Ab initio NCSM/RGM for three-body cluster systems and application to ^{4}He+n+n

Carolina Romero-Redondo

Petr Navrátil and Sofia Quaglioni

TRIUMF

Outline

Introduction: NCSM/RGM

Extension of the method to Three-body cluster states

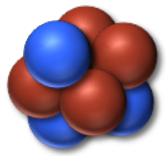
^{6}He : $^{4}He+n+n$

Summary and outlook

Introduction

Ab initio in nuclear physics

nucleus



Assumes nucleons as the effective degrees of freedom

Uses realistic interactions

The goal is to achieve a predictive theory for light nuclear systems to study:

- Exotic nuclei
- Reactions important in nuclear astrophysics
- Reactions important for energy production projects

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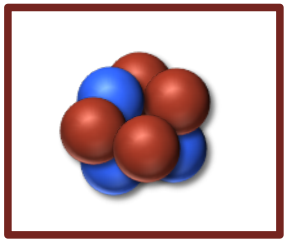
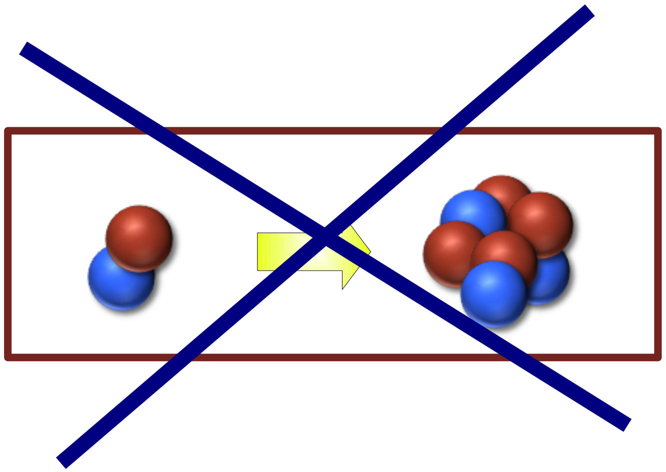
NCSM/RGM

No-core shell model (NCSM)

Is an ***ab initio*** method capable of studying light bound nuclei from an accurate Hamiltonian.

Is not able to deal with continuum states and therefore is not applicable to reactions.





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Resonating group method (RGM)

Microscopic cluster approach.

Permits studying the scattering of clusters

Non-realistic Hamiltonian

NCSM/RGM

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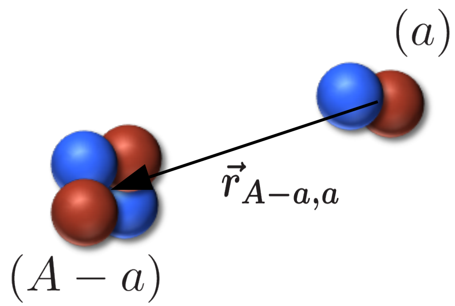
Non-realistic Hamiltonian

NCSM/RGM

Combines NCSM and RGM to obtain an *ab initio* formalism which uses an accurate nuclear Hamiltonian and is capable of studying both structure and scattering problems in light nuclear systems

NCSM/RGM

Binary clusters



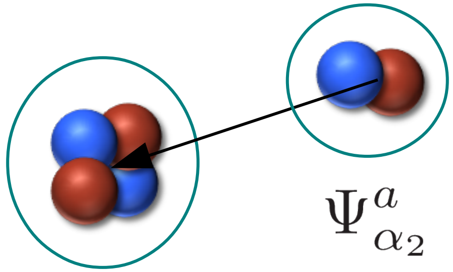
***A*-nucleon system**

S. Quaglioni and P. Navrátil

- PRL 101, 092501 (2008)
- PRC 79, 044606 (2009)

NCSM/RGM

Binary clusters



***A*-nucleon system**

$\Psi^{a}_{\alpha_2}$

$\Psi^{A-a}_{\alpha_1}$

NCSM wave functions

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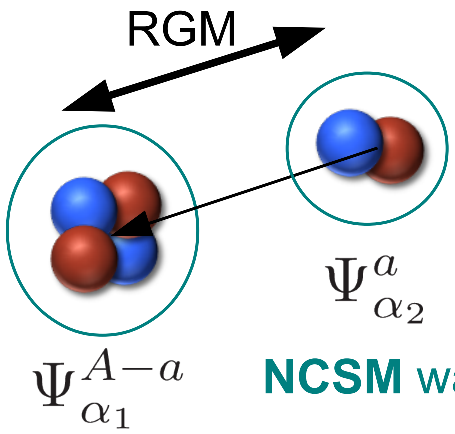

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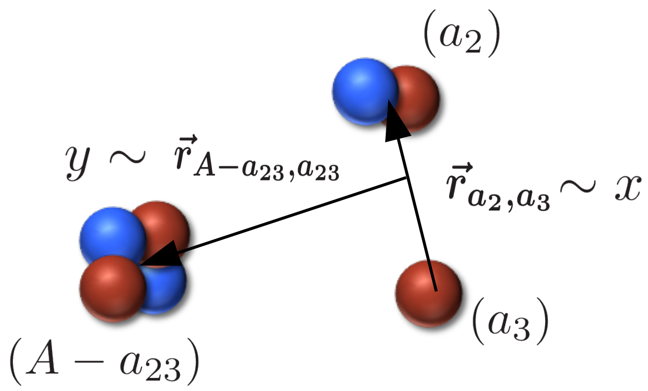
NCSM/RGM-3B

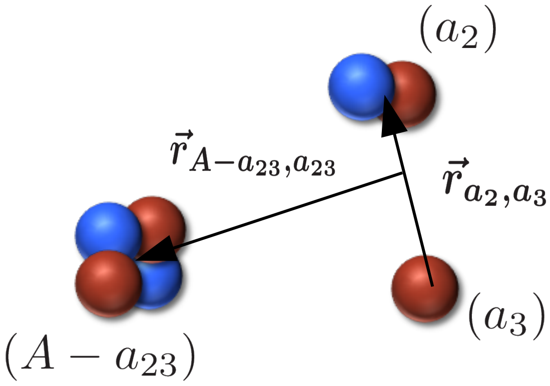


Extension to three-body cluster

C. Romero-Redondo

S. Quaglioni, P. Navrátil

NCSM/RGM-3B



Extension to three-body cluster

Why?

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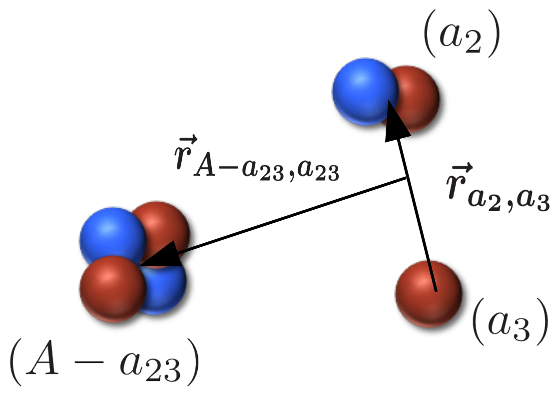
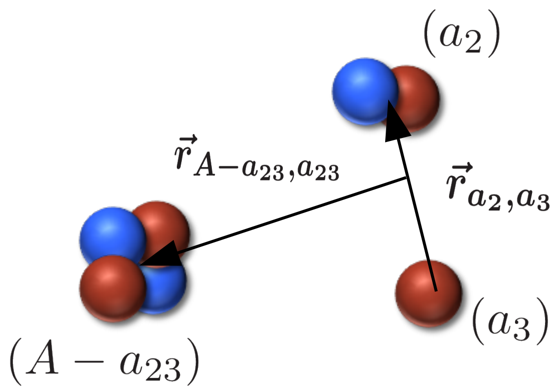
NCSM/RGM-3B



Extension to three-body cluster

Bound and resonant states:
2n Halo nuclei

NCSM/RGM-3B



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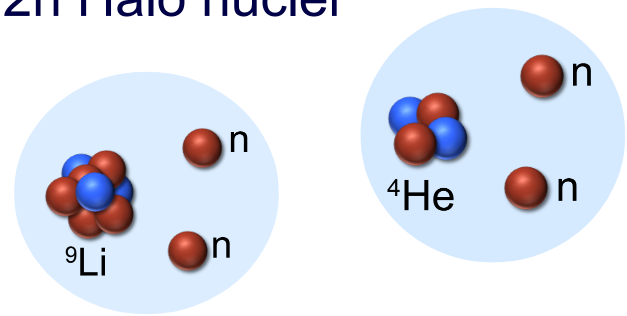

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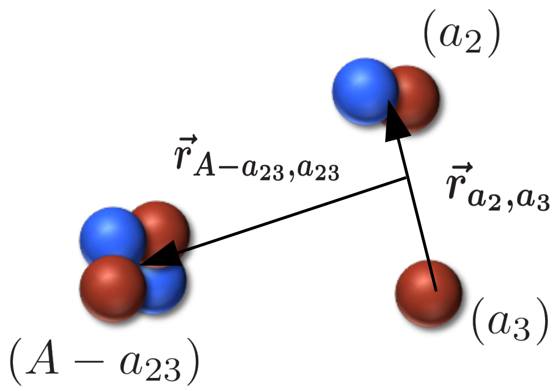

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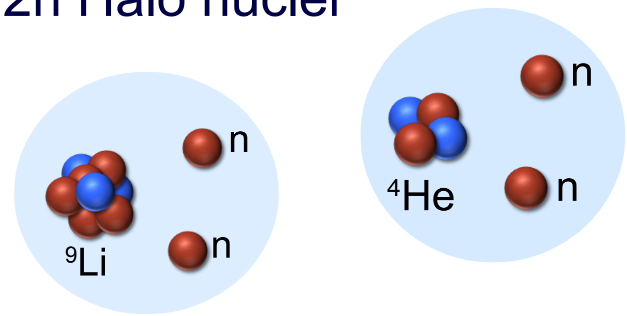
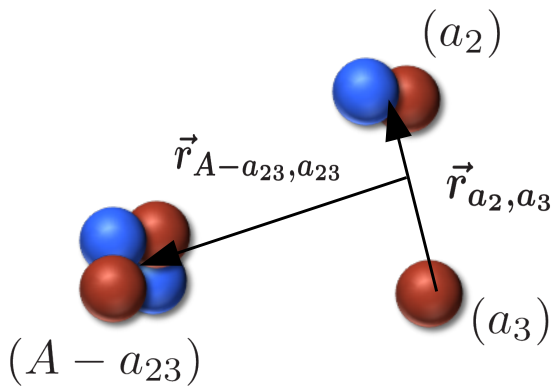


3-body continuum states:
Transfer reactions

NCSM/RGM-3B



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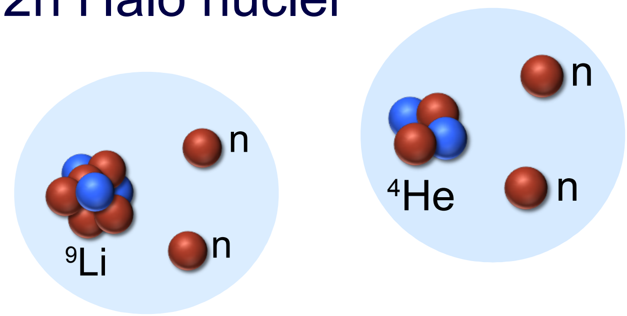

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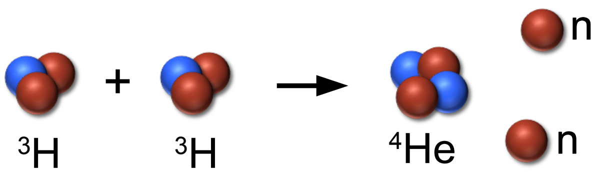
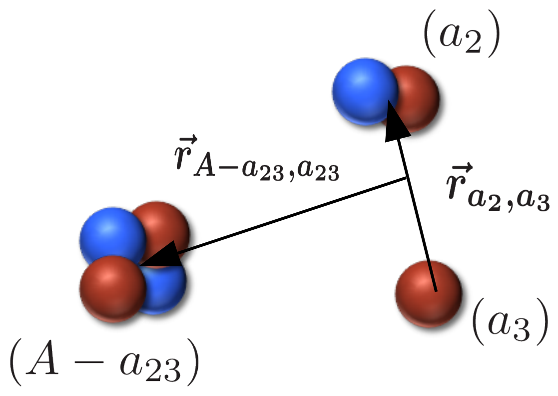


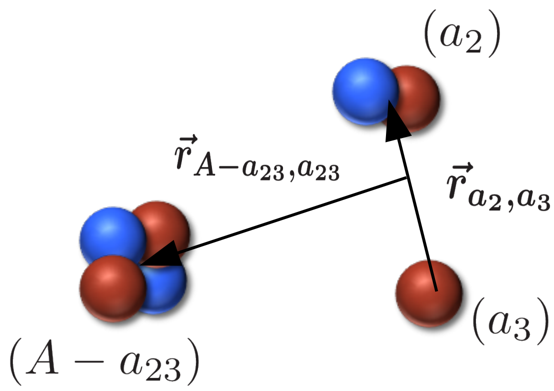
NCSM/RGM-3B



Basis

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx\,x^2 \int dy\,y^2 G_\nu^{J^\pi T}(x,y)\hat{A}_\nu |\boldsymbol{\Phi}^{J^\pi T}_{\nu x y}\rangle$$

NCSM/RGM-3B



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$$|\boldsymbol{\Phi^{J^\pi T}_{\nu r}}\rangle \sim \Psi^{A-a_{23}}_{\alpha_1} \Psi^{a_2}_{\alpha_2} \Psi^{a_3}_{\alpha_3} Y_{\ell_x}(\hat{r}_{a_2,a_3}) Y_{\ell_y}(\hat{r}_{A-a_{23},a_{23}}) \delta(r - r_{A-a_{23},a_{23}}) \delta(x - r_{a_2,a_3})$$

NCSM wave functions

NCSM/RGM-3B

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\boldsymbol{\Phi}_{\nu x y}^{J^\pi T}\rangle$$

Schrödinger equation

$$(\mathcal{H} - E)\,|\Psi^{J^\pi T}\rangle = 0$$

$$\sum_\nu \int dx dy x^2 y^2 \left[\mathcal{H}_{\nu'\nu}(x,y,x',y') - E\mathcal{N}_{\nu'\nu}(x,y,x',y')\right] G_\nu^{J^\pi T}(x,y) = 0$$

Hamiltonian Kernel

$$\langle \Phi_{\nu' r'}^{J^\pi T} | \hat{A}_{\nu'} \mathcal{H} \hat{A}_\nu | \Phi_{\nu r}^{J^\pi T}\rangle$$

Norm kernel

$$\langle \Phi_{\nu' r'}^{J^\pi T} | \hat{A}^2 | \Phi_{\nu r}^{J^\pi T}\rangle$$

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Relative movement wavefunction

NCSM/RGM-3B

$$|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx x^2 \int dy y^2 G_\nu^{J^\pi T}(x,y) \hat{A}_\nu |\boldsymbol{\Phi^{J^\pi T}_{\nu x y}}\rangle$$

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Orthogonalization

$$\sum_\nu \int dx dy x^2 y^2 \left[\mathbb{H}_{\nu'\nu}(x,y,x',y') - E\delta\nu'\nu \frac{\delta(x'-x)}{x'x}\frac{\delta(y'-y)}{y'y}\right] \chi_\nu^{J^\pi T}(x,y) = 0$$

NCSM/RGM-3B

$$\sum_{\nu} \int dx dy x^2 y^2 \left[\mathbb{H}_{\nu'\nu}(x,y,x',y') - E\delta\nu'\nu \frac{\delta(x'-x)}{x'x} \frac{\delta(y'-y)}{y'y} \right] \chi_{\nu}^{J^\pi T}(x,y) = 0$$

Hyperspherical coordinates: $\rho = \sqrt{x^2 + y^2}, \quad \alpha = \arctan(x/y)$

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$$\chi_{\nu}^{J^\pi T}(x,y) = \sum_k C_{k\nu}(\rho) \phi_k^{\ell_x \ell_y}(\alpha)$$

$$\phi_k^{\ell_x \ell_y}(\alpha) = N_k \sin^{\ell_x}(\alpha) \cos^{l_y}(\alpha) P_{k/2}^{\ell_x+1/2,\ell_y+1/2}(\cos 2\alpha)$$

NCSM/RGM-3B

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After changing to hyperspherical coordinates and integrating in α,α':

$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}_{\nu'\nu}^{k'k}(\rho', \rho) - E \frac{\delta(\rho - \rho')}{\rho^5} \delta_{\nu'\nu} \delta_{k'k} \right] C_{k\nu}^{J^\pi T}(\rho) = 0$$

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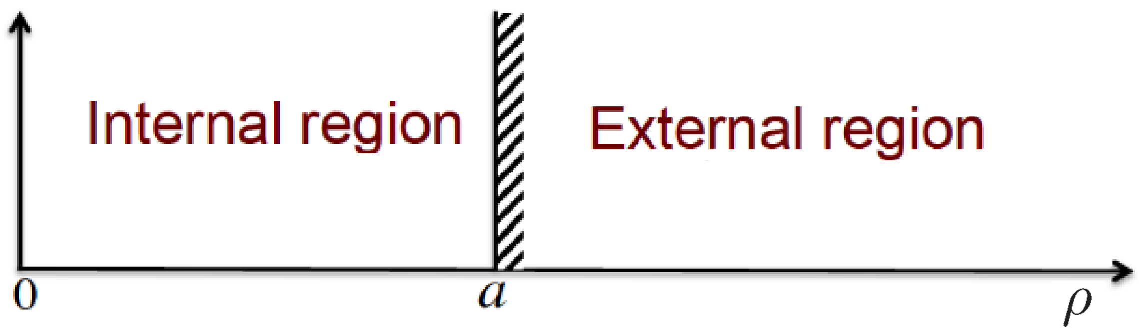
Coupled-channel microscopic R-matrix method on a Lagrange mesh

R-matrix

Internal region: expansion on a basis (ρ < a)

$$C_{k\nu}(\rho) = \sum_i \beta_{k\nu i} f_i(\rho)$$



External region: known asymptotic behaviour (ρ > a)

* Bound state: $C_{k\nu}(\rho) = A_{k\nu}\sqrt{\kappa\rho}K_{k+2}(\kappa\rho)$

* Continuum state: $C_{k\nu}(\rho) = A_{k\nu}\left[H_k^-(\kappa\rho)\delta_{\nu,\nu'}\delta_{k,k'} - S_{\nu k,\nu' k'}H_k^+(\kappa\rho)\right]$

R-matrix

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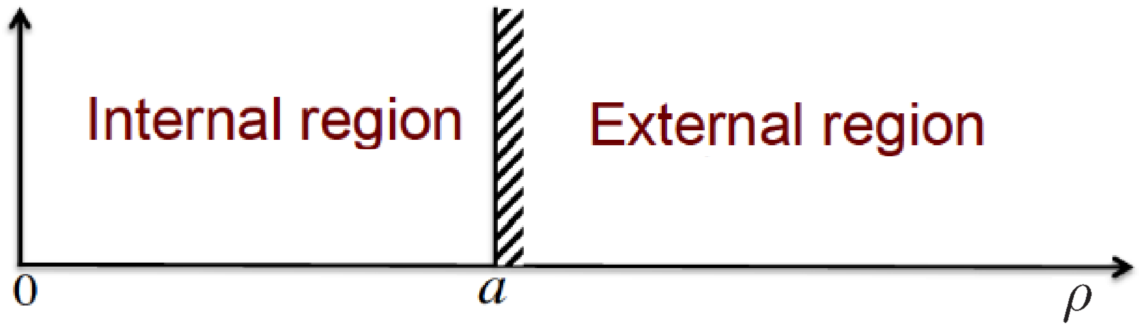

External region: known asymptotic behaviour (ρ > a)

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Scattering Matrix

* Continuum state: $$C_{k\nu}(\rho) = A_{k\nu}\left[H_k^-(\kappa\rho)\delta_{\nu,\nu'}\delta_{k,k'} - S_{\nu k,\nu' k'} H_k^+(\kappa\rho)\right]$$

R-matrix

$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}^{k' k}_{\nu' \nu}(\rho', \rho) - E \frac{\delta(\rho - \rho')}{\rho^5} \delta_{\nu' \nu} \delta_{k' k} \right] \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}} = 0$$

Matching conditions at ρ = a
Use of Bloch operator

$$\mathcal{L}_{k\nu} = \frac{\hbar^2}{2m} \delta(\rho - a) \frac{1}{\rho^{5/2}} \left(\frac{\partial}{\partial \rho} - \frac{L_{k\nu}}{\rho} \right) \rho^{5/2}$$ Bloch Operator

$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}^{k' k}_{\nu' \nu}(\rho', \rho) + \mathcal{L}_{k\nu} - E \frac{\delta(\rho - \rho')}{\rho^5} \delta_{\nu' \nu} \delta_{k' k} \right] \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}} = \sum_{\nu k} \int d\rho \rho^5 \mathcal{L}_{k\nu} \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}}$$

R-matrix

$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}^{k'k}_{\nu'\nu}(\rho', \rho) - E \frac{\delta(\rho - \rho')}{\rho^5} \delta_{\nu'\nu} \delta_{k'k} \right] \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}} = 0$$

Matching conditions at ρ = a
Use of Bloch operator

$$C_{k\nu}(\rho) = \sum_i \beta_{k\nu i} f_i(\rho)$$

* Bound state: Eigenvalue problem -> energy and $\beta_{k\nu i}$

* Continuum state: Scattering matrix

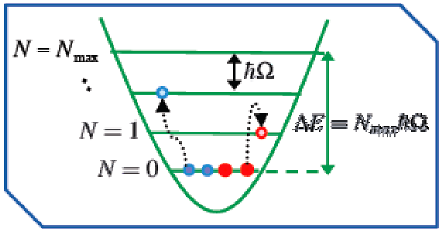
^{4}He+n+n

Input

- Accurate soft NN interaction: SRG-evolved chiral N^{3}LO potential with λ=1.5 fm^{-1}
  - Fits NN data with high accuracy
  - But: misses both chiral initial and SRG-induced NNN force
  - Fortunately: two effects mostly cancel each other

- ^{4}He ab initio wave function obtained within the NCSM

$$H^{(A-2)}\psi^{(A-2)}_{\beta_1}(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_{A-2}) = E^{(A-2)}_{\beta_1}\psi^{(A-2)}_{\beta_1}(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_{A-2})$$



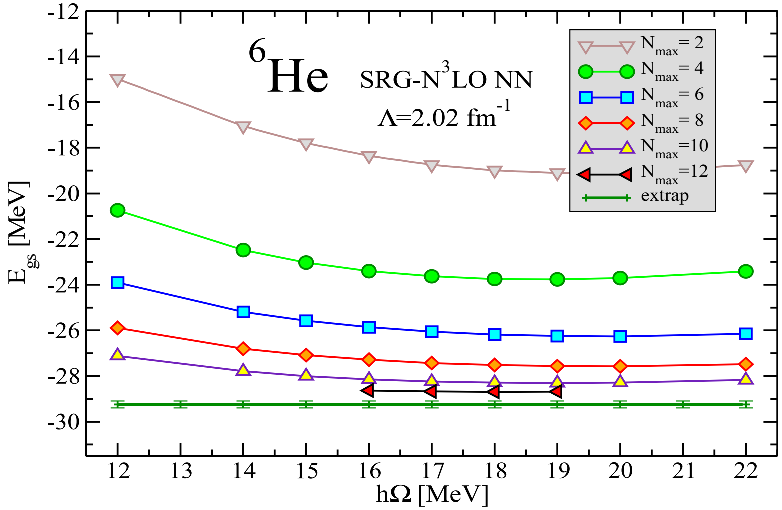
  - Large expansions in *A*-body harmonic oscillator (HO) basis
  - Preserves: 1) Pauli principle, and 2) translational invariance
  - Can include NNN interactions
  - ^{4}He binding energy close to experiment: 28.22 MeV (expt.: 28.3 MeV)

- Fully antisymmetric channel states: $\hat{A}_{\nu_3} = \sqrt{\frac{(A-2)!2!}{A!}}\left[1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right]\frac{1-\hat{P}_{A-1,A}}{\sqrt{2}}$

^{4}He+n+n

- Preliminary NCSM/RGM results
  - n+n+^{4}He(g.s.), N_{max} = 12, $h\Omega$ = 14 MeV
  - SRG-NN chiral with λ = 1.5 fm^{-1}
- Comparison with NCSM:
  - ~1 MeV difference in E_{gs} due to excitations of ^{4}He core, at present only included in the NCSM calculation
  - Contrary to NCSM, NCSM/RGM ^{4}He+n+n w.f. has the appropriate asymptotic behavior

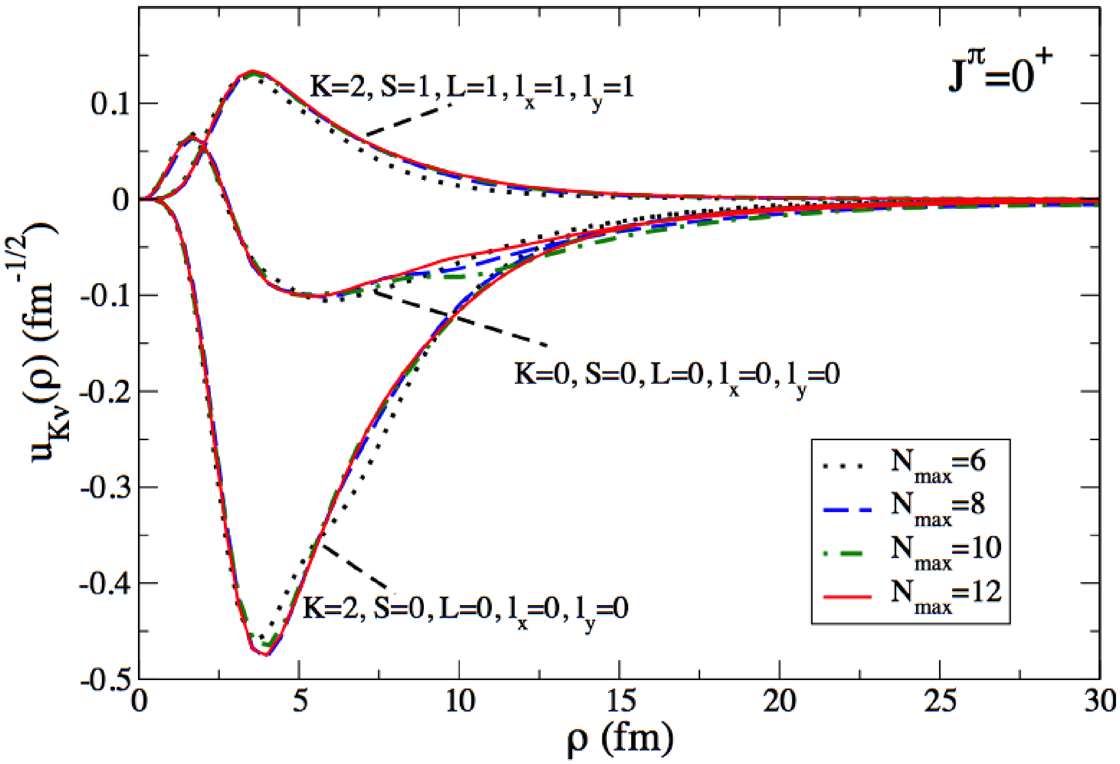
^{6}He ground state, NCSM



HO model space	Eg.s. (^{4}He) [MeV] (NCSM)	Eg.s. (^{6}He) [MeV] (NCSM)	Eg.s. (^{6}He) [MeV] (NCSM/RGM)
Nmax = 12	-28.22	-29.75	-28.70

^{4}He+n+n

^{6}He ground state wave function

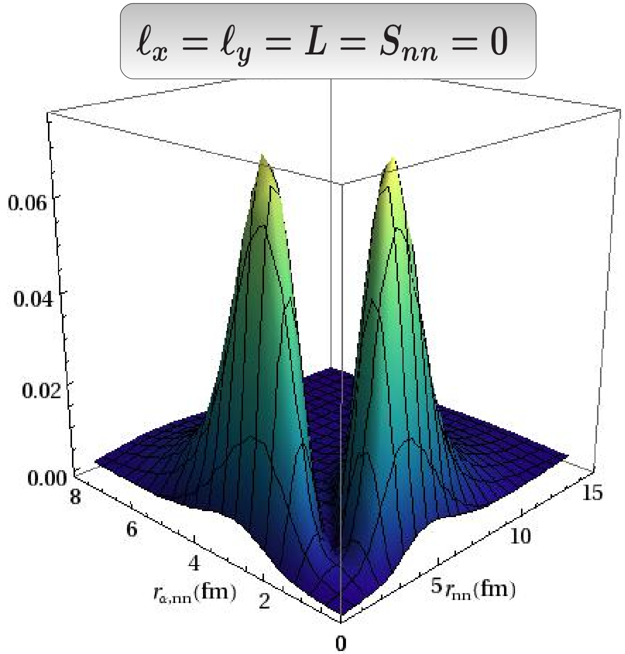


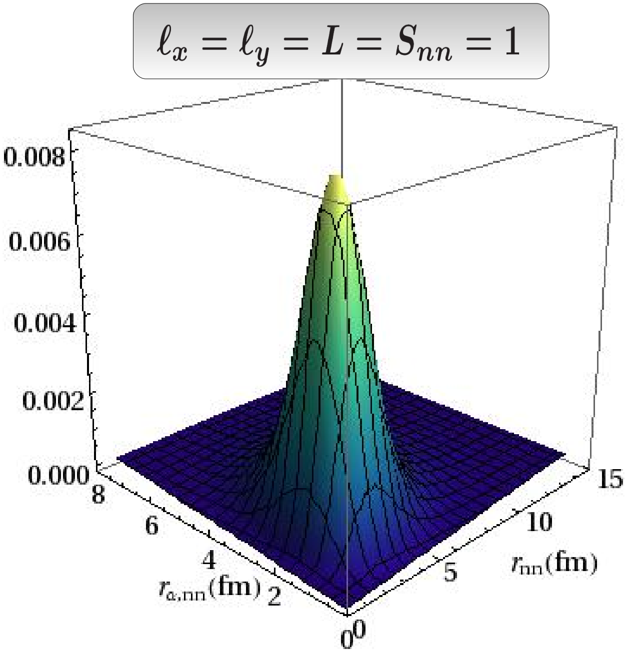
$$\chi_\nu^{J^\pi T}(x,y) = \sum_k C_{k\nu}(\rho)\phi_k^{\ell_x \ell_y}(\alpha)$$

^{4}He+n+n

^{6}He ground state wave function

Probability distribution





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^{4}He+n+n

^{6}He ground state wave function

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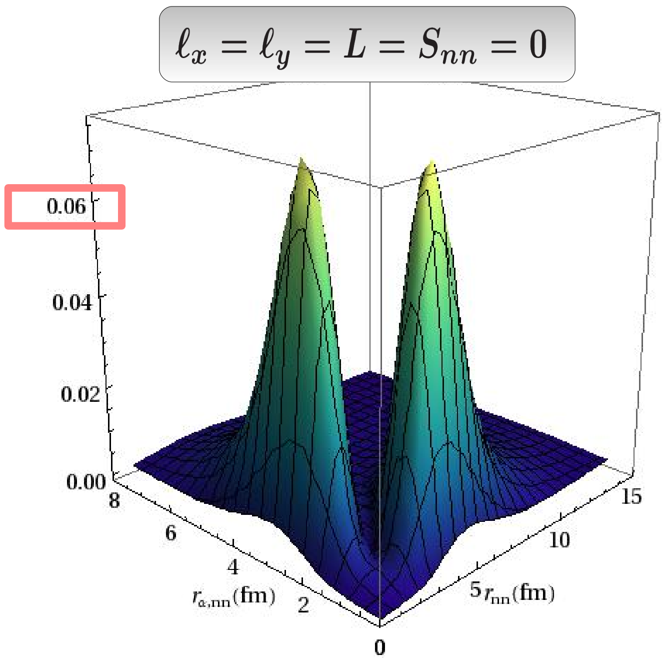

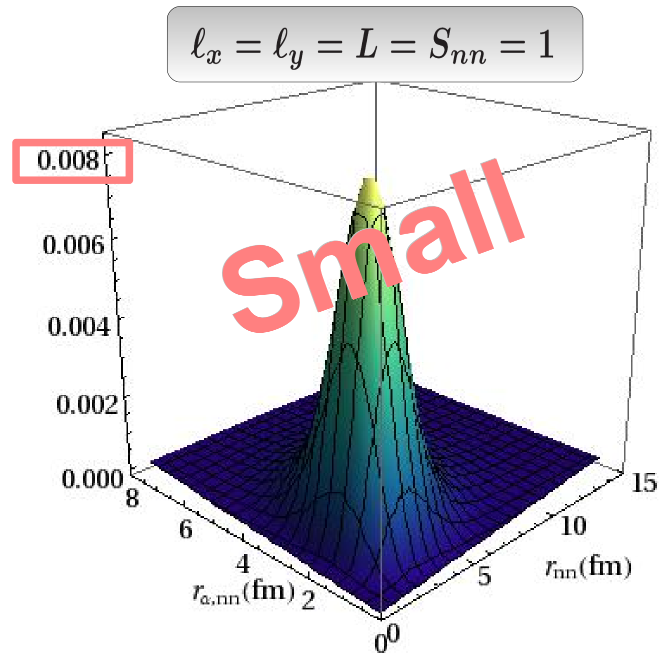

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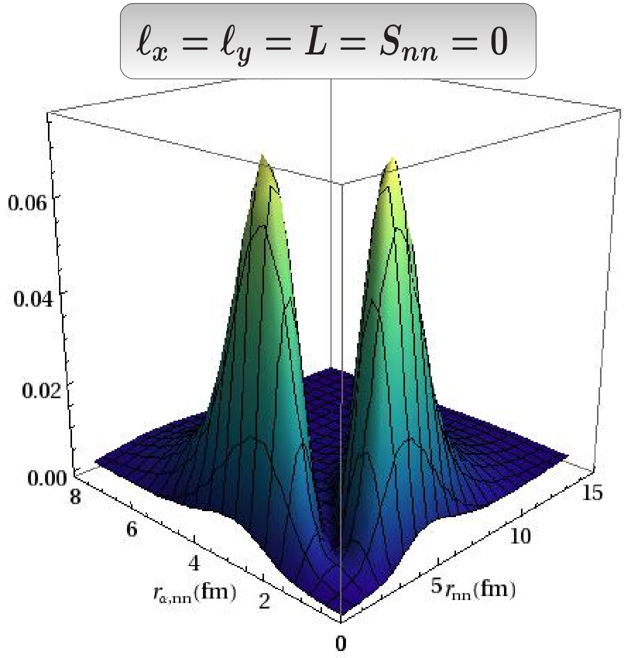
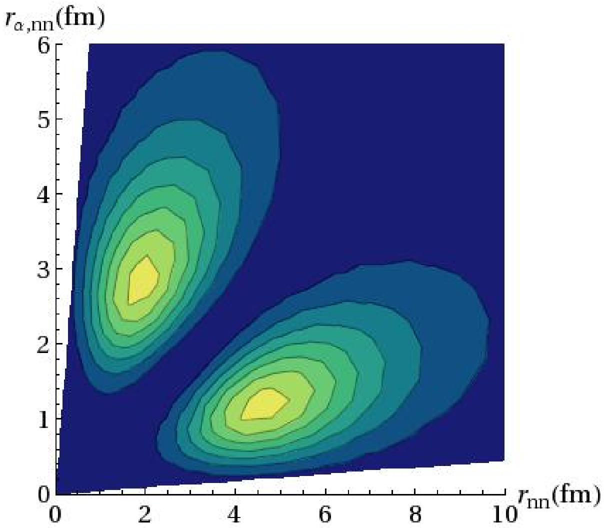
^{4}He+n+n

^{6}He ground state wave function

Probability distribution





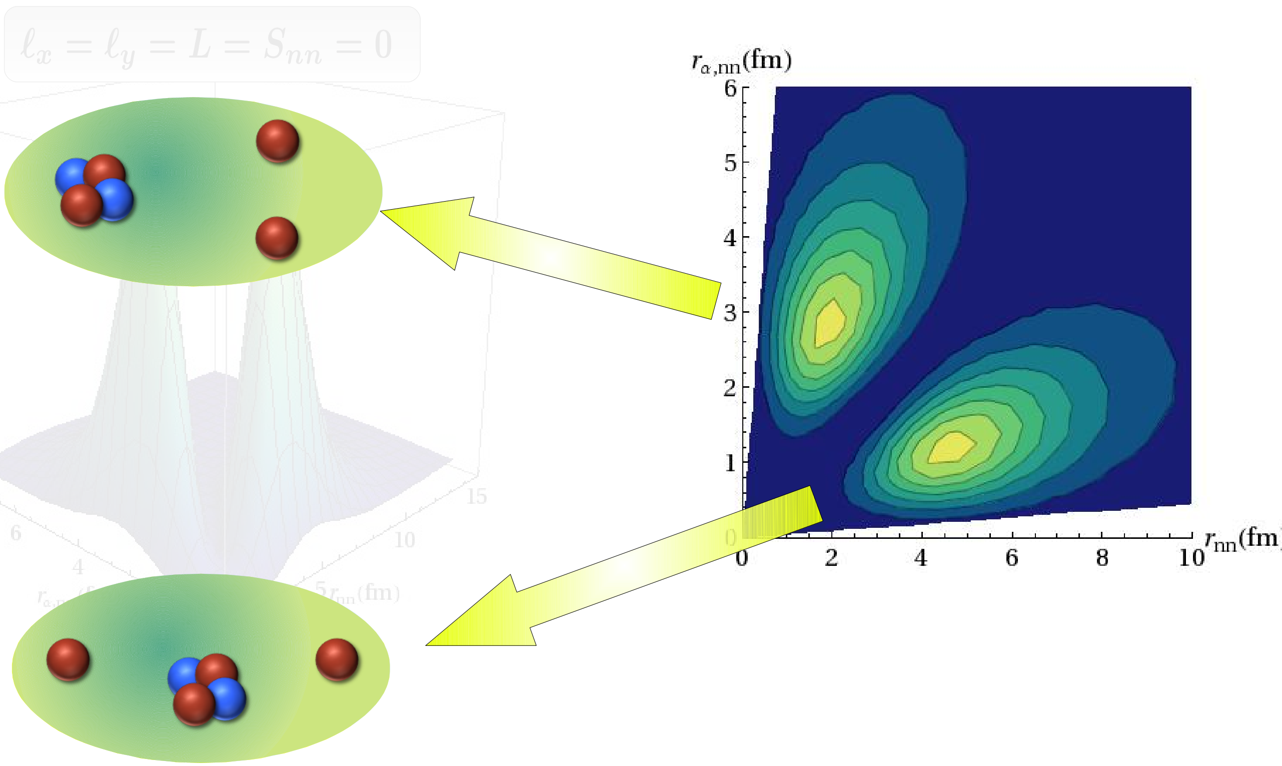
^{4}He+n+n

^{6}He ground state wave function

Probability distribution

Di-neutron

Cigar



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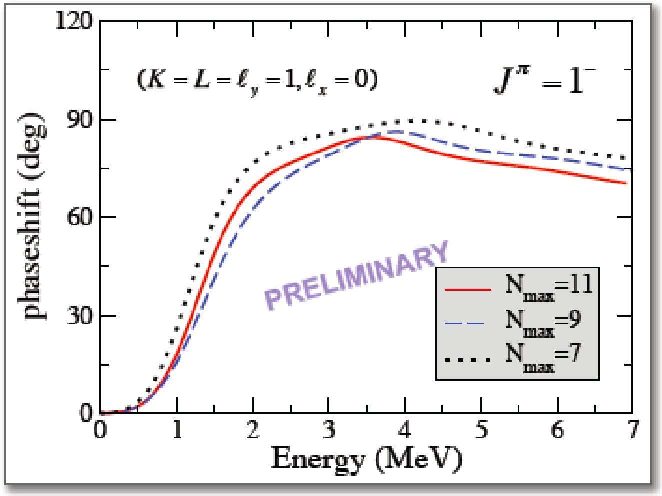
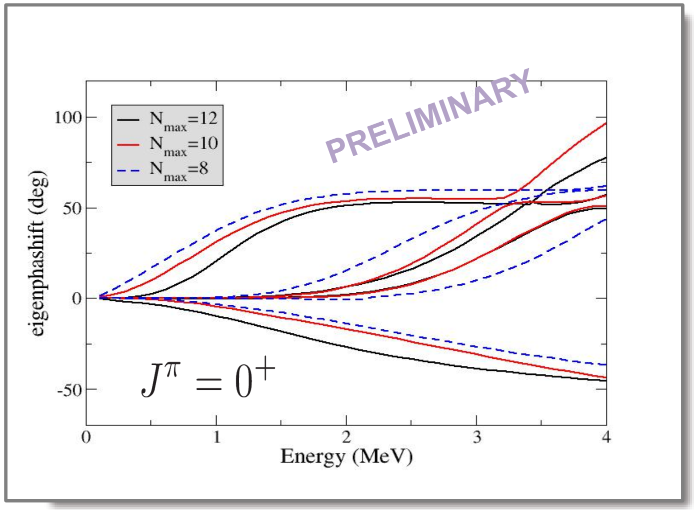
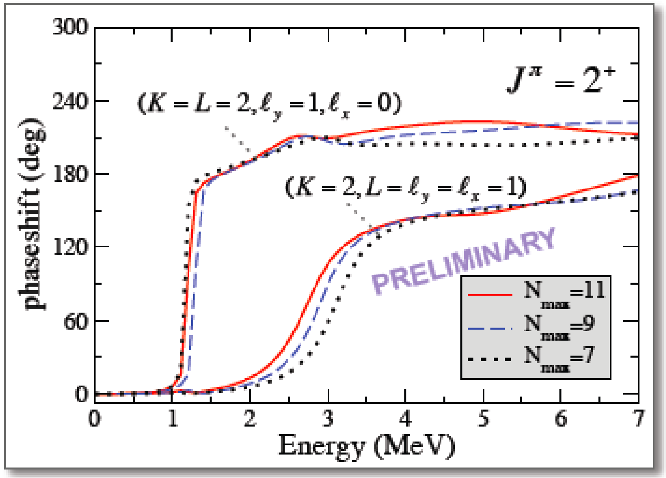
  - SRG-NN chiral with λ = 1.5 fm^{-1}
  - Matching at ρ_o = 30 fm (no propagation)
- Convergence of 1$^-$ and 2$^-$ is reasonable



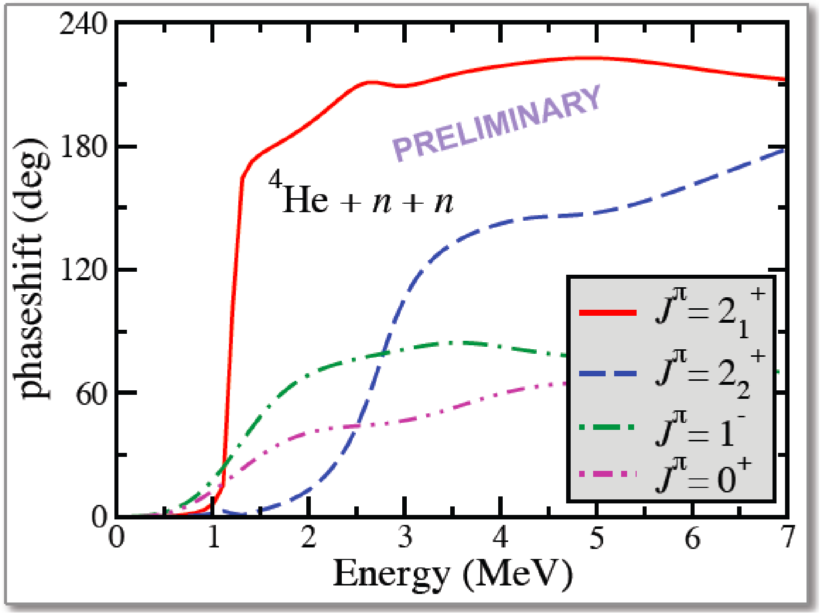


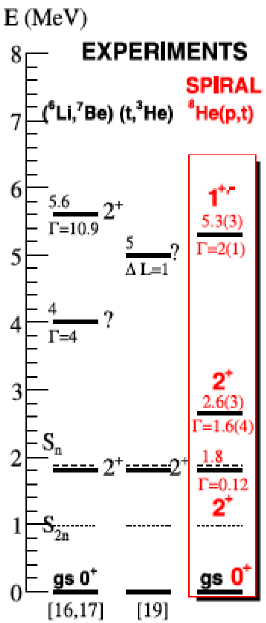


$^4He+n+n$

Preliminary results

Recent exp.: Phys. Lett. B 718 (2012) 441





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$$\Gamma = \frac{2}{\frac{d\delta(E)}{dE}}\Bigg|_{E=E_R}$$

Summary and Outlook

Approach is very useful for studying different types of nuclear systems

Bound and resonant states in structure problems

Continuum states for reaction problems

TRIUMF

Summary and Outlook

Approach is very useful for studying different types of nuclear systems

Bound and resonant states in structure problems

Continuum states for reaction problems

Results are very promising

Ground state of ^{6}He

Continuum ^{4}He+n+n

Work to do

Study more deeply the stability of the continuum results

Introduce ^4He core excitations by coupling the ^4He-n-n basis to ^6He NCSM eigenstates

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Transfer reactions, i.e, $^3H(^3H,2n)^4He$

Derive and calculate couplings between two and three body clusters

TRIUMF: Alberta | British Columbia | Calgary | Carleton | Guelph | Manitoba | McMaster | Montréal | Northern British Columbia | Queen's | Regina | Saint Mary's | Simon Fraser | Toronto | Victoria | Winnipeg | York

Thank you!

NCSM/RGM-3B

$$\sum_{\nu} \int dx dy x^2 y^2 \left[\mathbb{H}_{\nu'\nu}(x,y,x',y') - E\delta\nu'\nu \frac{\delta(x'-x)}{x'x} \frac{\delta(y'-y)}{y'y} \right] \chi_{\nu}^{J^\pi T}(x,y) = 0$$

Hyperspherical coordinates: $\rho = \sqrt{x^2+y^2}, \quad \alpha = \arctan(x/y)$

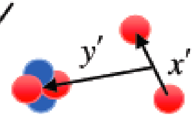
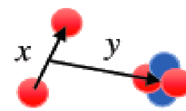
$$\bar{\mathcal{H}}_{\nu'\nu}^{k'k}(\rho',\rho) = \int d\alpha d\alpha' \sin^2\alpha \cos^2\alpha \sin^2\alpha' \cos^2\alpha' \phi_k^{\ell_x \ell_y}(\alpha) \mathbb{H}_{\nu'\nu}(x,y,x',y') \phi_{k'}^{\ell'_x \ell'_y}(\alpha')$$

After changing to hyperspherical coordinates and integrating in α,α':

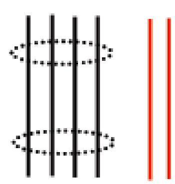
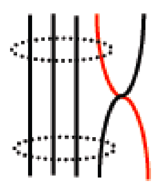
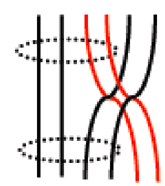
$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}_{\nu'\nu}^{k'k}(\rho',\rho) - E \frac{\delta(\rho-\rho')}{\rho^5} \delta_{\nu'\nu} \delta_{k'k} \right] \frac{C_{k\nu}^{J^\pi T}(\rho)}{\rho^{5/2}} = 0$$

Coupled-channel microscopic R-matrix method on a Lagrange mesh

Norm Kernel

$$\Big\langle$$  $$\Bigg|\left(1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right)\left(1-\hat{P}_{A-1,A}\right)\Bigg|$$  $$\Big\rangle$$

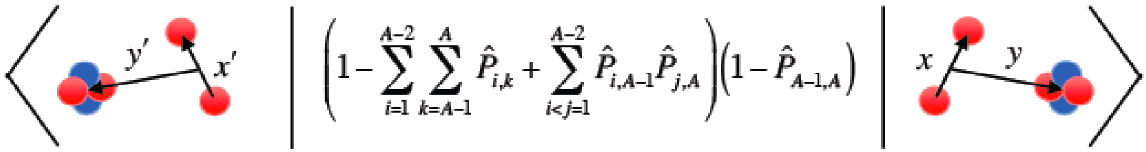
$$N_{\nu_3'\nu_3}(x',y',x,y)=\frac{1}{2}\left[1-(-1)^{\ell_x'+S_{23}'+T_{23}'}\right]\left[1-(-1)^{\ell_x+S_{23}+T_{23}}\right]\times\Bigg\{\delta_{\nu_3'\nu_3}\frac{\delta(x'-x)}{x'x}\frac{\delta(y'-y)}{y'y}$$
$$-2(A-2)\sum_{n_x'n_y'}\sum_{n_xn_y}R_{n_x'\ell_x'}(x')R_{n_y'\ell_y'}(y')\left\langle\Phi_{\nu_3'n_x'n_y'}\right|P_{A-2,A}\left|\Phi_{\nu_3n_xn_y}\right\rangle R_{n_x\ell_x}(x)R_{n_y\ell_y}(y)$$
$$+\frac{(A-2)(A-3)}{2}\sum_{n_x'n_y'}\sum_{n_xn_y}R_{n_x'\ell_x'}(x')R_{n_y'\ell_y'}(y')\left\langle\Phi_{\nu_3'n_x'n_y'}\right|P_{A-3,A-1}P_{A-2,A}\left|\Phi_{\nu_3n_xn_y}\right\rangle R_{n_x\ell_x}(x)R_{n_y\ell_y}(y)\Bigg\}$$

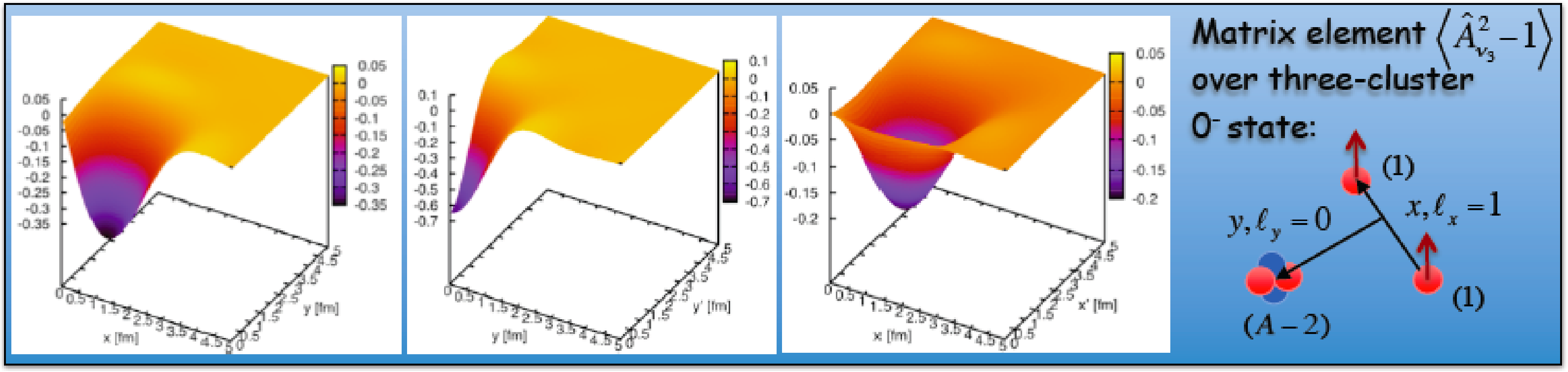
 **- 2(A-2) ×**  **+ (A-2)(A-3)/2 ×** 

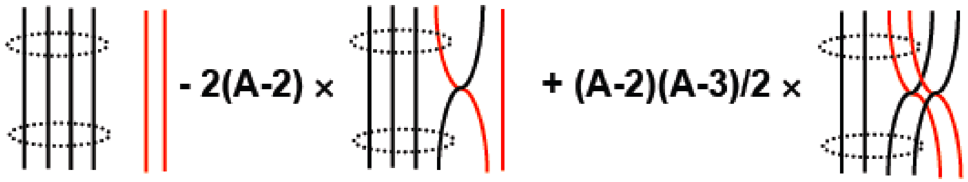
$${}_{SD}\left\langle\psi_{\mu_1}^{(A-2)}\right|a^+a\left|\psi_{\nu_1}^{(A-2)}\right\rangle_{SD}$$

$${}_{SD}\left\langle\psi_{\mu_1}^{(A-2)}\right|a^+a^+\,a\,a\left|\psi_{\nu_1}^{(A-2)}\right\rangle_{SD}$$

Norm Kernel



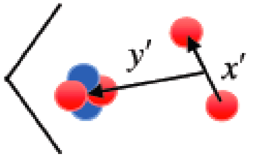




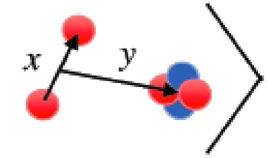
$${}_{SD}\left\langle \psi_{\mu_1}^{(A-2)} \middle| a^{+} a \middle| \psi_{\nu_1}^{(A-2)} \right\rangle_{SD}$$

$${}_{SD}\left\langle \psi_{\mu_1}^{(A-2)} \middle| a^{+} a^{+} a\, a \middle| \psi_{\nu_1}^{(A-2)} \right\rangle_{SD}$$

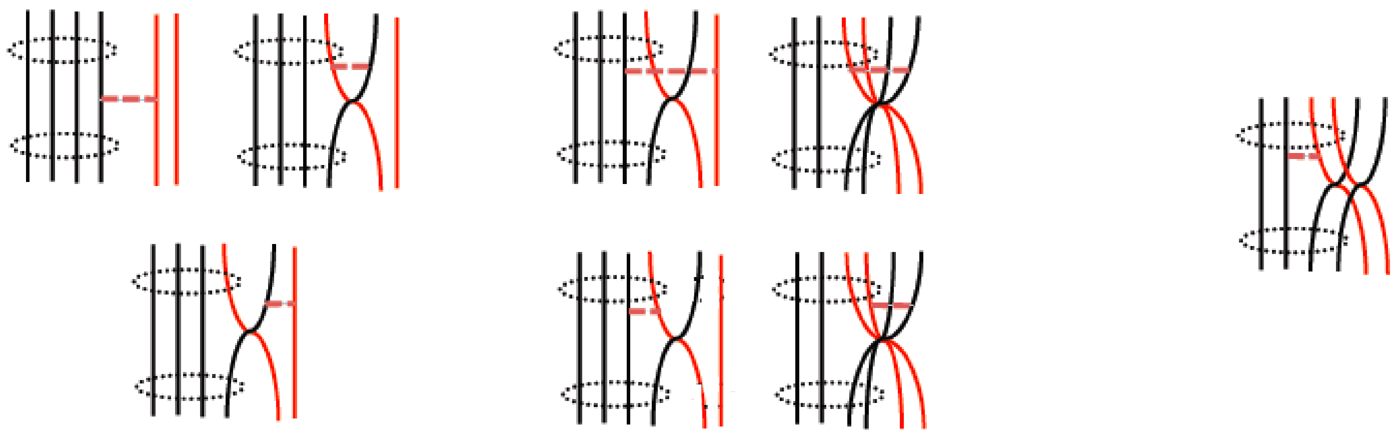
Hamiltonian Kernel



$$\left\langle \cdots \middle| \left(\sum_{l=1}^{A-2}\sum_{m=A-1}^{A} V_{lm} + V_{A-1A}\right)\left(1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k}+\sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right)\left(1-\hat{P}_{A-1,A}\right) \middle| \cdots \right\rangle$$



$$= V(x)N_{\nu_3'\nu_3}(x',y',x,y)+$$



$${}_{SD}\left\langle \psi_{\mu_1}^{(A-2)} \middle| a^+ a \middle| \psi_{\nu_1}^{(A-2)} \right\rangle_{SD}$$

$${}_{SD}\left\langle \psi_{\mu_1}^{(A-2)} \middle| a^+ a^+ a\, a \middle| \psi_{\nu_1}^{(A-2)} \right\rangle_{SD}$$

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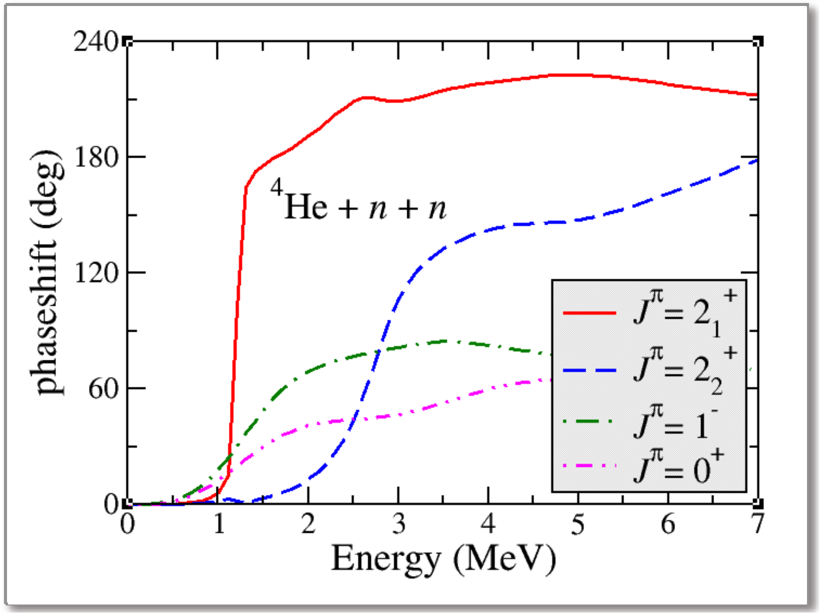
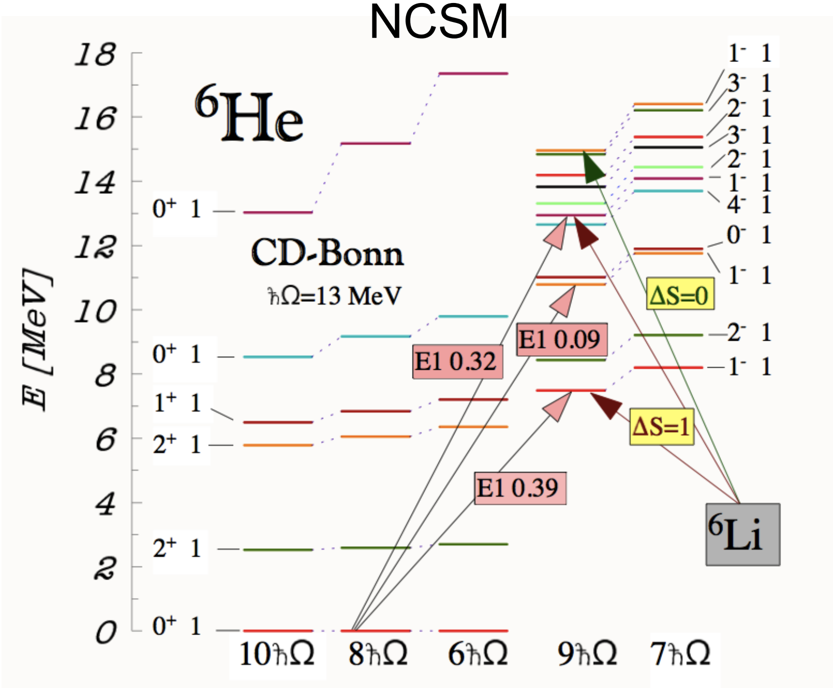
TRIUMF

^{4}He+n+n



NCSM



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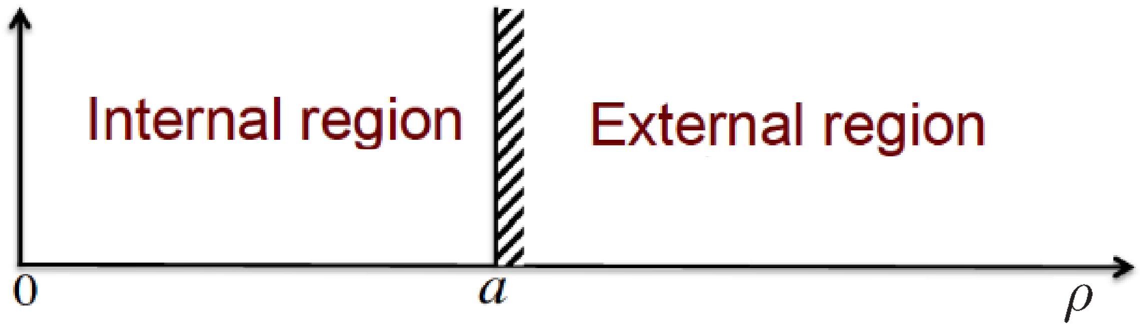
$$\Gamma = \left. \frac{2}{\frac{d\delta(E)}{dE}} \right|_{E=E_R}$$

R-matrix

Internal region: expansion on a basis ($\rho < a$)

$$C_{k\nu}(\rho) = \sum_i \beta_{k\nu i} f_i(\rho)$$



External region: known asymptotic behaviour ($\rho > a$)

* Bound state: $$C_{k\nu}(\rho) = A_{k\nu}\sqrt{\kappa\rho}K_{k+2}(\kappa\rho)$$

* Continuum state: $$C_{k\nu}(\rho) = A_{k\nu}\left[H_k^-(\kappa\rho)\delta_{\nu,\nu'}\delta_{k,k'} - S_{\nu k,\nu' k'}H_k^+(\kappa\rho)\right]$$

R-matrix

$$\sum_{\nu k} \int d\rho\rho^5 \left[\bar{\mathcal{H}}^{k'k}_{\nu'\nu}(\rho',\rho) - E\frac{\delta(\rho-\rho')}{\rho^5}\delta_{\nu'\nu}\delta_{k'k}\right] \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}} = 0$$

Matching conditions at ρ = a
Use of Bloch operator

$$\mathcal{L}_{k\nu} = \frac{\hbar^2}{2m}\delta(\rho-a)\frac{1}{\rho^{5/2}}\left(\frac{\partial}{\partial\rho} - \frac{L_{k\nu}}{\rho}\right)\rho^{5/2}$$ Bloch Operator

$$\sum_{\nu k} \int d\rho\rho^5 \left[\bar{\mathcal{H}}^{k'k}_{\nu'\nu}(\rho',\rho) + \mathcal{L}_{k\nu} - E\frac{\delta(\rho-\rho')}{\rho^5}\delta_{\nu'\nu}\delta_{k'k}\right] \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}} = \sum_{\nu k} \int d\rho\rho^5 \mathcal{L}_{k\nu} \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}}$$

R-matrix

$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}^{k' k}_{\nu' \nu}(\rho', \rho) - E \frac{\delta(\rho - \rho')}{\rho^5} \delta_{\nu' \nu} \delta_{k' k} \right] \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}} = 0$$

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Bloch Operator

$$\sum_{\nu k} \int d\rho \rho^5 \left[\bar{\mathcal{H}}^{k' k}_{\nu' \nu}(\rho', \rho) + \mathcal{L}_{k\nu} - E \frac{\delta(\rho - \rho')}{\rho^5} \delta_{\nu' \nu} \delta_{k' k} \right] \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}} = \sum_{\nu k} \int d\rho \rho^5 \mathcal{L}_{k\nu} \frac{C^{J^\pi T}_{k\nu}(\rho)}{\rho^{5/2}}$$

Projection on Lagrange basis element $f_{n'}(\rho)$

R-matrix

$$\mathcal{C}_{k\nu i,k'\nu' i'} = \left\langle f_i \middle| \bar{\mathcal{H}}^{k'k}_{\nu'\nu}(\rho',\rho) + \mathcal{L}_{k\nu} \middle| f_{i'} \right\rangle$$

Bound States

$$L_{k\nu}(E) = a\kappa \frac{C'^{ext}_{k\nu}(\kappa a)}{C^{ext}_{k\nu}(\kappa a)}$$

$$\mathcal{C}\beta = E\beta$$

Eigenvalue problem

Few iterations to converge

$$C_{k\nu}(\rho) = \sum_i \beta_{k\nu i} f_i(\rho)$$

R-matrix

$$\mathcal{C}_{k\nu i,k'\nu' i'} = \left\langle f_i \middle| \bar{\mathcal{H}}^{k'k}_{\nu'\nu}(\rho',\rho) + \mathcal{L}_{k\nu} \middle| f_{i'} \right\rangle$$

Continuum states $L_{k\nu} = 0$

$$\sum_{i'\nu'k} (\mathcal{C}_{k\nu i,k'\nu' i'} - E\delta_{i,i'}\delta_{\nu,\nu'}\delta_{k,k'})\beta_{k'\nu' i'} = \frac{A_{k\nu}\kappa\hbar}{2m}\left[H'^{-}_{k}(\kappa a)\delta_{\nu,\nu'}\delta_{k,k'} - S_{\nu k,\nu' k'} H'^{+}_{k}(\kappa a)\right]$$

R-matrix

$$\mathcal{C}_{k\nu i,k'\nu' i'} = \left\langle f_i \middle| \bar{\mathcal{H}}^{k'k}_{\nu'\nu}(\rho',\rho) + \mathcal{L}_{k\nu} \middle| f_{i'} \right\rangle$$

Continuum states

$$R_{k\nu,k'\nu'}(a) = \frac{\hbar}{2ma} \sum_{ii'} f_i(a) (\mathcal{C} - E\mathbb{I})^{-1}_{k\nu i,k'\nu' i'} f_{i'}(a)$$

$$Z_{k\nu,k'\nu'} = H^-_{k\nu}(\kappa a)\delta_{\nu,\nu'}\delta_{kk'} - \kappa a (H^-_{k\nu}(\kappa a))' R_{k\nu,k'\nu'}(a)$$

$$S = (Z^*)^{-1} Z$$

TRIUMF

n-n-^{4}He

Norm Kernel $\mathcal{N}_{\nu'\nu}(x, y, x', y')$