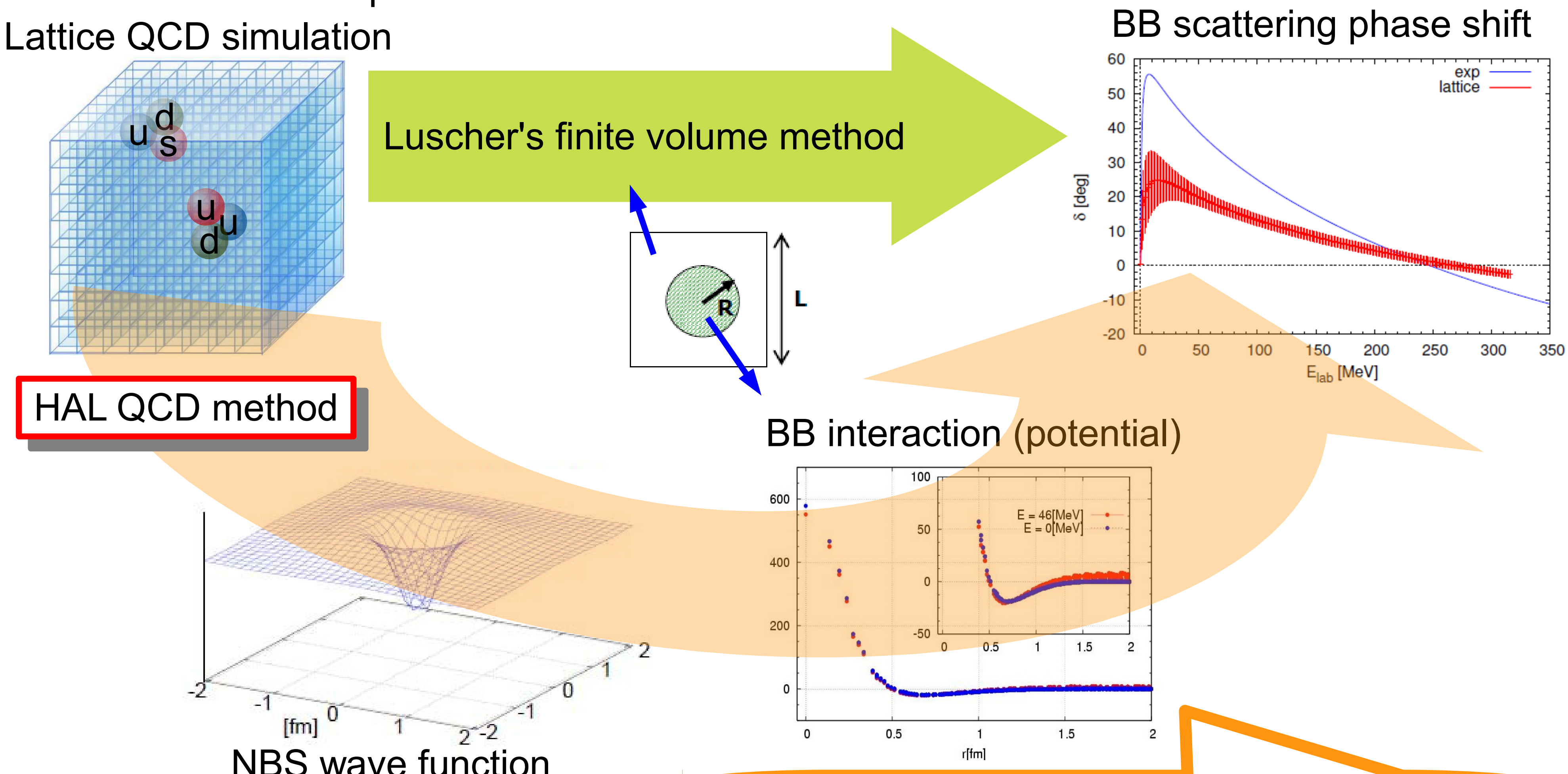
Coupled channel treatment is indespensable!

HAL QCD strategy

QCD to hadronic interactions

Lattice QCD simulation can connect the fundamental QCD with nuclear physics

$$L_{QCD} = \bar{q}(i\gamma_\mu D^\mu - m)q + \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$



N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99** (2007) 022001

The potential is proper for the phase shift by QCD

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi_\nu(E, t-t_0, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x}+\vec{r}) B_j(t, \vec{x}) | E, \nu, t_0 \rangle$$

E : Total energy of system

ν : other observables which needs to form the complete set

Four point correlator

$$F_{B_1 B_2}(\vec{r}, t) = \langle 0 \lceil T \left[B_1(\vec{r}, t) B_2(0, t) (\bar{B}_2 \bar{B}_1)_{t_0} \right] \rceil \rangle = \sum_n A_n \Psi_n e^{-E_n t}$$

Local composite interpolating operators

$$p = udu \quad n = udd \quad \Xi^0 = sus \quad \Xi^- = sds$$

$$\Lambda = \sqrt{\frac{1}{6}} [dsu + sud - 2uds]$$

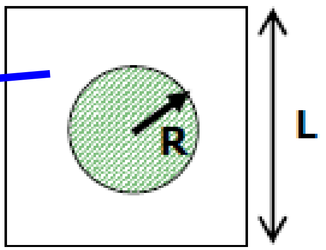
$$B = \epsilon^{abc} (q_a^T C \gamma_5 q_b) q_c$$

$$\Sigma^+ = -usu \quad \Sigma^0 = -\sqrt{\frac{1}{2}} [dsu + usd] \quad \Sigma^- = -dsd$$

NBS wave function has a same asymptotic form with quantum mechanics.

(NBS wave function is characterized from phase shift)

$$\Psi(t-t_{0,} \vec{r}) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$

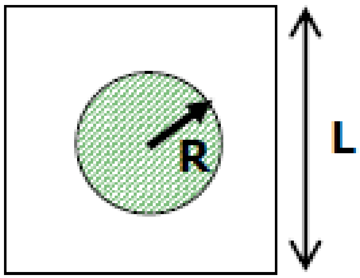


N. Ishii *et al.* [HAL QCD Collab.], Phys. Lett. B **712** (2012) 437

Schrödinger equation

- Define the energy-independent potential in Schrödinger equation (most general form)

$$\left(\frac{k^2}{2\mu}-H_0\right)\Psi(\vec{x})=\int U(\vec{x},\vec{y})\Psi(\vec{y})d^3y$$



- Recent development : Time dependent method.

We replace ψ to R defined below

$$\partial_t R_\alpha(\vec{x},E) \equiv \partial_t\left(\frac{A\Psi_\alpha(\vec{x},E)\mathrm{e}^{-Et}}{\mathrm{e}^{-m_A t}\mathrm{e}^{-m_B t}}\right) \propto -\frac{p_\alpha^2}{2\mu_\alpha}R_\alpha(\vec{x},E)$$

- Performing the derivative expansion for the interaction kernel

$$\left(-\frac{\partial}{\partial t}-H_0\right)R(\vec{x})=\int U(\vec{x},\vec{y})R(\vec{y})d^3y$$

- Taking the leading order of derivative expansion of non-local potential

$$U(\vec{x},\vec{y}) \simeq V_0(\vec{x})\delta(\vec{x}-\vec{y}) + V_1(\vec{x},\nabla)\delta(\vec{x}-\vec{y}) \cdots$$

- Finally local potential was obtained as

$$V(\vec{x}) = -\frac{\partial_t R(\vec{r})}{R(\vec{v})}+\frac{1}{2\mu}\frac{\nabla^2 R(\vec{x})}{R(\vec{x})}$$

Coupled channel Schrödinger equation

Preparation for the NBS wave function

Two-channel coupling case

$$\Psi^{\alpha}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\alpha}(t,\vec{r}) | E \rangle$$
$$\Psi^{\beta}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\beta}(t,\vec{x}) | E \rangle$$

The same “in” state

Inside the interaction range

In the *leading order of velocity expansion* of non-local potential,

Coupled channel Schrödinger equation.

Factorization of interaction kernel

$$\left(\frac{p_\alpha^2}{2\mu_\alpha} + \frac{\nabla^2}{2\mu_\alpha}\right)\psi^\alpha(\vec{x},E) = V^\alpha{}_\alpha(\vec{x})\psi^\alpha(\vec{x},E) + V^\alpha{}_\beta(\vec{x})\psi^\beta(\vec{x},E)$$

μ_α *: reduced mass*

p_α : asymptotic momentum.

Asymptotic momentum are replaced by the time-derivative of *R*.

$$R_I^{B_1 B_2}(t,\vec{r}) = \sum_{\vec{x}} \langle 0 | B_1(t,\vec{x}+\vec{r}) B_2(t,\vec{x}) \bar{I}(0) | 0 \rangle e^{(m_1+m_2)t}$$

$$\begin{pmatrix} V^\alpha{}_\alpha(\vec{r}) & V^\alpha{}_\beta(\vec{r})x \\ V^\beta{}_\alpha(\vec{r})x^{-1} & V^\beta{}_\beta(\vec{r}) \end{pmatrix} = \begin{pmatrix} (\frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t})R^\alpha_{I1}(\vec{r},E) & (\frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t})R^\beta_{I2}(\vec{r},E) \\ (\frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t})R^\alpha_{I1}(\vec{r},E) & (\frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t})R^\beta_{I2}(\vec{r},E) \end{pmatrix} \begin{pmatrix} R^\alpha_{I1}(\vec{r},E) & R^\beta_{I1}(\vec{r},E) \\ R^\alpha_{I2}(\vec{r},E) & R^\beta_{I2}(\vec{r},E) \end{pmatrix}^{-1}$$

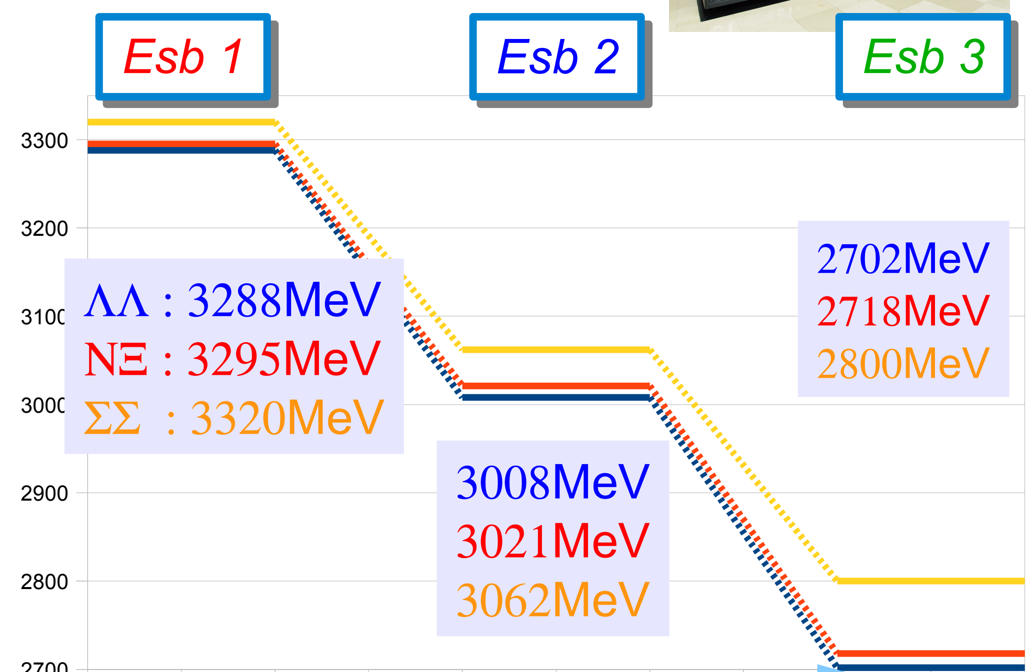
$$x = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

Numerical setup

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved Wilson quark action
  - β = 1.90, a^{-1} = 2.176 [*GeV*], 32^3x64 lattice, L = 2.902 [*fm*].
  - κ_s = 0.13640 is fixed, κ_{ud} = 0.13700, 0.13727 and 0.13754 are chosen.
- Flat wall source is considered to produce S-wave B-B state.
- The KEK computer system A resources are used.

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter



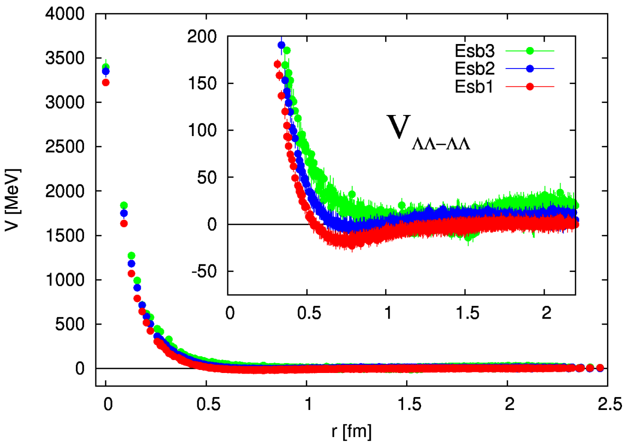
SU(3) breaking effects becomes larger

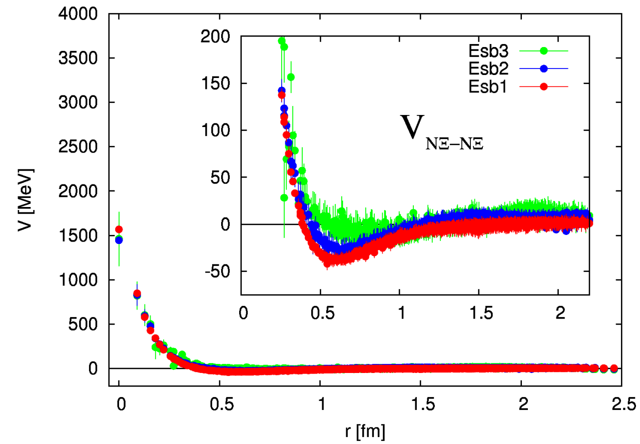
Strangeness S=-2 BB potential

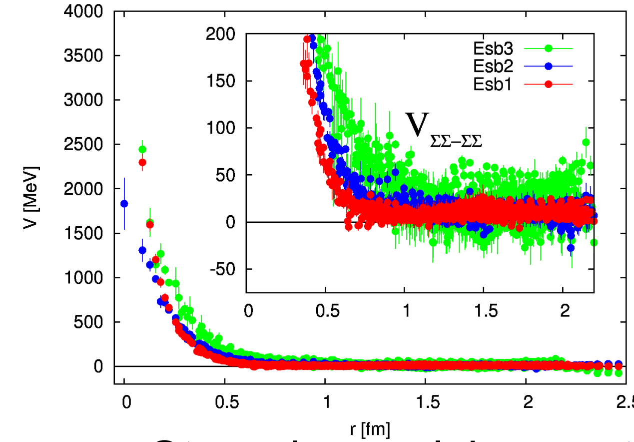
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

Diagonal elements



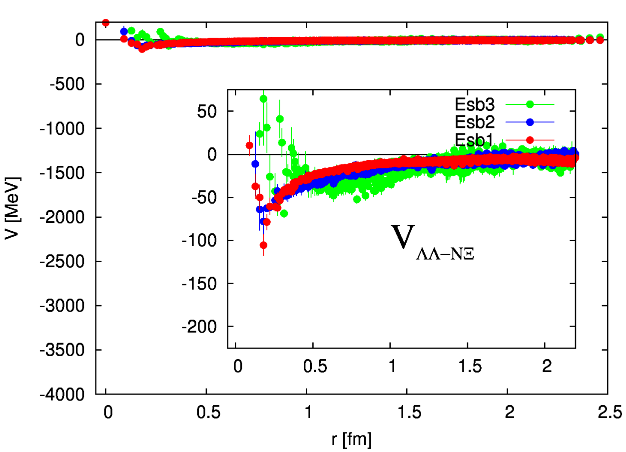


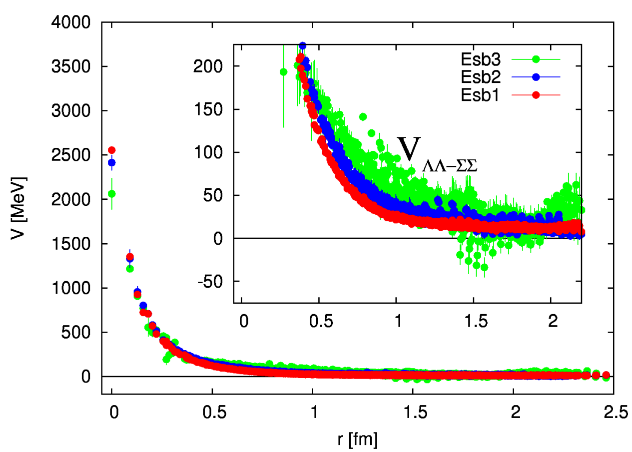


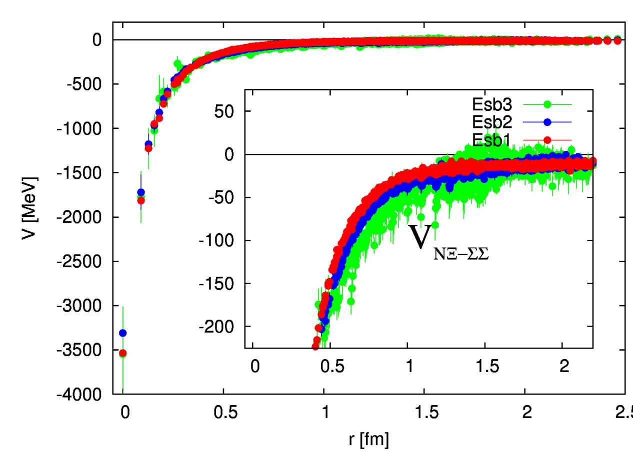
shallow attractive pocket
Deeper attractive pocket
Strongly repulsive

All channels have repulsive core

Off-diagonal elements







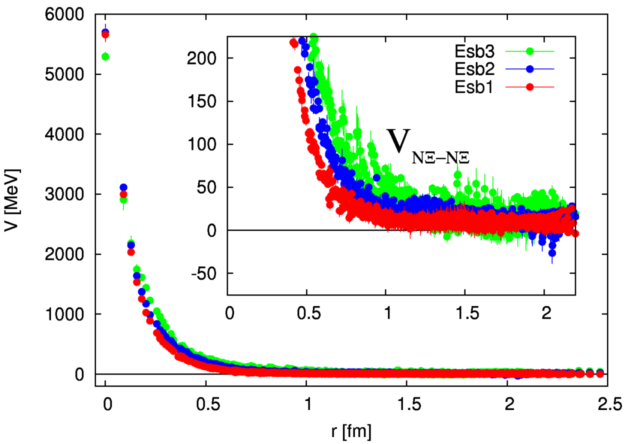
In this channel, our group found the "H-dibaryon" in the SU(3) limit.

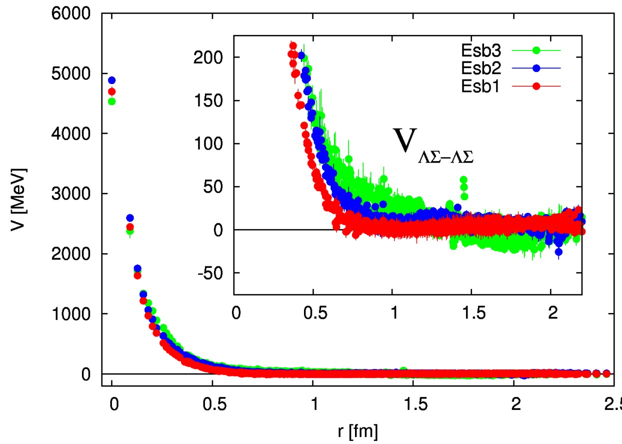
$N\Xi$, $\Lambda\Sigma$ (I=1) $^1S_0$ channel

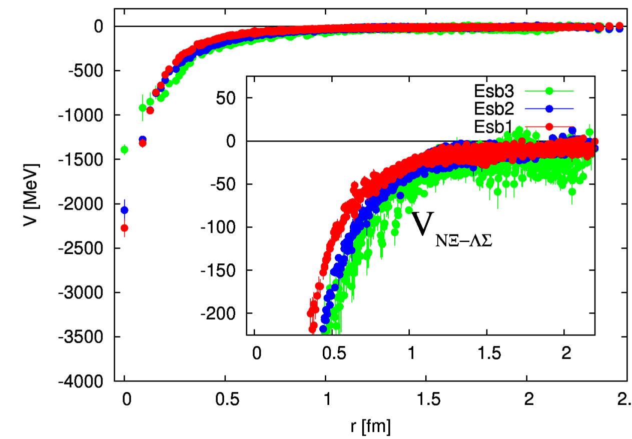
Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

Diagonal elements

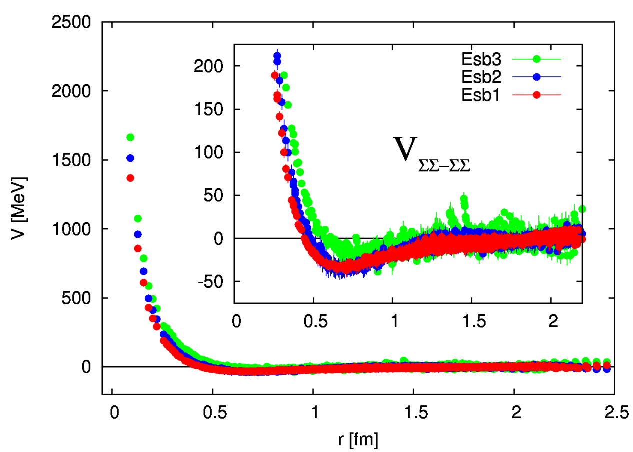
Off-diagonal elements





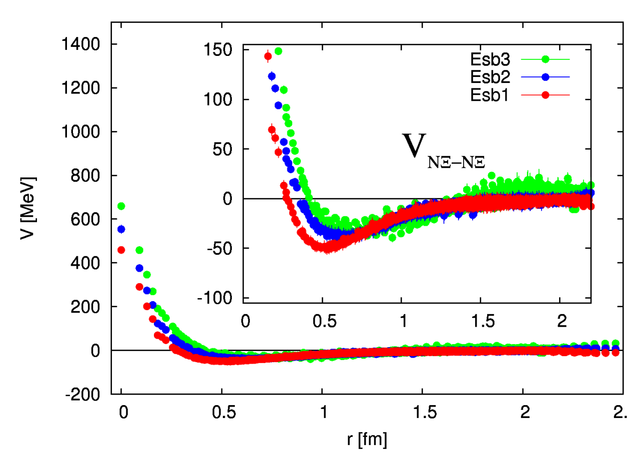


$\Sigma\Sigma$ (I=2) $^1S_0$ channel



Potential shape is similar to NN potential

$N\Xi$ (I=0) $^3S_1$ channel

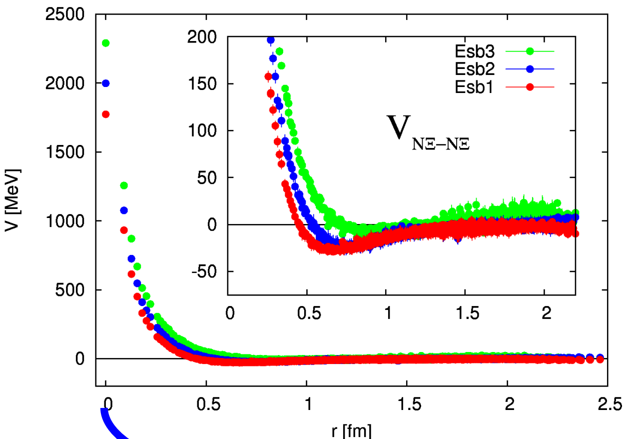


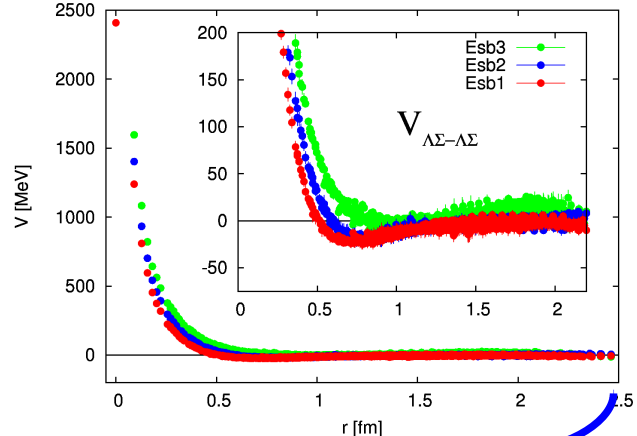
Small repulsive core
Deep attractive pocket

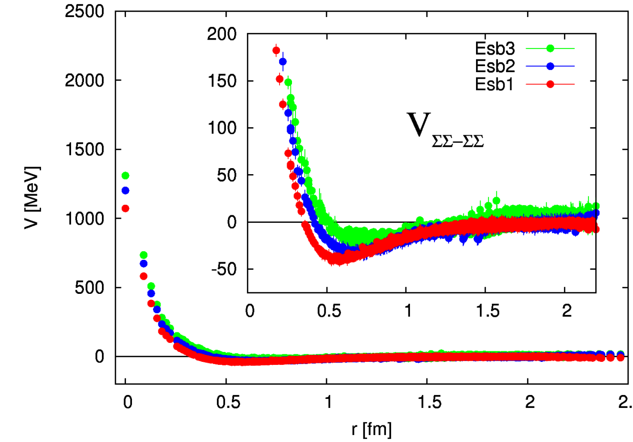
$N\Xi$, $\Sigma\Sigma$, $\Lambda\Sigma$ (I=1) $^3S_1$ channel

Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

Diagonal elements





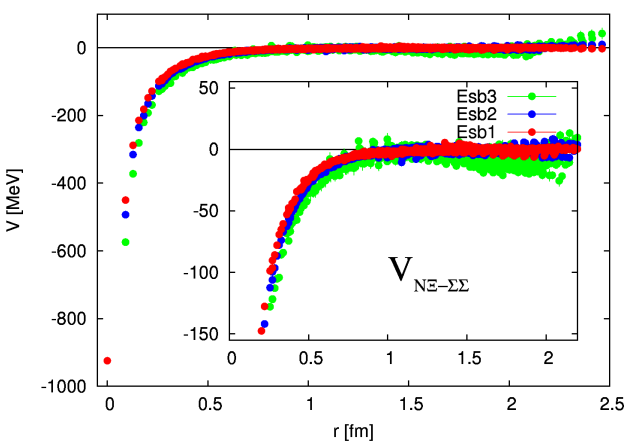


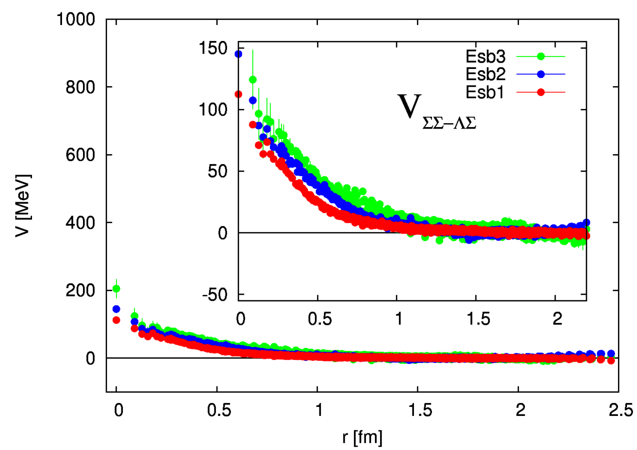
Most attractive

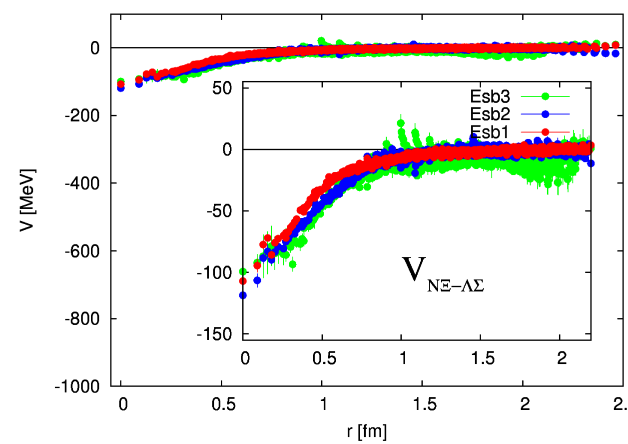
Attractive pocket becomes shallower as a lighter quark mass

All channels have repulsive core

Off-diagonal elements







Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

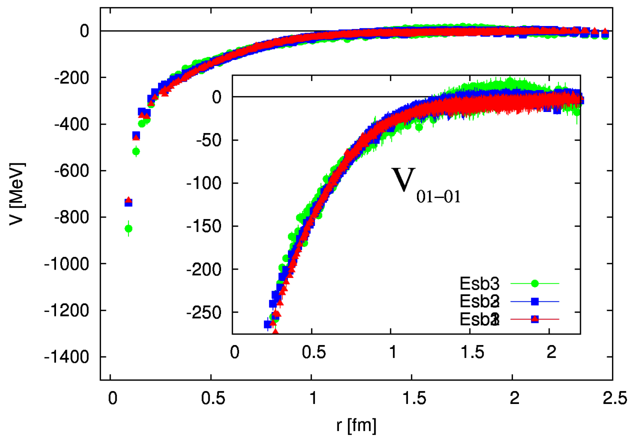
In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effectual measure of the SU(3) breaking effect.

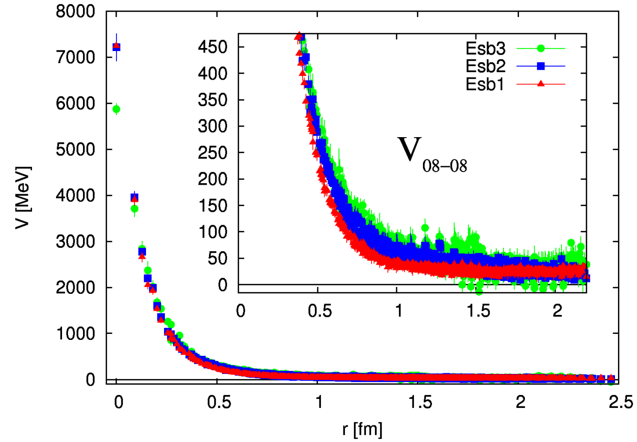
We will see how the SU(3) symmetry of potential will be broken
by changing the u,d quark masses lighter.

1, 8_s, 27 (I=0) $^1S_0$ channel

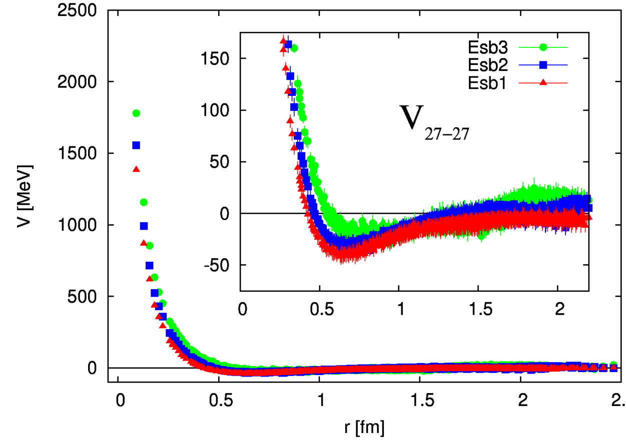
Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

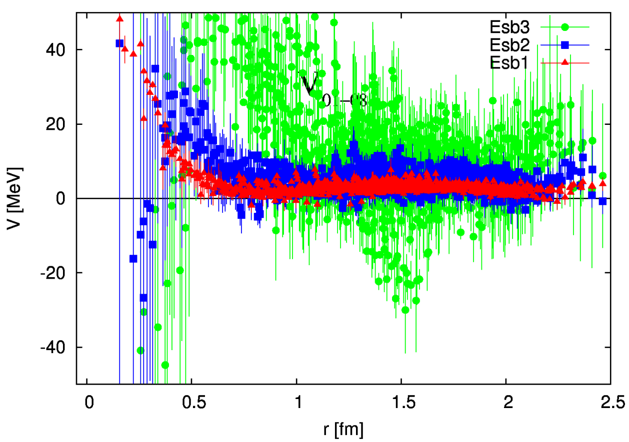


Strongly attractive
H-dibaryon channel

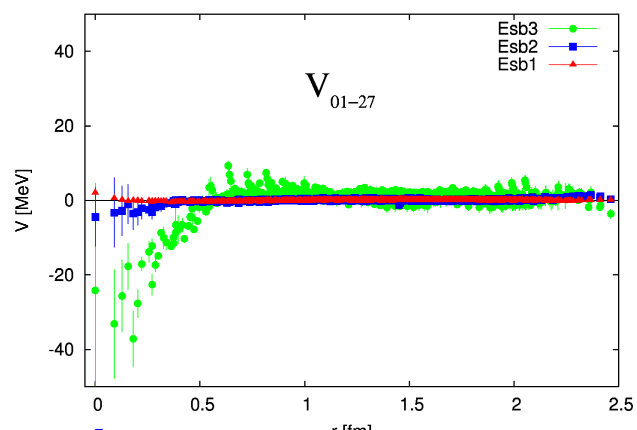


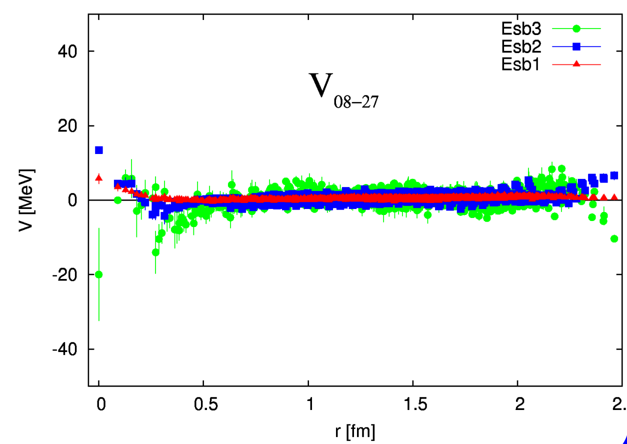
Pauli blocking effect





Mixture of singlet and octet
Is relatively larger than the others

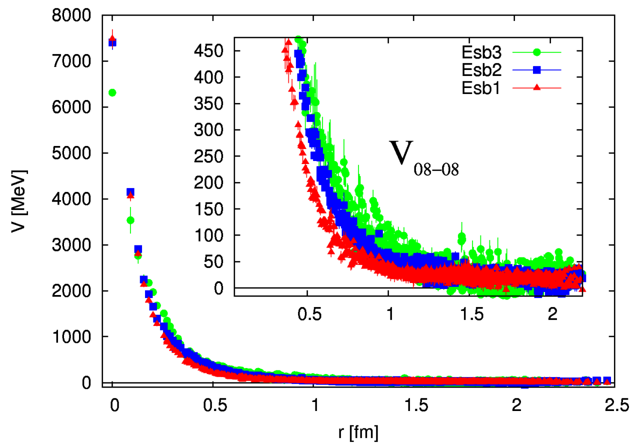




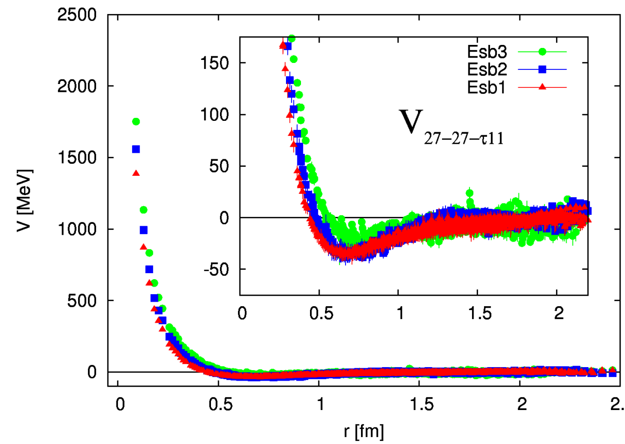
27 plet does not mix so much to the other representations

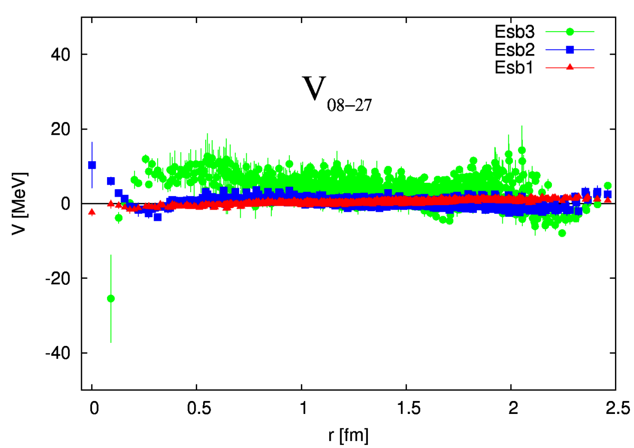
8_s, 27 (I=1) $^1S_0$ channel

Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV



Strongly repulsive

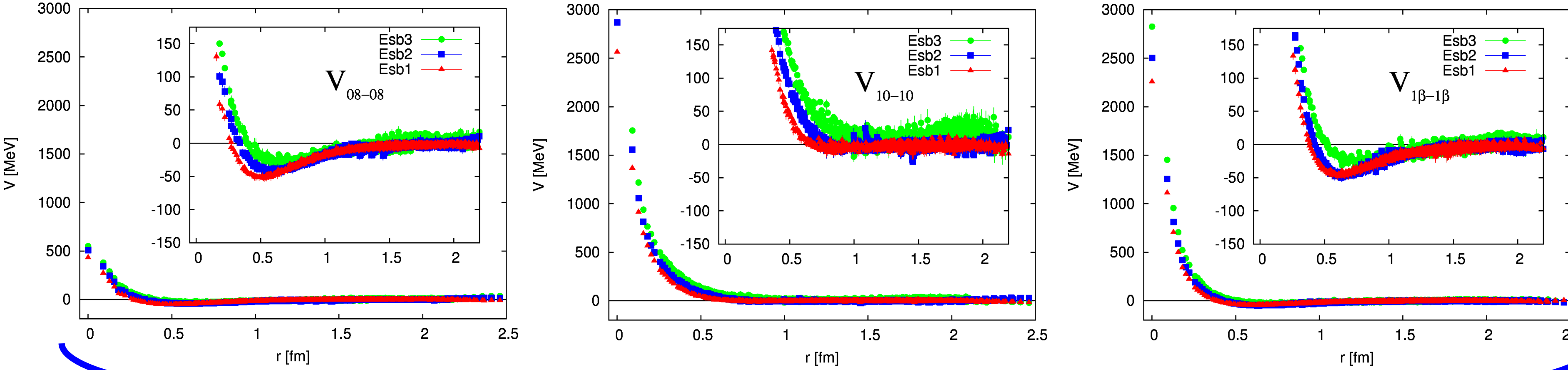




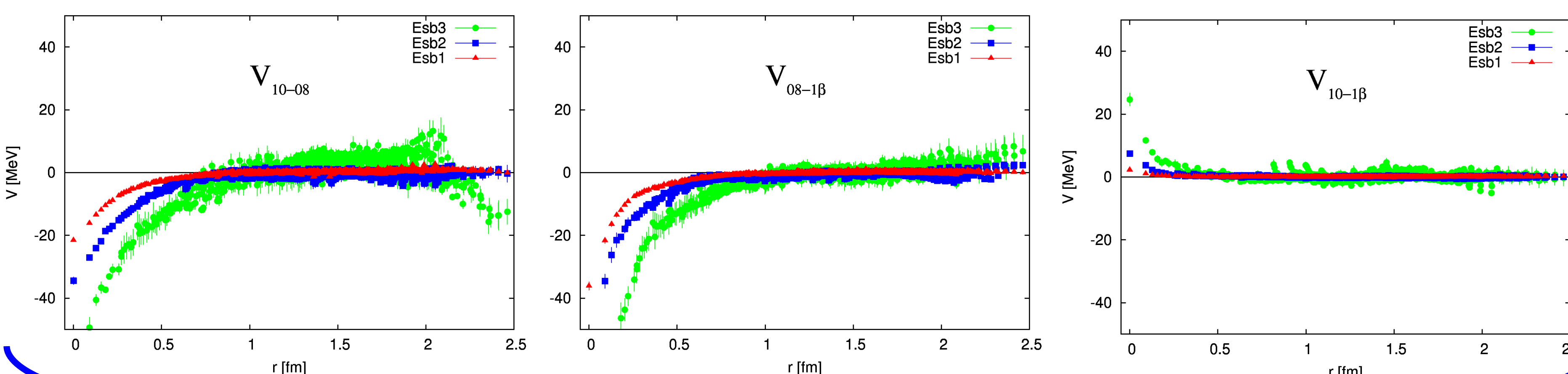
8_s and 27plet mixing is similar strength to the I=1 $^1S_0$ case

8a, 10, 10* (I=1) $^3S_1$ channel

Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

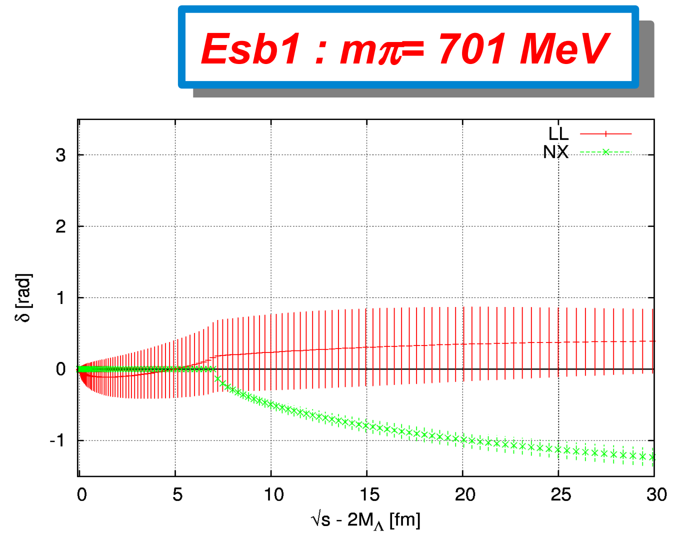


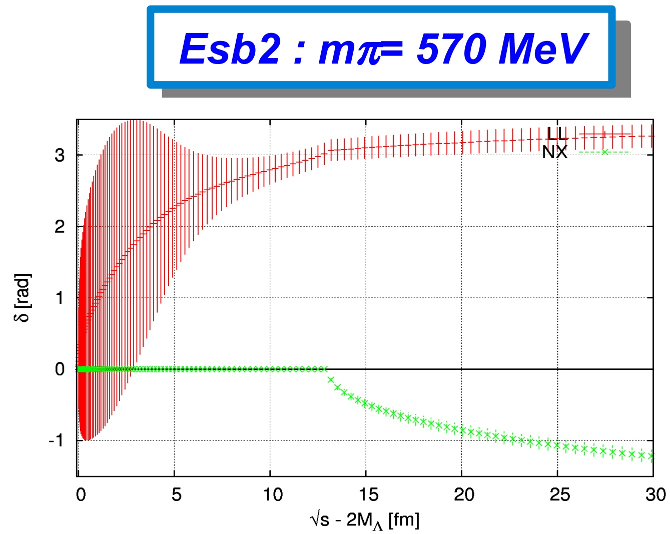
Repulsive core grow as decreasing quark mass

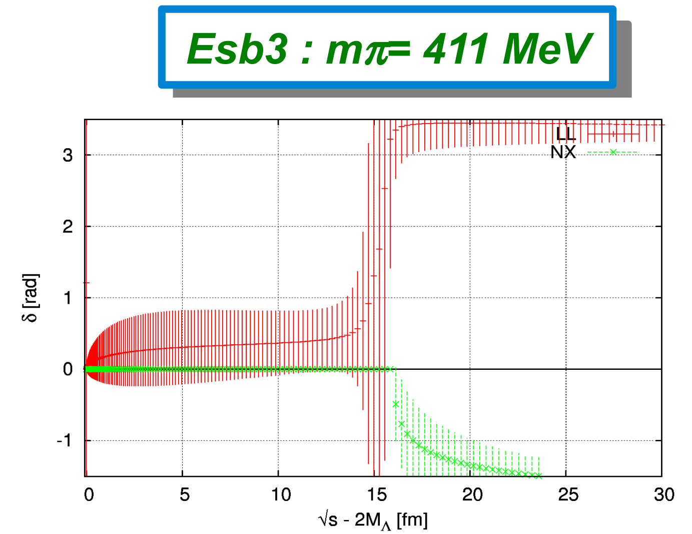


Mixture of these three IRs get larger and larger as decreasing quark mass.

ΛΛ and NΞ phase shifts







Preliminary!

- Esb1:
  - Bound H-dibaryon
- Esb2:
  - H-dibaryon is near the ΛΛ threshold
- Esb3:
  - The H-dibaryon resonance energy is close to NΞ threshold..

- We can see the clear resonance shape in ΛΛ phase shifts for Esb2 and 3.
- The "binding energy" of H-dibaryon from NΞ threshold becomes smaller as decreasing of quark masses.

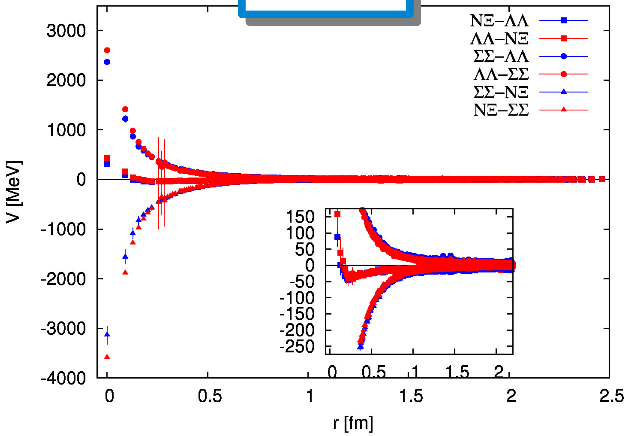
Summary and outlook

- We have investigated the S=-2 BB system from lattice QCD.
- In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.
- Potentials are derived from NBS wave functions calculated with PACS-CS configurations
- Quark mass dependence of potentials can be seen not in long range region but in shot distances as an enhancement of repulsive core.
- Small mixture between different SU(3) irreps can be seen as the flavor SU(3) breaking effect.
- SU(3) breaking effects are still small even in m_π/m_K=0.65 situation but it would be change drastically at physical situation m_π/m_K=0.28.

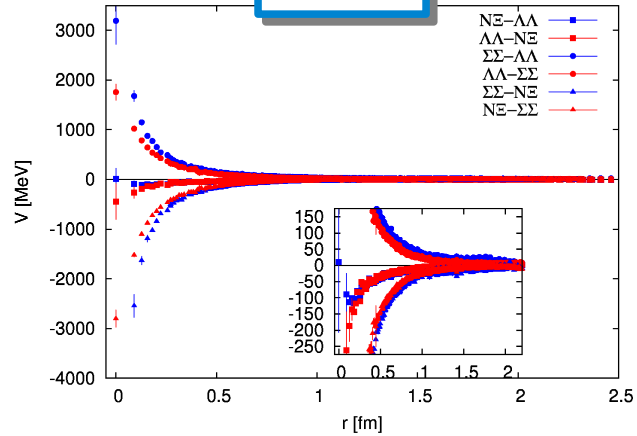
Backup slides

Harmiticity check for $^1S_0$ I=0

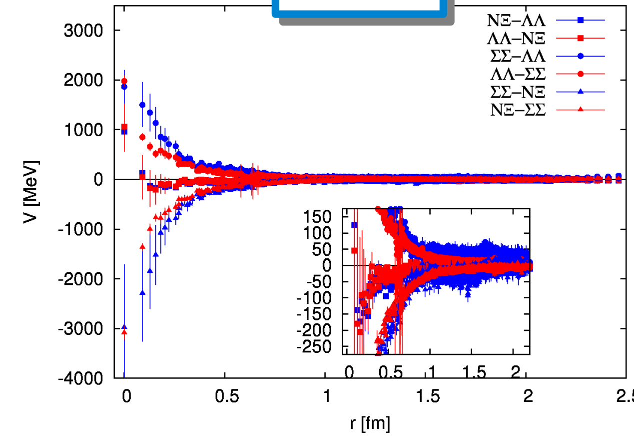
Esb 1



Esb 2

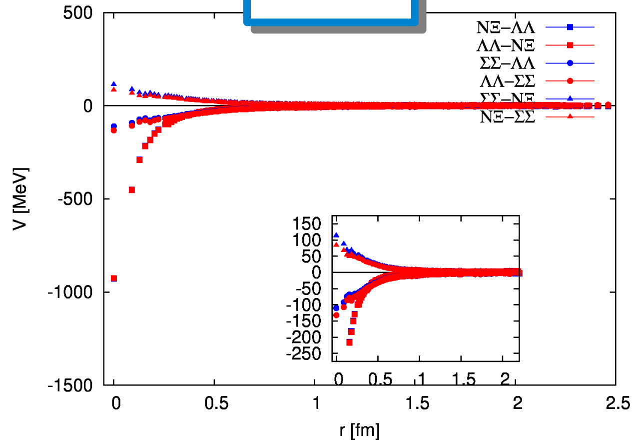


Esb 3

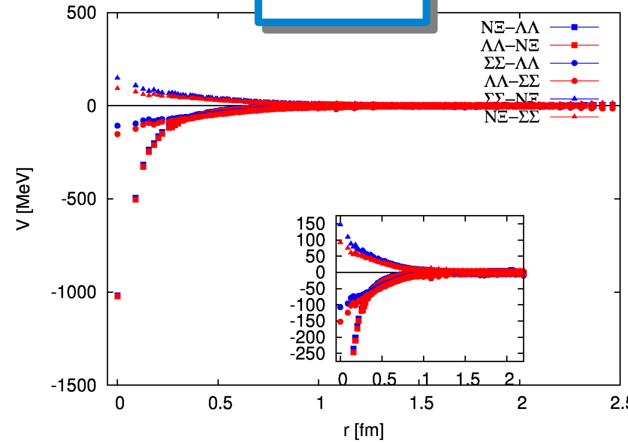


Harmiticity check for $^3S_1$ I=1

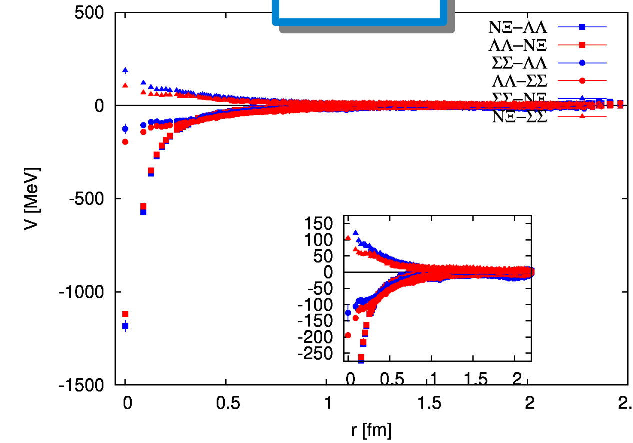
Esb 1



Esb 2



Esb 3



Hermiticity is roughly fine, but we need more statistics

Baryon operators

$$p_\alpha = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\, u(\xi_1)d(\xi_2)u(\xi_3)$$
$$n_\alpha = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\, u(\xi_1)d(\xi_2)d(\xi_3)$$

$$\Sigma_\alpha^+ = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\, u(\xi_1)s(\xi_2)u(\xi_3)$$
$$\Sigma_\alpha^0 = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\sqrt{\frac{1}{2}}\left[d(\xi_1)s(\xi_2)u(\xi_3)+u(\xi_1)s(\xi_2)d(\xi_3)\right]$$
$$\Sigma_\alpha^- = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\, d(\xi_1)s(\xi_2)d(\xi_3)$$

$$\Lambda_\alpha = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\sqrt{\frac{1}{6}}\left[d(\xi_1)s(\xi_2)u(\xi_3)+s(\xi_1)u(\xi_2)d(\xi_3)-2u(\xi_1)d(\xi_2)s(\xi_3)\right]$$

$$\Xi_\alpha^0 = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\, s(\xi_1)u(\xi_2)s(\xi_3)$$
$$\Xi_\alpha^- = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3\alpha}\, s(\xi_1)d(\xi_2)s(\xi_3)$$

- With corrected phase $\quad \bar{1} = -\epsilon^{123} = -(ds - sd) = sd - ds$

Irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$

with

$$\overline{\Sigma}\,\overline{\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^{+}}\,\overline{\Sigma^{-}} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^{0}}\,\overline{\Sigma^{0}} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^{-}}\,\overline{\Sigma^{+}}$$

$$\overline{N}\,\overline{\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^{-}} + \sqrt{\frac{1}{4}}\,\overline{\Xi^{-}}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^{0}} - \sqrt{\frac{1}{4}}\,\overline{\Xi^{0}}\,\overline{n}$$

$$\overline{BB^{(10^*)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} - \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{\Sigma^{+}} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^{+}}\,\overline{p}$$

$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^{-}} - \sqrt{\frac{1}{4}}\,\overline{\Xi^{-}}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^{0}} + \sqrt{\frac{1}{4}}\,\overline{\Xi^{0}}\,\overline{n}$$

Isospin combinations of BB operator

$\Lambda\Lambda,\ p\Xi^-,\ n\Xi^0,\ \Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0$

I=0 operstors

$$\Sigma\Sigma = +\sqrt{\frac{1}{3}}\Sigma^+\Sigma^- - \sqrt{\frac{1}{3}}\Sigma^0\Sigma^0 + \sqrt{\frac{1}{3}}\Sigma^-\Sigma^+$$

$$N\Xi = +\sqrt{\frac{1}{2}}p\Xi^- - \sqrt{\frac{1}{2}}n\Xi^0$$

I=1 operstors

$$\Sigma\Sigma = +\sqrt{\frac{1}{2}}\Sigma^+\Sigma^- - \sqrt{\frac{1}{2}}\Sigma^-\Sigma^+$$

$$N\Xi = +\sqrt{\frac{1}{2}}p\Xi^- + \sqrt{\frac{1}{2}}n\Xi^0$$

I=2 operstors

$$\Sigma\Sigma = +\sqrt{\frac{1}{6}}\Sigma^+\Sigma^- + \sqrt{\frac{4}{6}}\Sigma^0\Sigma^0 + \sqrt{\frac{1}{6}}\Sigma^-\Sigma^+$$

Effective Schrodinger equation with E-independent potential

$K(\vec{x};E) \equiv \left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E)$ **[START] local but E-dep pot. ($L^3 x L^3$ dof)**

(1) We assume ψ(x; E) for different E is linearly independent with each other.

(2) ψ(x; E) has a "left inverse" as an integration operator as

$E \equiv 2\sqrt{m_N^2 + k^2}$

$$\int d^3x \tilde{\psi}(\vec{x};E')\psi(\vec{x};E) = 2\pi\delta(E-E')$$

(3) K(x; E) can be factorized as

$$K(\vec{x};E) = \int \frac{dE'}{2\pi} K(\vec{x};E') \times \int d^3y \tilde{\psi}(\vec{y};E')\psi(\vec{y};E)$$

$$= \int d^3y \left\{ \sum_\alpha \int \frac{dE'}{2\pi} K(\vec{x};E')\tilde{\psi}(\vec{y};E') \right\} \psi(\vec{y};E)$$

$\equiv m_N U(\vec{x},\vec{y})$

(4) We are left with an effective Schrodinger equation with an E–independent potential U.

$$\left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E) = m_N \int d^3y\, U(\vec{x},\vec{y})\psi(\vec{y};E)$$

Intuitive understanding

[GOAL] non-local but E-indep pot. ($L^3 x L^3$ dof)

Asymptotic form of BS wave function

[C.-J.D.Lin et al., NPB619,467(2001)]

For simplicity, we consider BS wave function of two pions

$$\psi_{\vec{q}}(\vec{x}) \equiv \left\langle 0 \middle| N(\vec{x})\, N(\vec{0}) \middle| N(\vec{q}) N(-\vec{q}), in \right\rangle$$

complete set

$$1 = \int \frac{d^3p}{(2\pi)^3 2E_N(\vec{p})} |N(\vec{p})\rangle\langle N(\vec{p})| + \cdots$$

$$= \int \frac{d^3p}{(2\pi)^3 2E_N(\vec{p})} \langle 0 | N(\vec{x}) | N(\vec{p}) \rangle \left\langle N(\vec{p}) \middle| N(\vec{0}) \middle| N(\vec{q}) N(-\vec{q}), in \right\rangle + I(\vec{x})$$

$Z^{1/2} e^{i\vec{p}\cdot\vec{x}}$

$$disc. + Z^{1/2} \frac{T(\vec{p};\vec{q})}{m_N^2 - (2E_N(\vec{q}) - E_N(\vec{p}))^2 + \vec{p}^2 - i\varepsilon}$$

$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{(2\pi)^3} \int \frac{d^3p}{2E_N(\vec{p})} \frac{T(\vec{p};\vec{q})}{4E_N(\vec{q}) \cdot (E_N(\vec{p}) - E_N(\vec{q}) - i\varepsilon)} e^{i\vec{p}\cdot\vec{x}} \right)$$

Integral is dominated by the on-shell contribution $E_N(\vec{p}) \approx E_N(\vec{q})$

➔ T-matrix becomes the on-shell T-matrix

$$T^{(s-wave)}(s) = \frac{E(\vec{q})}{2|\vec{q}|}(-i)\left(e^{2i\delta_0(s)} - 1\right)$$

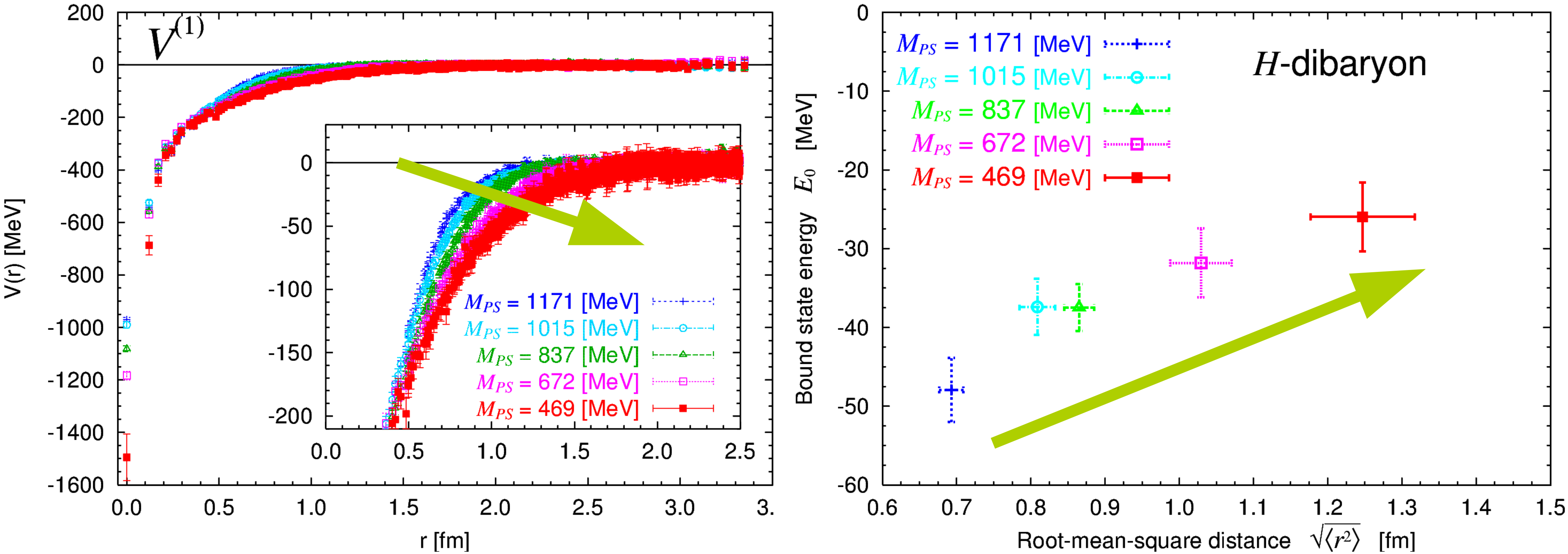
$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{2i}\left(e^{2i\delta_0(s)} - 1\right)\frac{e^{iqr}}{qr} \right) + \cdots$$

The asymptotic form

$$\psi_{\vec{q}}(\vec{x}) = Z e^{i\delta_0(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \quad \text{(s-wave)}$$

This is analogous to a non-rela. wave function

Looking for H-dibaryon in SU(3) limit



Growth of kinetic energy of baryon pair could be quicker than enhancement of attraction.

- Potential in flavor singlet channel is getting more attractive as decreasing quark masses
- Ground state energies in all different quark masses are below the free BB threshold.
- There is a 6q bound state in this mass range with SU(3) symmetry.

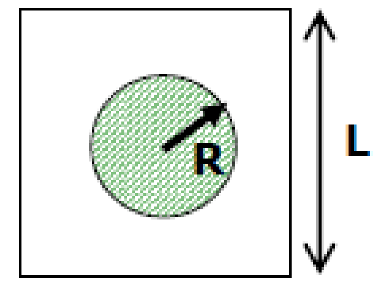
Energy indep. potential in coupled channel S.E.

Inside the interaction range, we can define the interaction kernel

$$\begin{pmatrix} p^2+\nabla & q^2+\nabla \\ p^2+\nabla & q^2+\nabla \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{x},E) & \psi_a^b(\vec{x},E) \\ \psi_a^b(\vec{x},E) & \psi_a^b(\vec{x},E) \end{pmatrix} = \begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix} \xrightarrow{|x|>R} 0$$

Factorization of the interaction kernel

$$\begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix} = \int dy \begin{pmatrix} U_a^a(\vec{x},\vec{y}) & U_b^a(\vec{x},\vec{y}) \\ U_a^b(\vec{x},\vec{y}) & U_b^b(\vec{x},\vec{y}) \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{y},E) & \psi_b^a(\vec{y},E) \\ \psi_a^b(\vec{y},E) & \psi_b^b(\vec{y},E) \end{pmatrix}$$



$$\int dx \begin{pmatrix} \tilde{\psi}_a^a(\vec{x},E') & \tilde{\psi}_b^a(\vec{x},E') \\ \tilde{\psi}_a^b(\vec{x},E') & \tilde{\psi}_b^b(\vec{x},E') \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{x},E) & \psi_b^a(\vec{x},E) \\ \psi_a^b(\vec{x},E) & \psi_b^b(\vec{x},E) \end{pmatrix} = 2\pi\delta(E-E')$$

$$\begin{pmatrix} U_a^a(\vec{x},\vec{y}) & U_b^a(\vec{x},\vec{y}) \\ U_a^b(\vec{x},\vec{y}) & U_b^b(\vec{x},\vec{y}) \end{pmatrix} = \int \frac{dE}{2\pi}\begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix}\begin{pmatrix} \tilde{\psi}_a^a(\vec{y},E) & \tilde{\psi}_b^a(\vec{y},E) \\ \tilde{\psi}_a^b(\vec{y},E) & \tilde{\psi}_b^b(\vec{y},E) \end{pmatrix}$$

Energy independent potential in Schrödinger equation.