Properties of Baryons as Relativistic Three-Quark Systems

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Motivation for Resorting to Quark Models

To be able to describe/understand

- in a consistent manner
- on the microscopic level
- in accordance with the properties of low-energy QCD such phenomena like

- **hadron spectra**: ground states & excitations
- **hadron structure**: r_E, μ, g_A ; G_E, G_M, G_A, G_P, ...
  i.e. electroweak form factors etc.
- **resonance excitations**: $\gamma N \rightarrow N^*$, $e^- N \rightarrow N^*$, ...
- **resonance decays**:
  $\rho \rightarrow \pi\pi$, $\omega \rightarrow \pi\pi\pi$, $N^* \rightarrow N\pi$, $\Delta \rightarrow N\pi$, $\Lambda^* \rightarrow KN$, ...
- **meson-baryon interactions**: $\pi - N$, $K - N$, ...
- **hyperon-hyperon interactions**: $N - N$, $N - Y$, ...
  etc. etc.

Outline

Low-Energy QCD

Low-energy QCD of N_f flavors is characterized by:

- spontaneous breaking of chiral symmetry ($SB\chi S$):

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

$\to$ appearance of $(N_f^2 - 1)$ **Goldstone bosons** $\vec{\phi}$
$\to$ generation of quasiparticles with dynamical mass, i.e. **constituent quarks** ψ

- thus (effective) interaction Lagrangian:

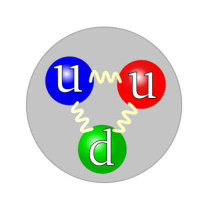
$$\mathcal{L}_{\text{int}} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^f \cdot \vec{\phi}\psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189
E.V. Shuryak: Phys. Rep. **115**, 151 (1984)
L.Ya. Glozman and D.O. Riska: Phys. Rep. **268**, 263 (1996)
see also:
S. Weinberg: Phys. Rev. Lett. **105**, 261601 (2010)

Baryons

Baryons are considered as colorless bound states of three constituent quarks.

Here the proton:



- 'Constituent' quarks are quasiparticles with dynamical mass, NOT the original QCD d.o.f. (i.e. 'current' quarks).
- 'Constituent' quarks are confined and interact via hyperfine interactions associated with $SB\chi S$, i.e. Goldstone-boson exchange.

Relativistic quantum mechanics (RQM)

i.e. **quantum theory** respecting **Poincaré invariance**

(theory on a Hilbert space $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M}\,|P,J,\Sigma\rangle = M\,|P,J,\Sigma\rangle \quad , \qquad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu\,|P,J,\Sigma\rangle = P^\mu\,|P,J,\Sigma\rangle \quad , \qquad \hat{P}^\mu = \hat{M}\hat{V}^\mu$$

Relativistic Constituent-Quark Model (RCQM)

Interacting mass operator

$$\begin{aligned}
\hat{M} &= \hat{M}_{free} + \hat{M}_{int} \\
\hat{M}_{free} &= \sqrt{\hat{H}^2_{free} - \hat{\vec{P}}^2_{free}} \\
\hat{M}^{rest\ frame}_{int} &= \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} [\hat{V}^{conf}_{ij} + \hat{V}^{hf}_{ij}]
\end{aligned}$$

fulfilling the **Poincaré algebra**

$$\begin{array}{lll}
[\hat{P}_i, \hat{P}_j] = 0, & [\hat{J}_i, \hat{H}] = 0, & [\hat{P}_i, \hat{H}] = 0, \\
[\hat{K}_i, \hat{H}] = -i\hat{P}_i & [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k & [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k, \\
[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}
\end{array}$$

$\hat{H}, \hat{P}_i$ ... time and space translations,
$\hat{J}_i$ ... rotations, $\hat{K}_i$ ... Lorentz boosts

Universal GBE RCQM

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

$$\Rightarrow \quad H_{free} = \sum_{i=1}^{3} \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn: Phys. Rev. D **58**, 094030 (1998)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

Universal GBE RCQM Parametrization

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V_\beta(\vec{r}_{ij}) = \frac{g_\beta^2}{4\pi}\frac{1}{12 m_i m_j}\left\{\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi\delta(\vec{r}_{ij})\right\}$$

$$= \frac{g_\beta^2}{4\pi}\frac{1}{12 m_i m_j}\left\{\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2 \frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}}\right\}$$

$$B = -402 \text{ MeV}, \quad C = 2.33 \text{ fm}^{-2}$$

$$\beta = 24: \quad \frac{g_{24}^2}{4\pi} = 0.7, \quad \mu_{24} = \mu_\pi = 139 \text{ MeV}, \quad \Lambda_{24} = 700.5 \text{ MeV}$$

$$\beta = 0: \quad \left(\frac{g_0}{g_{24}}\right)^2 = 1.5, \quad \mu_0 = \mu_{\eta'} = 958 \text{ MeV}, \quad \Lambda_0 = 1484 \text{ MeV}$$

$$m_u = m_d = 340 \text{ MeV}, \quad m_s = 480 \text{ MeV},$$
$$m_c = 1675 \text{ MeV}, \quad m_b = 5055 \text{ MeV}$$

Solution of Mass-Operator EV Problem

$$\hat{M}\,|P,J,\Sigma,F_{abc}\rangle = M\,|P,J,\Sigma,F_{abc}\rangle$$
$$= M\,|M,V,J,\Sigma,F_{abc}\rangle$$

$\rightarrow$ baryon wave functions (initially in rest frame)

$$\Psi_{PJ\Sigma F_{abc}}(\vec{\xi},\vec{\eta}) = \left\langle \vec{\xi},\vec{\eta}\,\middle|\,P,J,\Sigma,F_{abc}\right\rangle ,$$

where $\vec{\xi}$ and $\vec{\eta}$ are the usual Jacobi coordinates and

P	.....	momentum eigenvalues
$(M, V$	.....	mass resp. velocity eigenvalues)
J	.....	intrinsic spin $\hat{=}$ total angular momentum)
Σ	.....	z-component of J
F_{abc}	.....	flavor content

Advanced Few-Body Methods – 1

A) **Stochastic Variational Method** (SVM)

$$\Psi_{PJ\Sigma F_{abc}}(\mathbf{x}) = \sum_i c_i \left\{ e^{-\frac{1}{2}\tilde{\mathbf{x}}A\mathbf{x}} \left[\Theta_{LM_L}(\hat{\mathbf{x}})\chi_S\right]_{J\Sigma} \phi_{F_{abc}} \right\}_i$$

with linear and nonlinear variational parameters

$$c_i\,, \qquad A = \{\beta, \delta, \nu, n, \lambda, l, L, s, S, F_{abc}, d\}$$

searched by a generalized Rayleigh-Ritz principle through a stochastic selection of basis states

V.I. Kukulin and V.M. Krasnopol'sky: J. Phys. G **3**, 795 (1977)

Y. Suzuki and K. Varga: *Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems* (Springer, Berlin, 1998)

B) Modified **Faddeev Integral Equations**

$$\begin{aligned} H &= H_0 + v_\alpha + v_\beta + v_\gamma = \\ &\quad H_0 + v_\alpha^{\mathrm{conf}} + v_\beta^{\mathrm{conf}} + v_\gamma^{\mathrm{conf}} + \tilde{v}_\alpha + \tilde{v}_\beta + \tilde{v}_\gamma = \\ &\quad H^{\mathrm{conf}} + \tilde{v}_\alpha + \tilde{v}_\beta + \tilde{v}_\gamma \,, \\ \text{with} &\quad H^{\mathrm{conf}} = H_0 + v_\alpha^{\mathrm{conf}} + v_\beta^{\mathrm{conf}} + v_\gamma^{\mathrm{conf}} \end{aligned}$$

$$\begin{aligned} \Psi_{PJ\Sigma F_{abc}}(\mathbf{k}) &= \left(\tilde{\psi}_\alpha + \tilde{\psi}_\beta + \tilde{\psi}_\gamma\right)_{PJ\Sigma F_{abc}}(\mathbf{k}) \\ \tilde{\psi}_\alpha &= G_\alpha^{\mathrm{conf}}(E)\tilde{v}_\alpha\left(\tilde{\psi}_\beta + \tilde{\psi}_\gamma\right) \\ & G_\alpha^{\mathrm{conf}}(E) = \left(E - H^{\mathrm{conf}} - \tilde{v}_\alpha\right)^{-1} \end{aligned}$$

Z. Papp: Few-Body Syst. **26**, 99 (1999)

Z. Papp, A. Krassnigg, and W. Plessas: Phys. Rev. C **62**, 044004 (2000)

J. McEwen, J. Day, A. Gonzalez, Z. Papp, and W. Plessas: Few-Body Syst. **47**, 225 (2010)

Solution Accuracy

Baryon	J^P	Faddeev GBE	Faddeev OGE	SVM GBE	SVM OGE	Experiment
N(939)	$\frac{1}{2}^+$	939	940	939	939	938-940
N(1440)	$\frac{1}{2}^+$	1459	1578	1459	1577	1420-1470
N(1520)	$\frac{3}{2}^-$	1520	1521	1519	1521	1515-1525
N(1535)	$\frac{1}{2}^-$	1520	1521	1519	1521	1525-1545
N(1650)	$\frac{1}{2}^-$	1646	1686	1647	1690	1645-1670
N(1675)	$\frac{5}{2}^-$	1646	1686	1647	1690	1670-1680
Δ(1232)	$\frac{3}{2}^+$	1240	1229	1240	1231	1231-1233
Δ(1600)	$\frac{3}{2}^+$	1718	1852	1718	1854	1550-1700
Δ(1620)	$\frac{1}{2}^-$	1640	1618	1642	1621	1600-1660
Δ(1700)	$\frac{3}{2}^-$	1640	1618	1642	1621	1670-1750
Λ(1116)	$\frac{1}{2}^+$	1133	1127	1136	1113	1116
Λ(1405)	$\frac{1}{2}^-$	1561	1639	1556	1628	1401-1410
Λ(1520)	$\frac{3}{2}^-$	1561	1639	1556	1628	1519-1521
Λ(1600)	$\frac{1}{2}^+$	1607	1749	1625	1747	1560-1700
Λ(1670)	$\frac{1}{2}^-$	1672	1723	1682	1734	1660-1680
Λ(1690)	$\frac{3}{2}^-$	1672	1723	1682	1734	1685-1695

Z. Papp, A. Krassnigg, and W. Plessas: Phys. Rev. C **62**, 044004 (2000)

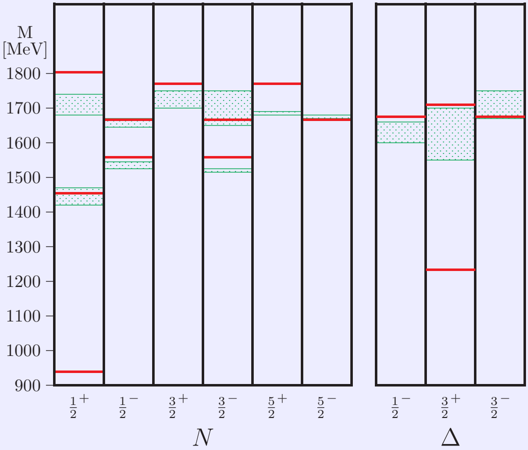
J.P. Day: PhD Thesis, Univ. Graz (2013)

Baryon **Excitation Spectra**

and

Mass-Operator **Eigenstates**

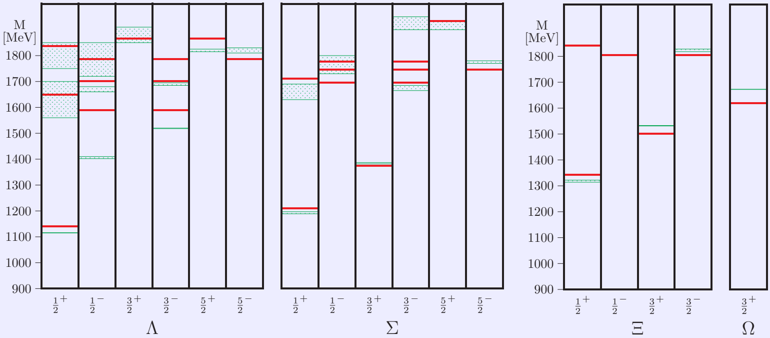
Light Baryon Spectra



red Universal GBE RCQM

green PDG 2013 (experiment)

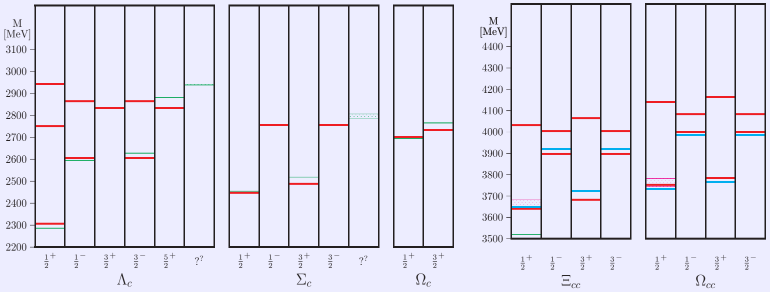
Strange Baryon Spectra



red Universal GBE RCQM

green PDG 2013 (experiment)

Charm Baryon Spectra



Left panel – single charm:

red Universal GBE RCQM prediction

green PDG 2013 (experiment)

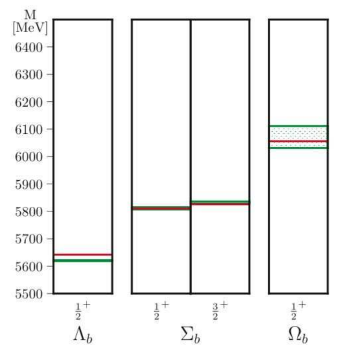
Right panel – double charm:

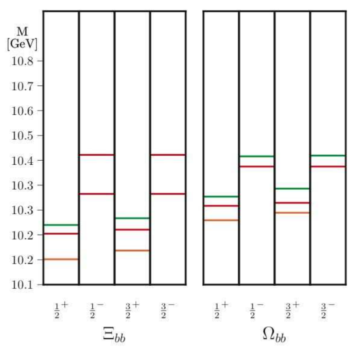
green M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

cyan S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

magenta L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

Bottom Baryon Spectra





Left panel – single bottom:

red Universal GBE RCQM prediction

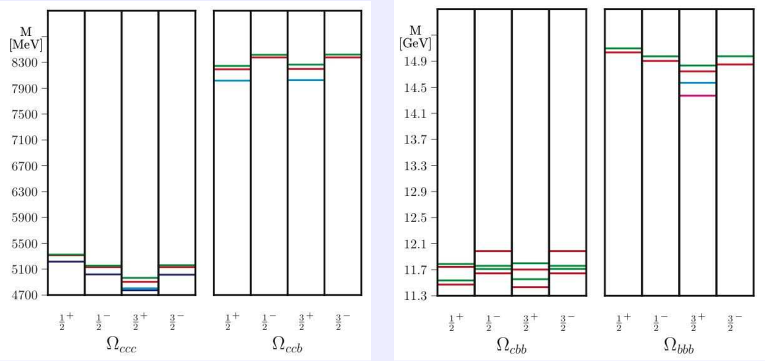
green PDG 2013 (experiment)

Right panel – single bottom:

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)

orange D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

Triple-Heavy Baryon Spectra



red Universal GBE RCQM

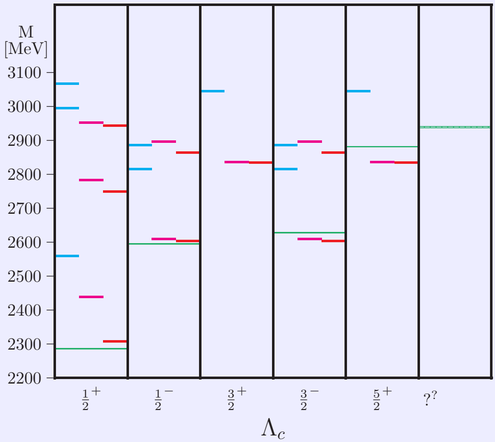
green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrelativistic one-gluon-exchange CQM)

blue S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

cyan A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)

magenta S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Influence of Light-Heavy Q-Q Interaction



leftmost cyan levels	confinement only
middle magenta levels	including only light-light GBE
rightmost red levels	including full GBE RCQM

Rest-Frame Baryon States

Mass operator eigenstates

$$\hat{M}\,|P,J,\Sigma,T,M_T\rangle = M\,|P,J,\Sigma,T,M_T\rangle$$

represented in configuration space

$$\left\langle \vec{\xi},\vec{\eta}\,\middle|\,P,J,\Sigma,T,M_T\right\rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi},\vec{\eta})$$

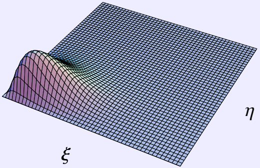
with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

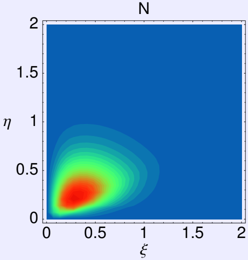
Picture the baryon wave functions through
spatial probability density distributions

$$\rho(\xi,\eta) = \xi^2\eta^2 \int d\Omega_\xi d\Omega_\eta \; \Psi^\star_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)$$

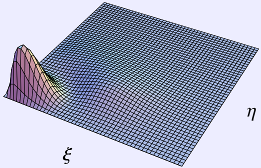
Pictures of Baryons (rest frame)

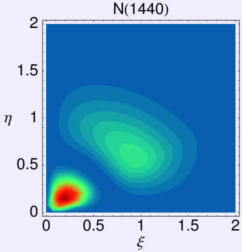
N GBE CQM





N(1440) GBE CQM

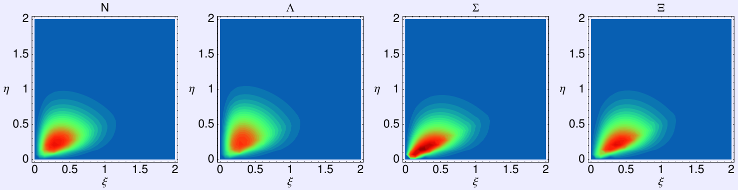




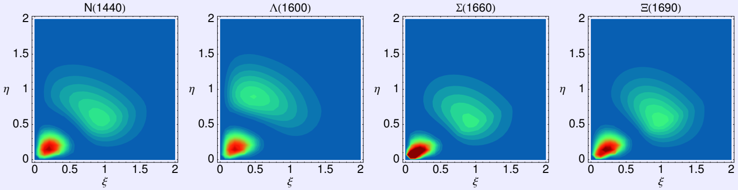
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi,\eta)$ for the $\frac{1}{2}^+$ **octet** baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:



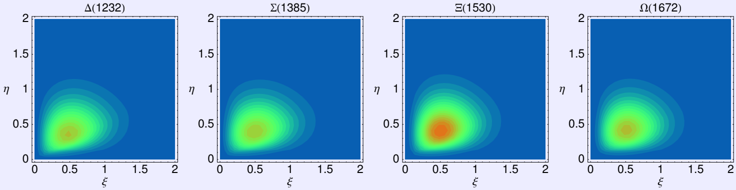
$\rho(\xi,\eta)$ for the $\frac{1}{2}^+$ **octet** baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:



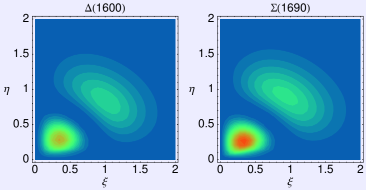
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi,\eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states **$\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, $\Omega(1672)$**:



$\rho(\xi,\eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states $\Delta(1600)$, $\Sigma(1690)$:



T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

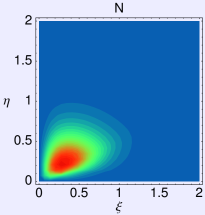
Root-Mean-Square Radii

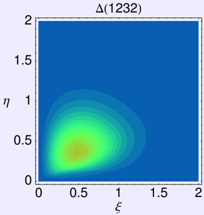
The **root-mean-square radius** (in the rest frame):

$$r_{\rm rms} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P=0, J, \Sigma \left| \hat{r}_i^2 \right| P=0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable**! Is NOT **relativistically invariant**!

$\rightarrow$ Idea about the spatial distribution of constituent quarks.





$r^N_{\rm rms}$=0.304 fm

r^p_E=0.905 fm, $(r^n_E)^2$=−0.128 fm^2

$r^\Delta_{\rm rms}$=0.390 fm

$r_E^{\Delta^{++}}$=$r_E^{\Delta^{+}}$=$r_E^{\Delta^{-}}$=0.656 fm, $r_E^{\Delta^{0}}$=0 fm

See: K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

New Quark-Model Classification

multiplet	$(LS)J^P$				
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(1690)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
octet	$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$	
octet	$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$	
octet	$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
octet	$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$	
octet	$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(1950)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$		
decuplet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$		
decuplet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$			
singlet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Lambda(1405)^{71}$			
singlet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Lambda(1520)^{71}$			
singlet	$(0\frac{1}{2})\frac{1}{2}^+$	$\Lambda(1810)^{92}$			

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)

See also the PDG: Phys. Rev. D **86**, 010001 (2012)

Various Baryon Reactions

Matrix elements of a transition operator $\hat{O}$ between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T_3', Y'|\hat{O}|P, J, \Sigma, T, T_3, Y\rangle$$

$\hat{O}$ ... $\hat{J}^\mu_{\mathrm{em}}$ $\rightarrow$ electromagnetic FF's

... $\hat{A}^\mu_{\mathrm{axial}}$ $\rightarrow$ axial FF's

... $\hat{S}$ $\rightarrow$ scalar FF

... $\hat{\Theta}^{\mu\nu}$ $\rightarrow$ gravitational/tensor FF's

... $\hat{D}^\mu_\lambda$ $\rightarrow$ hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3'; f_{i_1'}, f_{i_2'}, f_{i_3'}|\hat{O}|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3}\rangle$$

$\Uparrow$ boosted 3-body states $\Uparrow$ boosted 3-body states

Electromagnetic Nucleon Observables

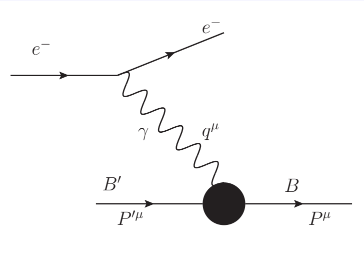
Covariant predictions for:

- **Electromagnetic** nucleon form factors
  $G_E^p(Q^2)$, $G_M^p(Q^2)$; $G_E^n(Q^2)$, $G_M^n(Q^2)$
- **Electric radii** and **magnetic moments**
  r_E^p, μ^p; r_E^n, μ^n

$\rightarrow$ Comparison to experiment

Electron Scattering and E.m. Form Factors

Elastic electron scattering:



Invariant form factors:

$$F^\nu_{\Sigma'\Sigma}(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}^\nu_{\rm em} | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2 \,; \qquad q^\mu = P^\mu - P'^\mu$$

Transition Matrix Elements in Point Form

Incoming baryon state: $|V, M, J, \Sigma\rangle \quad \hat{=} |P, J, \Sigma\rangle$

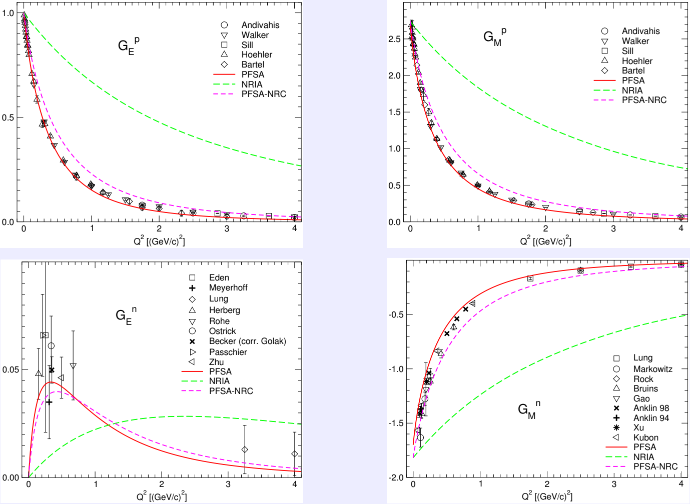
Outgoing baryon state: $|V', M', J', \Sigma'\rangle \quad \hat{=} |P', J', \Sigma'\rangle$

Transition operator: $\hat{O} = \hat{J}^{\mu}_{\rm em}$

$$
\begin{aligned}
&\langle V', M', J', \Sigma' | \hat{J}^{\mu}_{\rm em} | V, M, J, \Sigma\rangle = \\
&= \frac{2}{MM'} \sum_{\sigma_i \sigma_i'} \sum_{\mu_i \mu_i'} \int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}_2' d^3\vec{k}_3' \\
&\times \sqrt{\frac{\left(\sum_i \omega_i'\right)^3}{\prod_i 2\omega_i'}} \prod_{\sigma_i'} D^{\star \frac{1}{2}}_{\sigma_i' \mu_i'} \left\{R_W \left[k_i'; B\left(V'\right)\right]\right\} \Psi^{\star}_{M'J'\Sigma'} \left(\vec{k}_1', \vec{k}_2', \vec{k}_3'; \mu_1', \mu_2', \mu_3'\right) \\
&\times \langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' | \hat{J}^{\mu}_{rd} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle \\
&\times \sqrt{\frac{\left(\sum_i \omega_i\right)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i \mu_i} \left\{R_W \left[k_i; B\left(V\right)\right]\right\} \Psi_{MJ\Sigma} \left(\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3\right) \\
&\times 2MV_0 \delta^3 \left(M\vec{V} - M'\vec{V}' - \vec{q}\right)
\end{aligned}
$$

where $p_i = B_c(V)k_i$, $p_i' = B_c(V')k_i'$, and $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$

Covariant predictions of the GBE CQM:

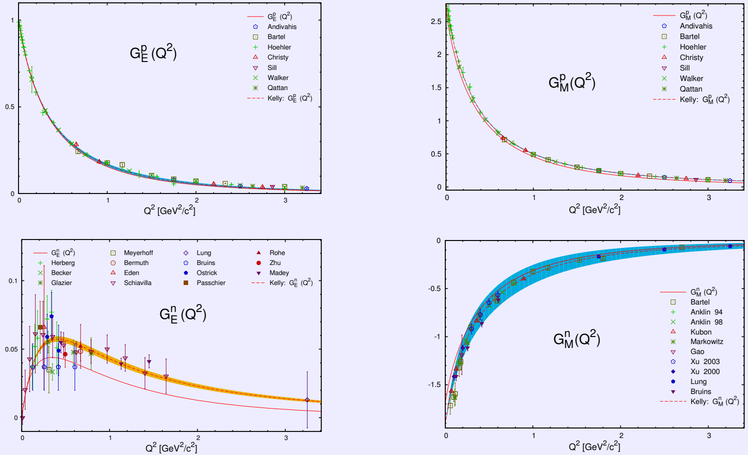


R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

Electromagnetic Nucleon Form Factors

Low-energy QCD
RCQM
Solution
Spectroscopy
Light, strange, charm, bottom
Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
Gravitational FF
Vertex FFs
πNN, $\pi N\Delta$
Summary

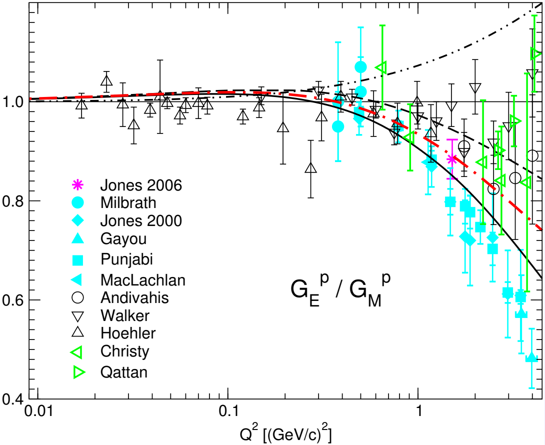
Covariant predictions of the GBE CQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

M. Rohrmoser: Diploma Thesis, Univ. of Graz, 2013

Proton Electric/Magnetic Form Factor Ratio



solid: GBE RCQM PFSM   dash-double-dot: GBE RCQM IFSM

T. Melde, K. Berger, L. Canton, W. Plessas, and R. F. Wagenbrunn: Phys. Rev. D **76**, 074020 (2007)

Nucleon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123[1)]
		0.70870 ± 0.00113[2)]
n	-0.13	-0.1161 ± 0.0022

[1)] CODATA value (PDG)

[2)] Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleon r_E^2 and μ – Nonrelativistic !!!

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	GBE NRIA	Experiment
p	0.82	0.10	0.7692 ± 0.0123 [1]
			0.70870 ± 0.00113 [2]
n	-0.13	-0.01	-0.1161 ± 0.0022

[1] CODATA value (PDG)   [2] Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

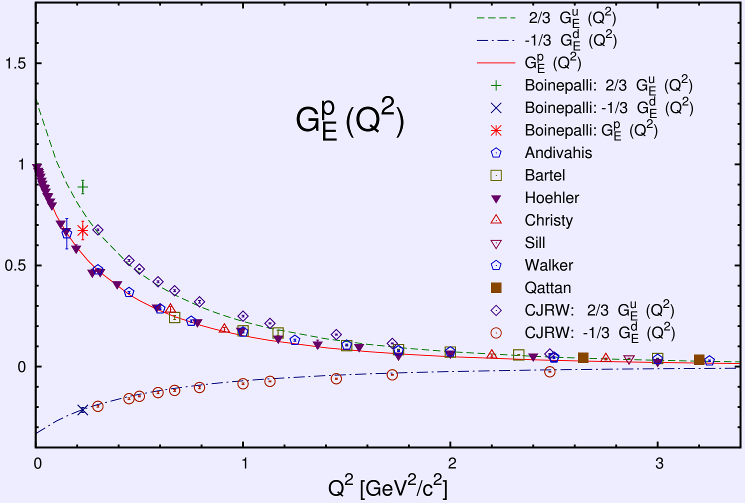
Baryon	GBE PFSM	GBE NRIA	Experiment
p	2.70	2.74	2.792847356
n	-1.70	-1.82	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleons N

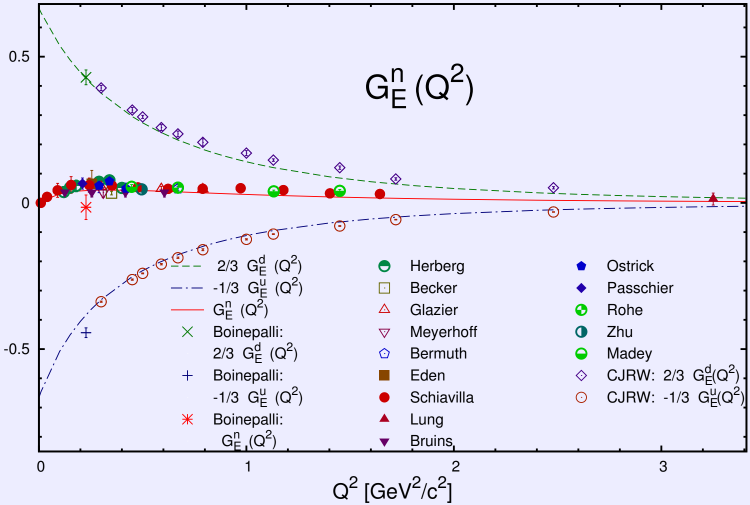
Proton Electric Form Factor

$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

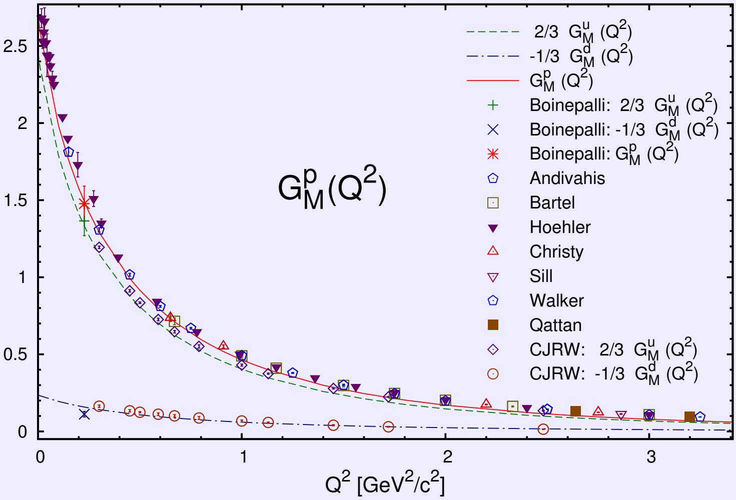
$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Proton Magnetic Form Factor

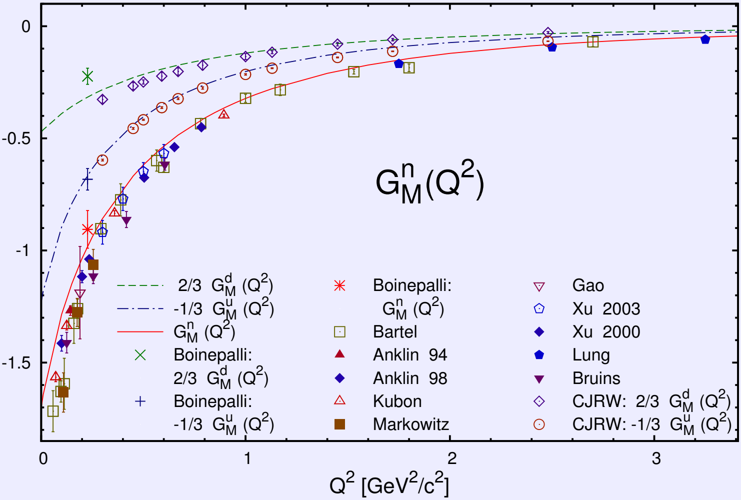
$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

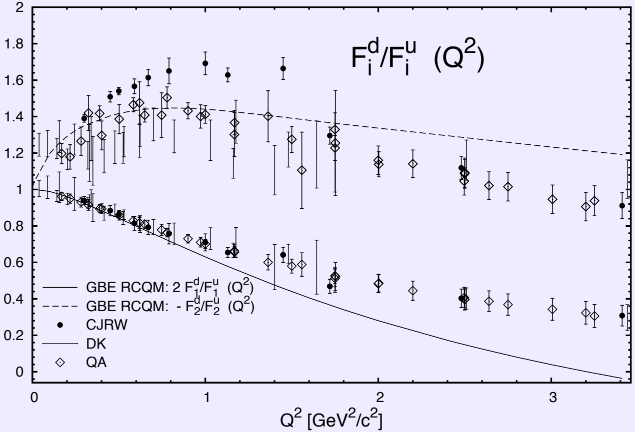
Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Ratios F_i^d / F_i^u of Flavor Contr. to F_1 and F_2



GBE RCQM prediction: M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Phenomenology:
- ● G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)
- | M. Diehl and P. Kroll: Europ. Phys. J. A **73**, 2397 (2013)
- ◇ I.A. Qattan and J. Arrington: Phys. Rev. C **86**, 065210 (2012)

Conclusions from Nucleon Flavor Analysis

- **Flavor analysis of nucleon e.m. form factors** in a relativistically invariant framework (point form).
- The **GBE RCQM** predicts flavor contributions in reasonable agreement with **experimental data**.
- The GBE RCQM relies on $\{QQQ\}$ degrees of freedom only; no explicit $\{QQQQ\bar{Q}\}$ etc.
- No explicit **meson-cloud effects** are included.
- No **strangeness content** in the nucleon for the low momentum transfers considered here.
- With respect to F_2^d/F_2^u three different phenomenological analyses give **distinct answers**.
- Details:

  M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

  W. Plessas: Mod. Phys. Lett. A **28**, 136022 (2013)

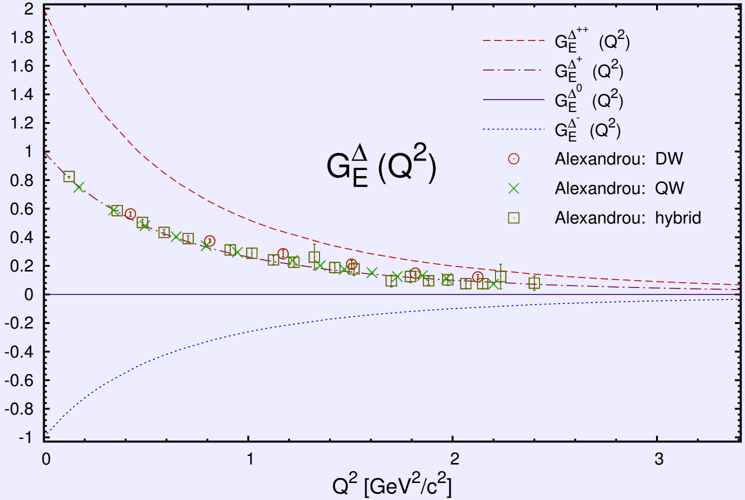
Δ and Hyperon E.m. Form Factors

$$\Delta$$

$$\Lambda, \Sigma, \Xi$$

$$\Sigma^*, \Xi^*, \Omega$$

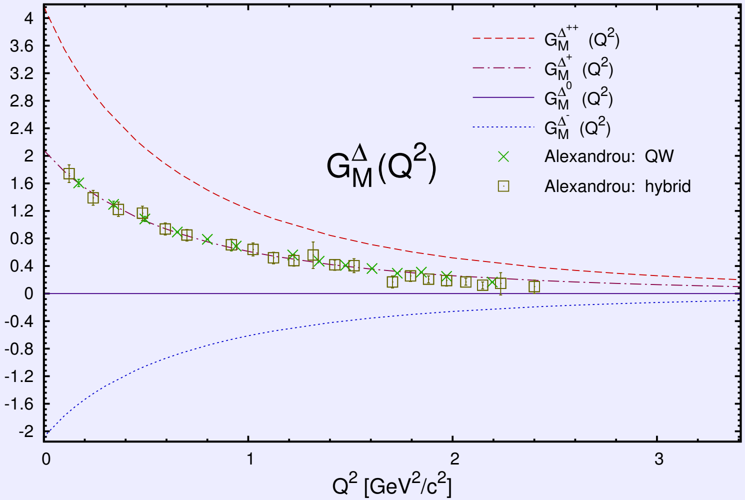
Electric Δ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

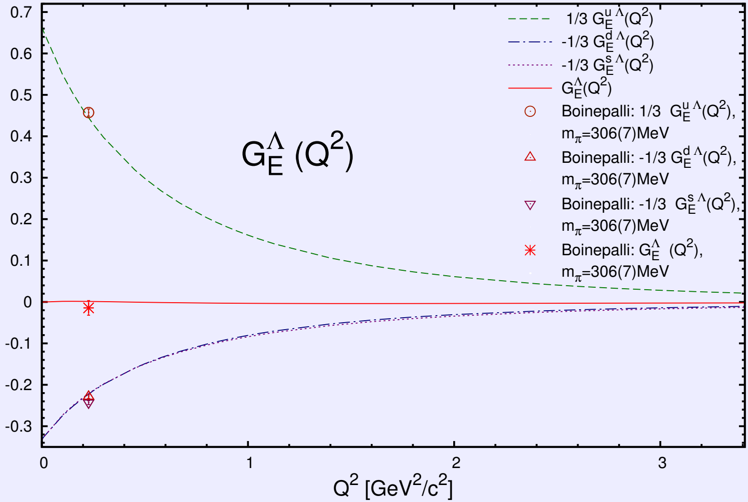
Magnetic Δ Form Factors



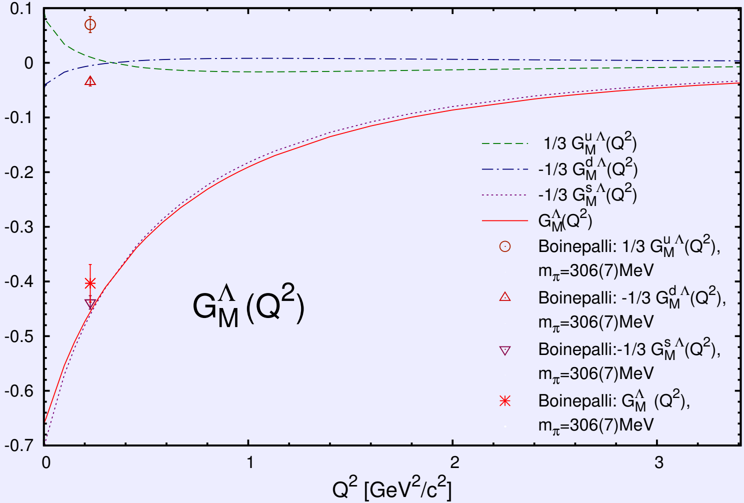
GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

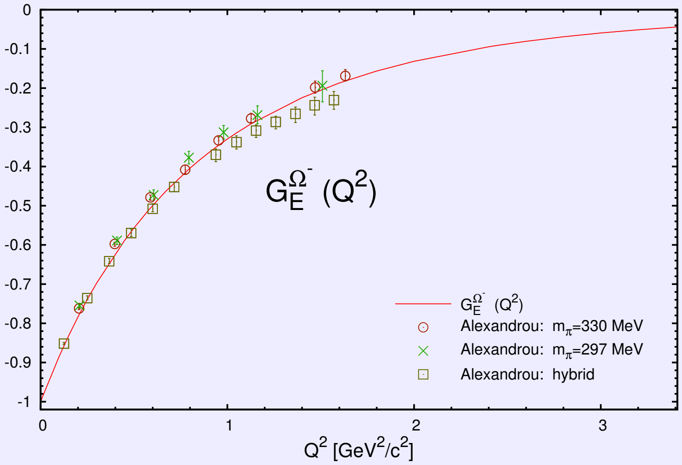
Octet $\Lambda(uds)$ Electric Form Factor



Octet $\Lambda(uds)$ Magnetic Form Factor

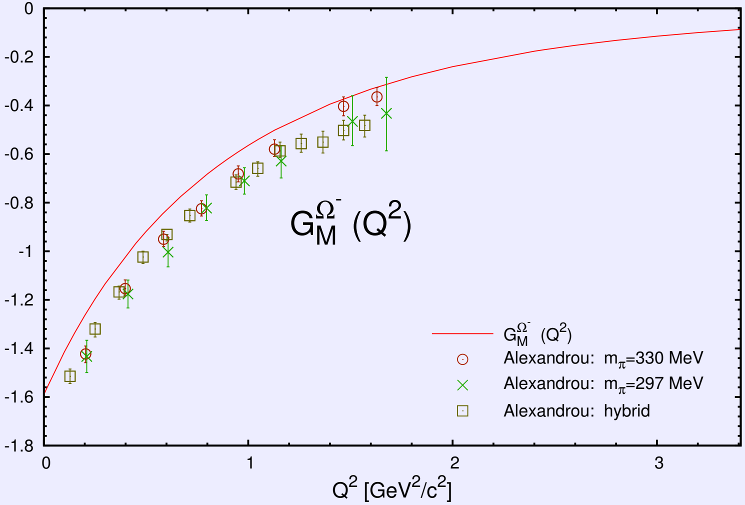


Decuplet $\Omega^-(sss)$ Electric Form Factor



Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Decuplet $\Omega^-(sss)$ Magnetic Form Factor

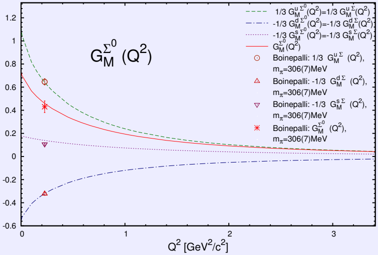


Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Octet $\Sigma^0(dds)$ vs. Decuplet $\Sigma^{*0}(dds)$

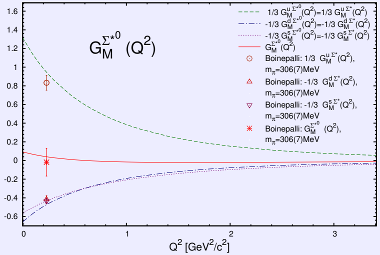
Octet

$$G_M^{\Sigma^0} = \frac{1}{3} G_M^{u,\Sigma} - \frac{1}{3} G_M^{d,\Sigma} - \frac{1}{3} G_M^{s,\Sigma}$$



Decuplet

$$G_M^{\Sigma^{*0}} = \frac{1}{3} G_M^{u,\Sigma^*} - \frac{1}{3} G_M^{d,\Sigma^*} - \frac{1}{3} G_M^{s,\Sigma^*}$$



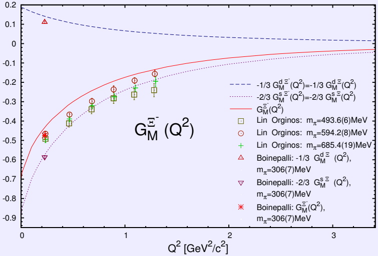
Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

S. Boinepalli et al.: Phys. Rev. D **80**, 054505 (2009)

Octet $\Xi^-(dss)$ vs. Decuplet Octet $\Xi^{*-}(dss)$

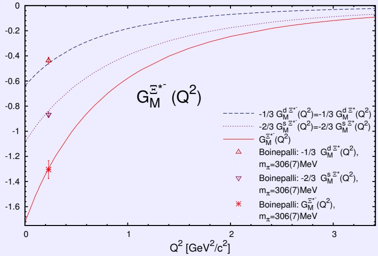
Octet

$$G_M^{\Xi^-} = -\frac{1}{3}G_M^{d,\Xi} - \frac{2}{3}G_M^{s,\Xi}$$



Decuplet

$$G_M^{\Xi^{*-}} = -\frac{1}{3}G_M^{d,\Xi^*} - \frac{2}{3}G_M^{s,\Xi^*}$$



Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

S. Boinepalli et al.: Phys. Rev. D **80**, 054505 (2009)

Baryon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123
n	-0.13	-0.1161 ± 0.0022
Σ^-	0.72	$0.61 \pm 0.12 \pm 0.09$

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427
Λ	-0.64	-0.613 ± 0.004
Σ^+	2.38	2.458 ± 0.010
Σ^-	-0.93	-1.160 ± 0.025
Ξ^0	-1.25	-1.250 ± 0.014
Ξ^-	-0.70	-0.6507 ± 0.0025
Δ^+	2.08	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$
Δ^{++}	4.17	$3.7 - 7.5$
Ω^-	-1.59	-2.020 ± 0.05

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Axial **Charges** and Axial **Form Factors**

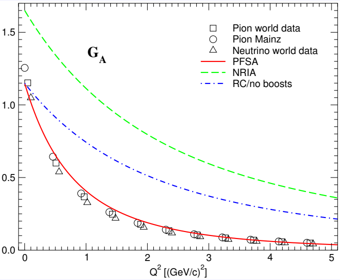
of

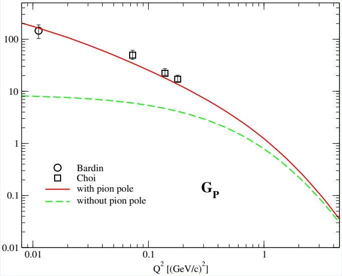
N Ground State and N^* Resonances

as well as

$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

Covariant predictions of the GBE RCQM:





$$g_A^{GBE} = 1.15 \quad \text{vs.} \quad g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Axial Charges of N and N^* Resonances

State	J^P	EGBE	Lattice QCD	GN	NR
N(939)	$\frac{1}{2}^+$	1.15	1.23∼1.26	1.66	1.65
N(1440)	$\frac{1}{2}^+$	1.16	?	1.66	1.61
N(1535)	$\frac{1}{2}^-$	0.02	∼0.00	-0.11	-0.20
N(1710)	$\frac{1}{2}^+$	0.35	?	0.33	0.42
N(1650)	$\frac{1}{2}^-$	0.51	∼0.55	0.55	0.64

EGBE — **E**xtended **GBE** RCQM covariant result

Lattice — **Lattice QCD** calculations by LHPC Collaboration and Takahashi-Kunihiro (Kyoto)

GN — **G**lozman-**N**efediev $SU(6) \times O(3)$ nonrelativistic QM

NR — **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

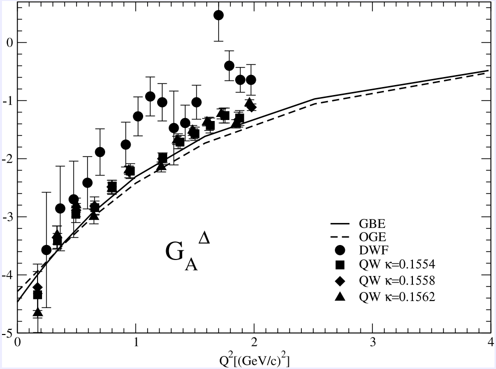
Axial Charges of N and N^* Resonances

		EGBE		psGBE		OGE	
State	J^P	Mass	g_A	Mass	g_A	Mass	g_A
N(939)	$\frac{1}{2}^+$	939	1.15	939	1.15	939	1.11
N(1520)	$\frac{3}{2}^-$	1524	-0.64	1519	-0.21	1520	-0.15
N(1440)	$\frac{1}{2}^+$	1464	1.16	1459	1.13	1578	1.10
N(1535)	$\frac{1}{2}^-$	1498	0.02	1519	0.09	1520	0.13
N(1680)	$\frac{5}{2}^+$	1689	0.89	1728	0.83	1858	0.70
N(1675)	$\frac{5}{2}^-$	1676	0.84	1647	0.83	1690	0.80
N(1710)	$\frac{1}{2}^+$	1757	0.35	1776	0.37	1860	0.32
N(1650)	$\frac{1}{2}^-$	1581	0.51	1647	0.46	1690	0.44
N(1720)	$\frac{3}{2}^+$	1746	0.35	1728	0.34	1858	0.25
N(1700)	$\frac{3}{2}^-$	1608	-0.10	1647	-0.50	1690	-0.47

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Axial Form Factor of the Δ

Covariant predictions of the GBE and OGE RCQMs:



Ki-Seok Choi: PhD Thesis, Graz, 2011

(Lattice QCD data from C. Alexandrou et al., PoS LATTICE2010, 141 (2010))

Axial Charges of $\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

	J^P	Exp	EGBE	LO	EOT	JT	NR
N	$\frac{1}{2}^+$	1.2695	1.15	1.18	1.314	1.18	1.65
Σ	$\frac{1}{2}^+$	-	0.65	0.636	0.686	0.73	0.93
Ξ	$\frac{1}{2}^+$	-	-0.21	-0.277	-0.299	-0.23	-0.32
Δ	$\frac{3}{2}^+$	-	-4.48	-	-	$\sim$-4.5	-6.00
Σ^*	$\frac{3}{2}^+$	-	-1.06	-	-	-	-1.41
Ξ^*	$\frac{3}{2}^+$	-	-0.75	-	-	-	-1.00

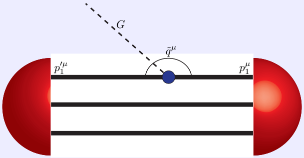
EGBE — **E**xtended **GBE** RCQM covariant result
LO — **L**in and **O**rginos lattice-QCD calculation
EOT — **E**rkol, **O**ka, and **T**akahashi lattice-QCD calculation
JT — **J**iang and **T**iburzi χPT calculation
NR — **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. D **82**, 014007 (2010)

Gravitational Form Factors

of

the Nucleon

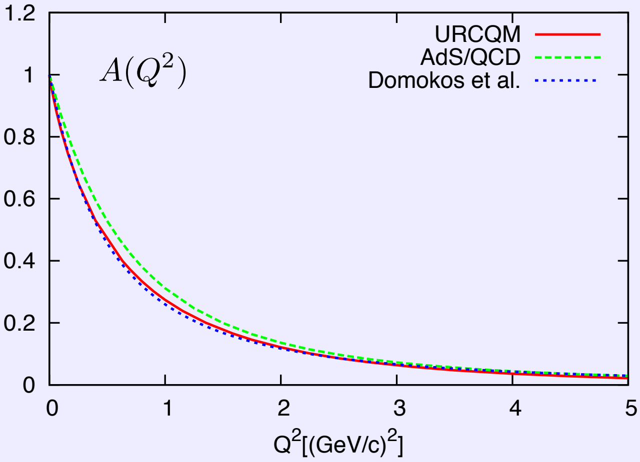


Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P'J\Sigma'|\hat{\Theta}^{\mu\nu}|PJ\Sigma\rangle = \bar{U}(P')\Big[\gamma^{(\mu}\bar{P}^{\nu)}A(Q^2) + \frac{i}{2M}\bar{P}^{(\mu}\sigma^{\nu)}B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M}C(Q^2)\Big]U(P)$$

$$A(Q^2) \sim \langle P'J\Sigma'|\Theta^{00}|PJ\Sigma\rangle$$

Nucleon Gravitational Form Factor $A(Q^2)$



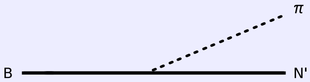
J.P. Day: PhD Thesis, Univ. Graz, August 2013

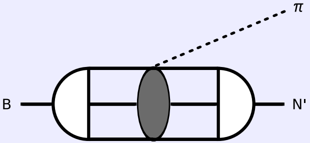
Microscopic Description

of

Meson-Baryon Interaction Vertices

Interaction vertices





$$F_{i\to f} = (2\pi)^4 \langle f|\, \mathcal{L}_I(0)\, |i\rangle \qquad \equiv \qquad \langle V', M', J', \Sigma'|\, \hat{D}^{\pi}_{\rm rd}\, |V, M, J, \Sigma\rangle$$

where

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \left| \hat{D}^{\pi}_{\rm rd} \right| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle =$$

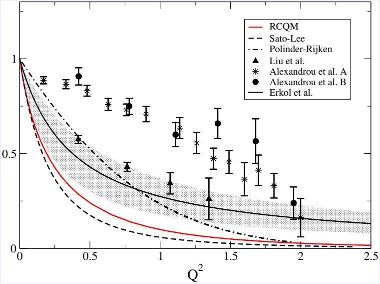
$$3\mathcal{N}_{\rm S} \frac{i g_{qqm}}{2m_1\,(2\pi)^{\frac{3}{2}}} \bar{u}(p_1', \sigma_1')\gamma_5\gamma_\mu\lambda_m u(p_1, \sigma_1)\tilde{q}^\mu 2p_{20}\delta(\vec{p}_2 - \vec{p}_2{}')2p_{30}\delta(\vec{p}_3 - \vec{p}_3{}')\delta_{\sigma_2\sigma_2'}\delta_{\sigma_3\sigma_3'}$$

and

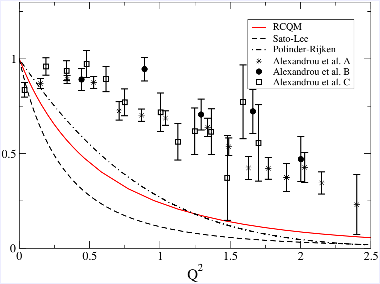
$$G_{\pi NN}\left(Q^2\right) = \frac{1}{f_{\pi NN}} \frac{m_\pi\sqrt{2\pi}}{\sqrt{2M_N}} \frac{\sqrt{E_N' + M_N'}}{E_N' + M_N' + \omega} \frac{F_{i\to f}}{Q_z}$$

$$G_{\pi N\Delta}\left(Q^2\right) = -\frac{1}{f_{\pi N\Delta}} \frac{3\sqrt{2\pi}}{2} \frac{m_\pi}{\sqrt{E_N' + M_N'}\sqrt{2M_\Delta}} \frac{F_{i\to f}}{Q_z}$$

πNN and $\pi N\Delta$ Interaction Vertices



$G_{\pi NN}$



$G_{\pi N\Delta}$

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Form-Factor Parametrizations

$$G\left(\vec{q}^{\,2}\right)=\frac{1}{1+\left(\frac{\vec{q}}{\Lambda_1}\right)^2+\left(\frac{\vec{q}}{\Lambda_2}\right)^4} \qquad G\left(Q^2\right)=\frac{1}{1+\left(\frac{Q}{\Lambda}\right)^2}$$

		RCQM	SL	PR		LIU	ERK	ALX
N	$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.075		0.0649	0.0481	0.0412
	Λ_1	0.451	0.453	0.940	Λ	0.747	0.614	1.65
	Λ_2	0.931	0.641	1.102		-	-	-
Δ	$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.478				
	Λ_1	0.594	0.458	0.853				
	Λ_2	0.998	0.648	1.014				

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Summary and Conclusions

- Surprisingly **good agreement** of predictions by GBE RCQM with experimental data (wherever such data are available)
- **Small deviations** left in some observables, such as electric radii and magnetic moments
- Surprisingly **good agreement** of predictions by GBE RCQM with lattice-QCD results
- Most important symmetries of GBE RCQM:
  - **SBχS**
  - **Lorentz invariance**
  - **time-reversal invariance**
  - **current conservation**
- The **non-relativistic** quark model **does not work** in any instance

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Thank you very much

for

your attention!