One- and two-neutron halos in effective field theory

arXiv:1001.1511, Nucl. Phys. A865 (2011), 17; Phys. Lett. B723 (2013), 196;
Nucl. Phys. A913 (2013), 105; in preparation

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Work done in collaboration with:
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- APS Topical Group on Few-body Problems (GFB): Please join!

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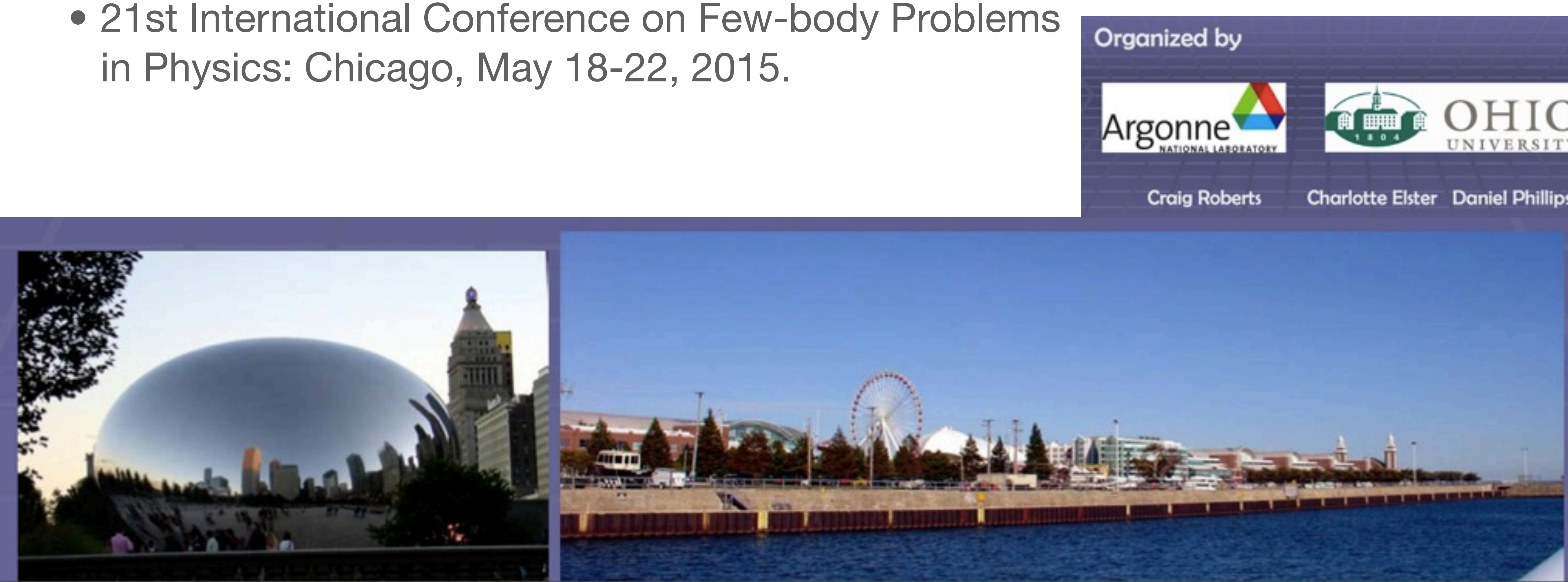
- 21st International Conference on Few-body Problems in Physics: Chicago, May 18-22, 2015.



Outline

- Generalities: halo nuclei, experimental techniques

Dissociation

- Example 1: Halo EFT for the one-neutron halo Carbon-19

Shallow S-wave state

Dissociation

- Example 2: Halo EFT for Beryllium-11 and Lithium-8

Capture

Shallow P-wave states

Role of P waves

The need for good input

- Example 3: Carbon-22 as a two-neutron halo

Matter radius-binding energy connection

- Outlook

Halo nuclei

- A halo nucleus as one in which a few (1, 2, 3, 4, ...) nucleons have a $\langle r^2 \rangle^{1/2}$ that is markedly larger than the range, R, of their interaction with the rest of the nucleus–the core.

- Typically $R \equiv R_{core} \sim 2$ fm. Define $R_{halo} = \langle r^2 \rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}.

- Usually produced in unstable beams, so neutron pickup reactions, e.g. (p,d), in inverse kinematics are one way to investigate.

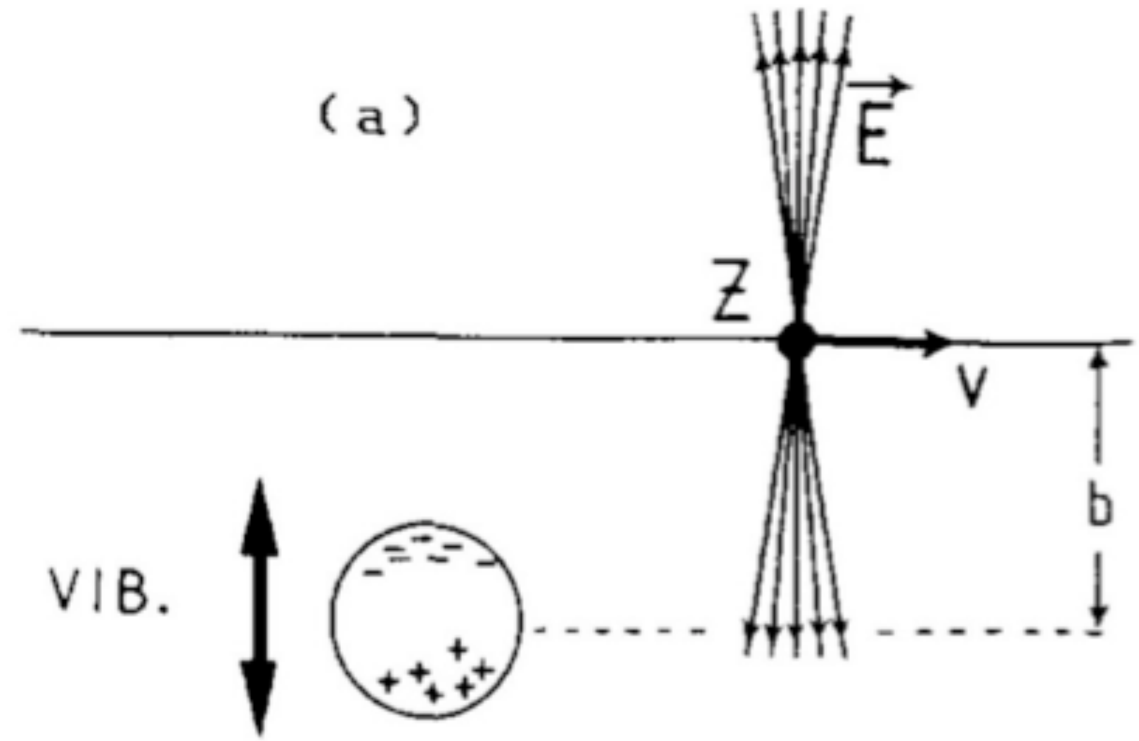
- Here my concern will be mostly with electromagnetic probes.

Probing halo nuclei

Bertulani, arXiv:0908.4307

- Coulomb dissociation: collide halo nucleus (we hope peripherally) with a high-Z nucleus

- Do with different Z, different nuclear sizes, different energies to test systematics



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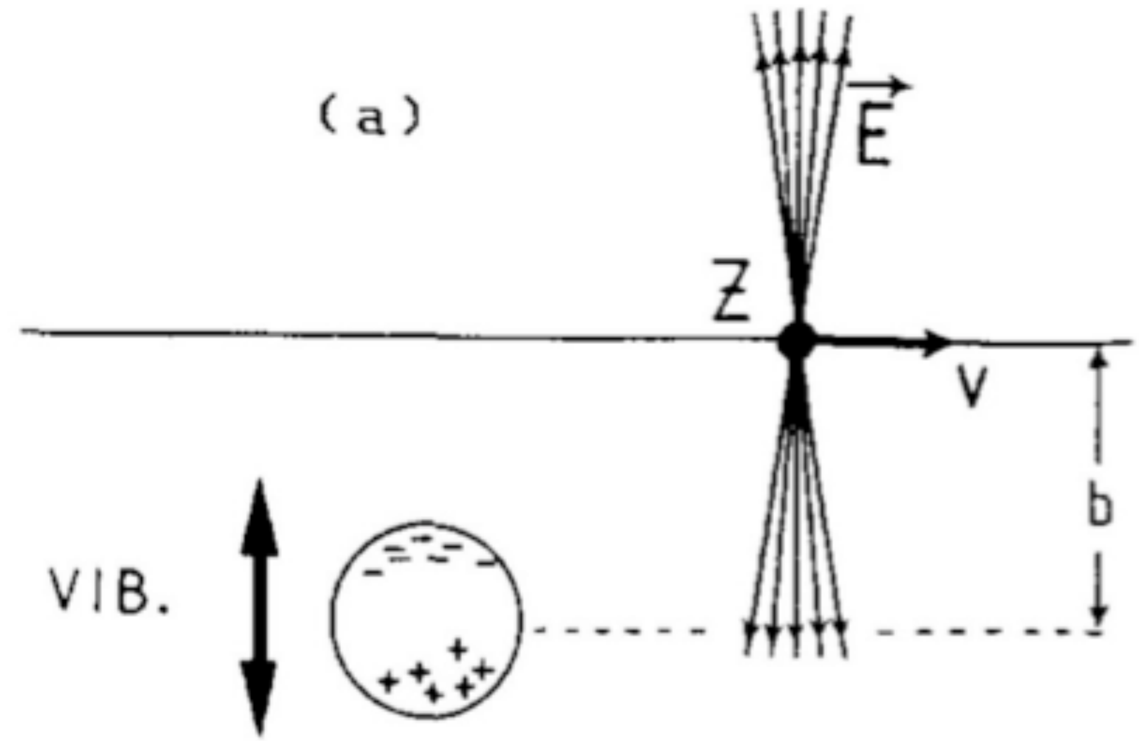

- Coulomb excitation dissociation cross section (p.v. b>>R_{target})

$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

  - $n_{\pi L}(E_\gamma, b)$ virtual photon numbers, dependent only on kinematic factors. Number of equivalent (virtual) photons that strike the halo nucleus.

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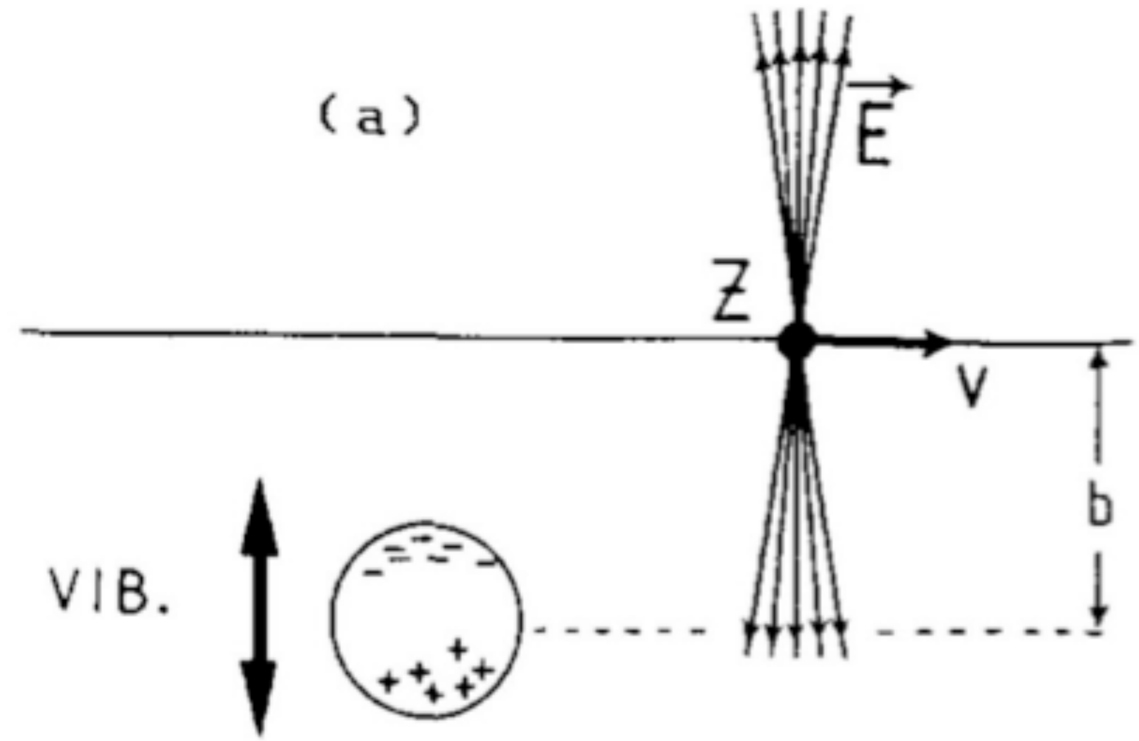

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  - $\sigma_\gamma^{\pi L}(E_\gamma)$ can then be extracted: it's the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity πL.

A first shot at Coulomb dissociation: Carbon-19

Acharya, Phillips. NPA 913, 103 (2013)

- ^{19}C neutron separation energy=580(90) keV. Ground state=$1/2^+$
- ^{18}C has 0^+ ground state; there is a 2^+ 1.62 MeV above the ground state
- Treat $1/2^+$ in ^{19}C as s-wave halo state: $|^{18}\mathrm{C}\rangle|n\rangle$.
- Expansion parameter $R_{core}/R_{halo} \approx 0.5$

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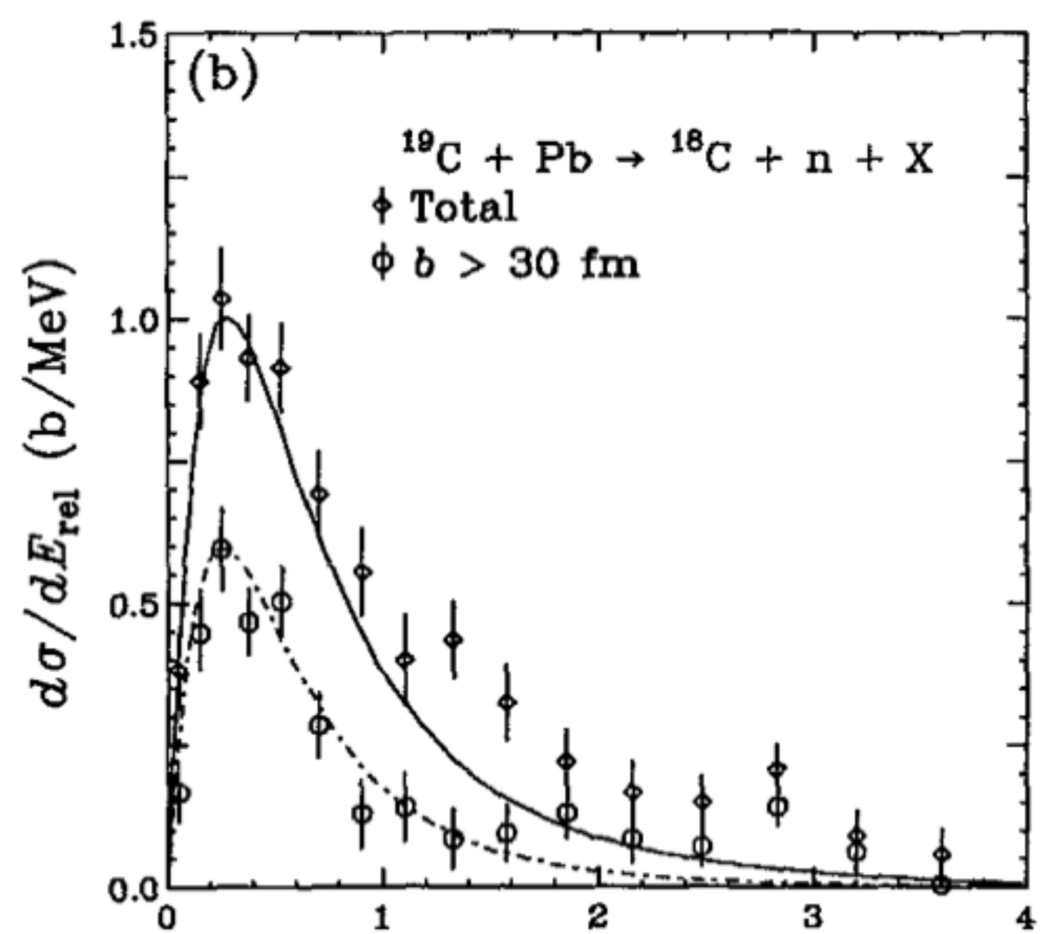
- Data, including cut on impact parameter



Nakamura et al. (2003)

Lagrangian: shallow S- and P-states

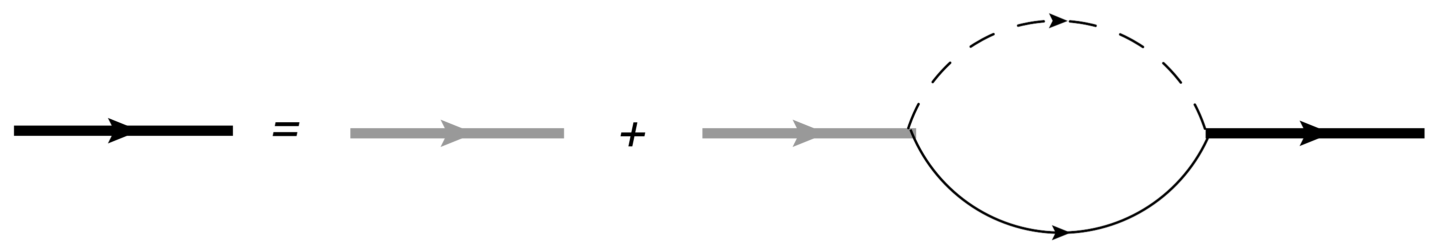
$$
\begin{aligned}
\mathcal{L} \;=\;& c^\dagger\left(i\partial_t+\frac{\nabla^2}{2M}\right)c+n^\dagger\left(i\partial_t+\frac{\nabla^2}{2m}\right)n\\
&+\sigma^\dagger\left[\eta_0\left(i\partial_t+\frac{\nabla^2}{2M_{nc}}\right)+\Delta_0\right]\sigma+\pi_j^\dagger\left[\eta_1\left(i\partial_t+\frac{\nabla^2}{2M_{nc}}\right)+\Delta_1\right]\pi_j\\
&-g_0\left[\sigma n^\dagger c^\dagger+\sigma^\dagger nc\right]-\frac{g_1}{2}\left[\pi_j^\dagger(n\; i\overleftrightarrow{\nabla}_j\; c)+(c^\dagger\; i\overleftrightarrow{\nabla}_j\; n^\dagger)\pi_j\right]\\
&-\frac{g_1}{2}\frac{M-m}{M_{nc}}\left[\pi_j^\dagger\; i\overrightarrow{\nabla}_j\;(nc)-i\overleftrightarrow{\nabla}_j\;(n^\dagger c^\dagger)\pi_j\right]+\ldots\,,
\end{aligned}
$$

- c, n: “core”, “neutron” fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings

Dressing the S-wave state

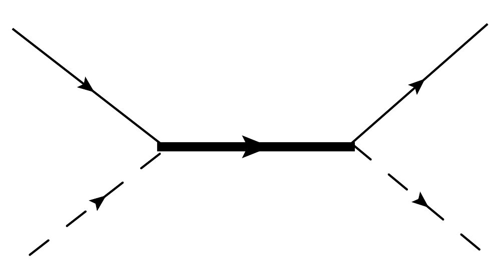
Kaplan, Savage, Wise; van Kolck; Gegelia; Birse, Richardson, McGovern

- σnc coupling g_0 of order R_{halo}, nc loop of order $1/R_{halo}$. Therefore need to sum all bubbles:



$$D_\sigma(p) = \frac{1}{\Delta_0 + \eta_0[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\sigma(p)}$$

$$\Sigma_\sigma(p) = -\frac{g_0^2 m_R}{2\pi}\left[\mu + i\sqrt{2m_R\left(p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + i\eta\right)}\right] \quad \text{(PDS)}$$



$$t = \frac{2\pi}{m_R}\frac{1}{\frac{1}{a_0} - \frac{1}{2}r_0 k^2 + ik}$$

$$D_\sigma(p) = \frac{2\pi\gamma_0}{m_R^2 g_0^2}\frac{1}{1 - r_0\gamma_0}\frac{1}{p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + B_0} + \text{regular}$$

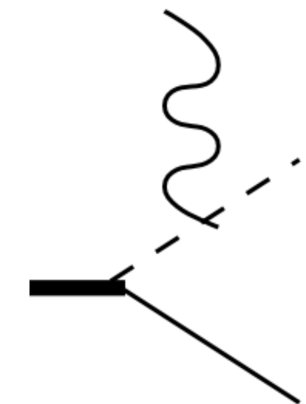
Counting in S waves: $a_0 \sim R_{halo} \sim 1/\gamma_0$; $r_0 \sim R_{core}$. $r_0 = 0$ at LO.

Predicting dissociation

c.f. Chen, Savage (1999)

- Leading order: no FSI, r_0=0⇒ γ_0 is only free parameter=0.16 fm^{-1}

$$\mathcal{M} = \frac{eQ_c g_0 2m_R}{\gamma_0^2 + \left(\mathbf{p}' - \frac{m}{M_{nc}}\mathbf{k}\right)^2}$$



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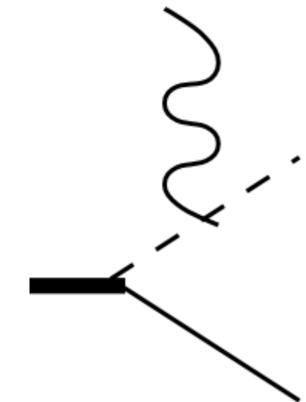

$$\frac{dB(E1)}{e^2 dE} = \frac{12 m_R}{\pi^2} Z_{eff}^2 \frac{\gamma_0}{1 - r_0\gamma_0} \frac{p^3}{(\gamma_0^2 + p^2)^4}$$

$Z_{eff}=6/19$

- Corresponds to $u_0(r)=A_0 \exp(-\gamma_0 r)$; $A_0^2 = \dfrac{2\gamma_0}{1 - r_0\gamma_0}$

Universal E1 strength formula for S-wave halos

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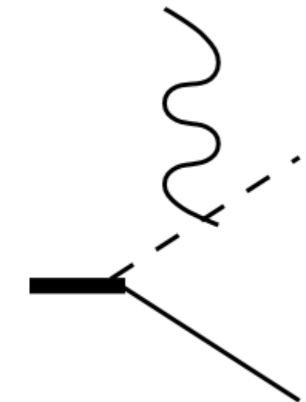

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Universal E1 strength formula for S-wave halos

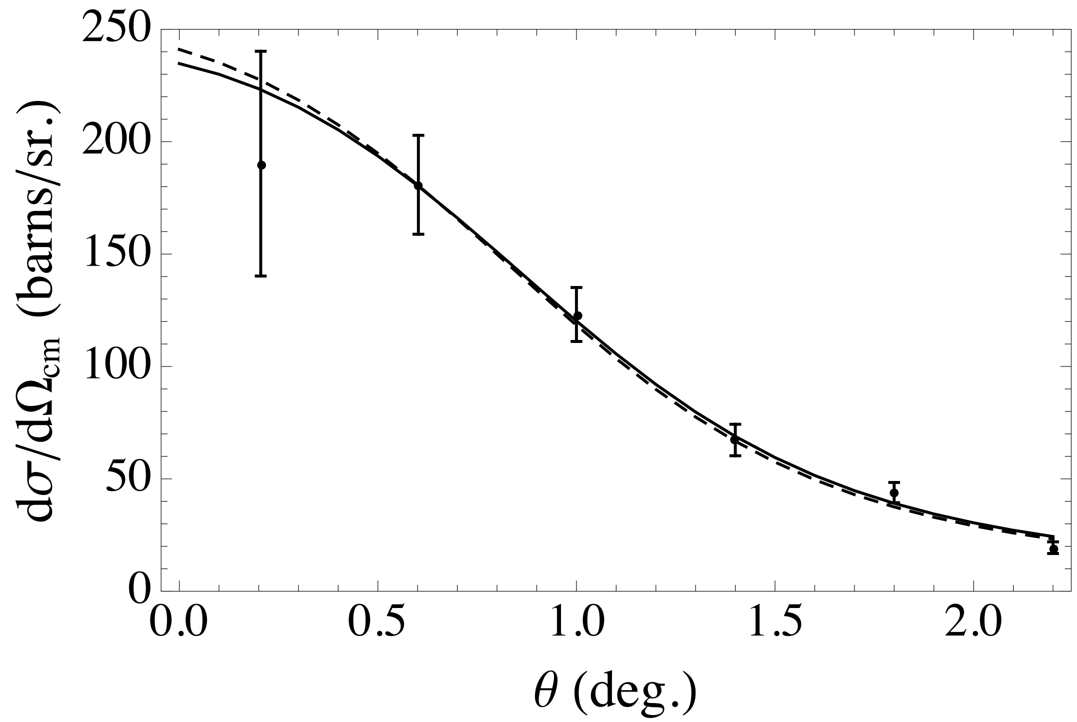
- Final-state interactions suppressed by $(R_{core}/R_{halo})^3$

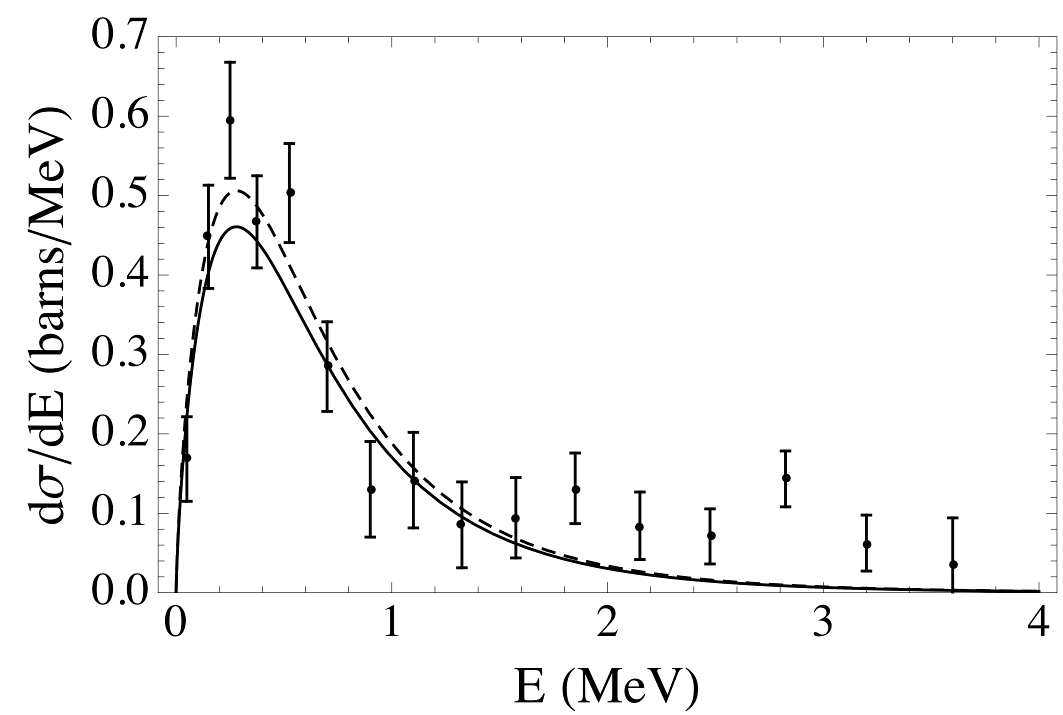
- First gauge-invariant contact operator: $L_{E1}\sigma^\dagger \mathbf{E} \cdot (n \overleftrightarrow{\nabla} c) + \mathrm{h.c.}$

Results

Data: Nakamura et al., 1999, 2003
Analysis: Acharya, Phillips. NPA, 2013

- Integrate this E1 strength for transition to a core + neutron state, per unit energy per unit solid angle, as function of energy of the outgoing nc pair over differential photon numbers and over angle, respectively.





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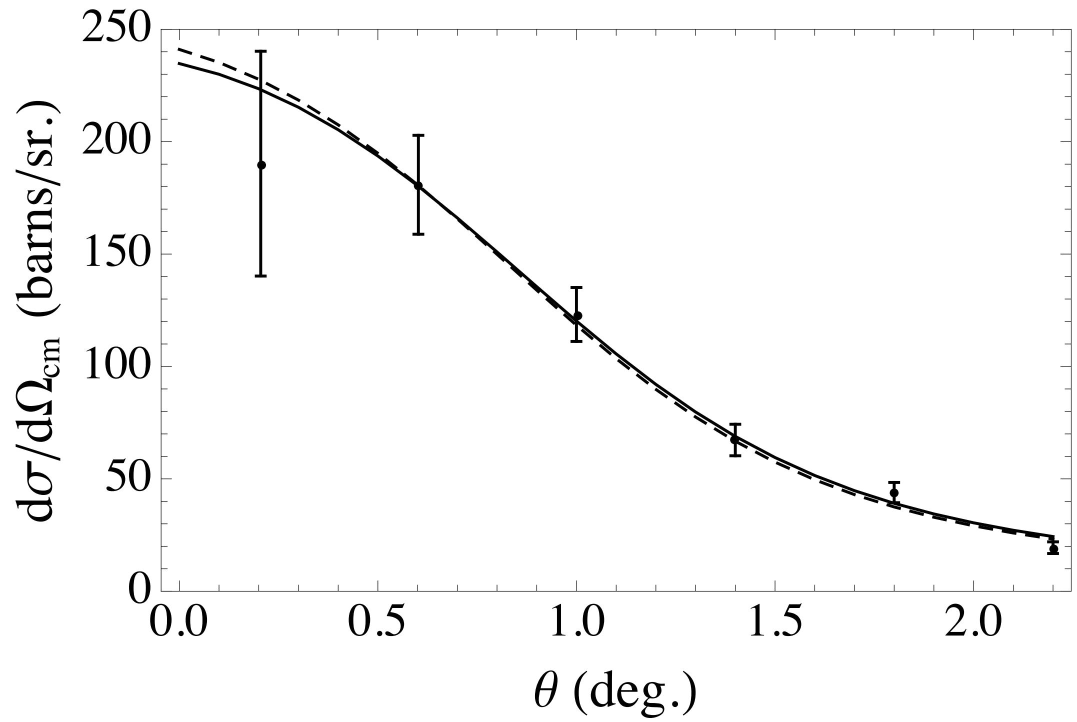

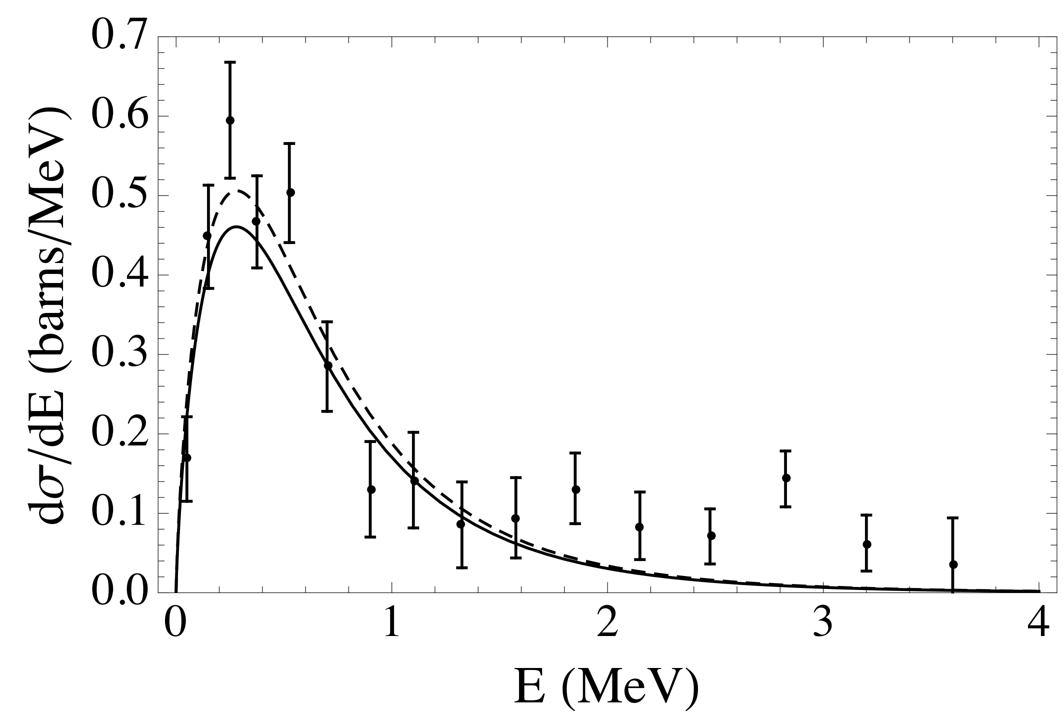

- $\gamma_0 \equiv a$ determines peak position and fall off of angular distribution

- r_0 fixed from fitting height of peak

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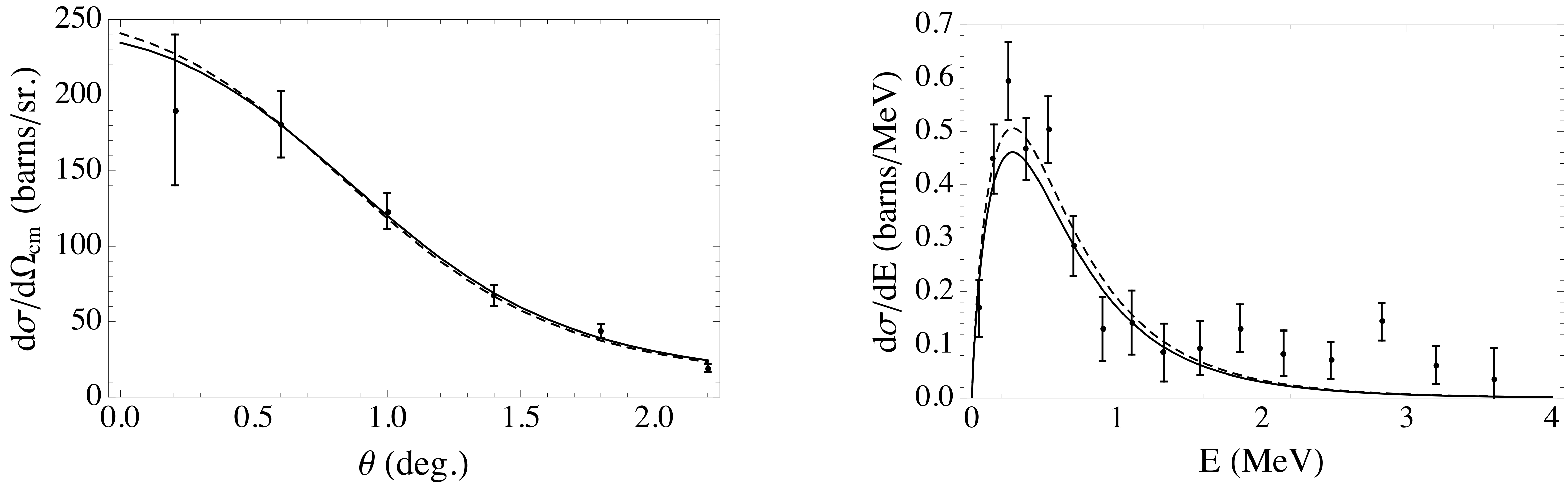

$$
\begin{aligned}
a &= (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT}))\ \text{fm}; \\
r_0 &= (2.6^{+0.6}_{-0.9}(\text{stat.}) \pm 0.1(\text{EFT}))\ \text{fm}.
\end{aligned}
$$

Determine S-wave ^{18}C-n scattering parameters from dissociation data.

Enter P-wave states: γ_{E1} + ^{11}Be$\rightarrow$^{10}Be + n

Typel & Baur, Phys. Rev. Lett. 93, 142502 (2004); Nucl. Phys. A759, 247 (2005); Eur. Phys. J. A 38, 355 (2008)

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$$k^3 \cot \delta_1 = -1/2\, r_1 (k^2 + \gamma_1^2) \Rightarrow \delta_1 \sim R_{core}/R_{halo} \text{ if } k \sim 1/R_{halo} \sim \gamma_1.$$

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

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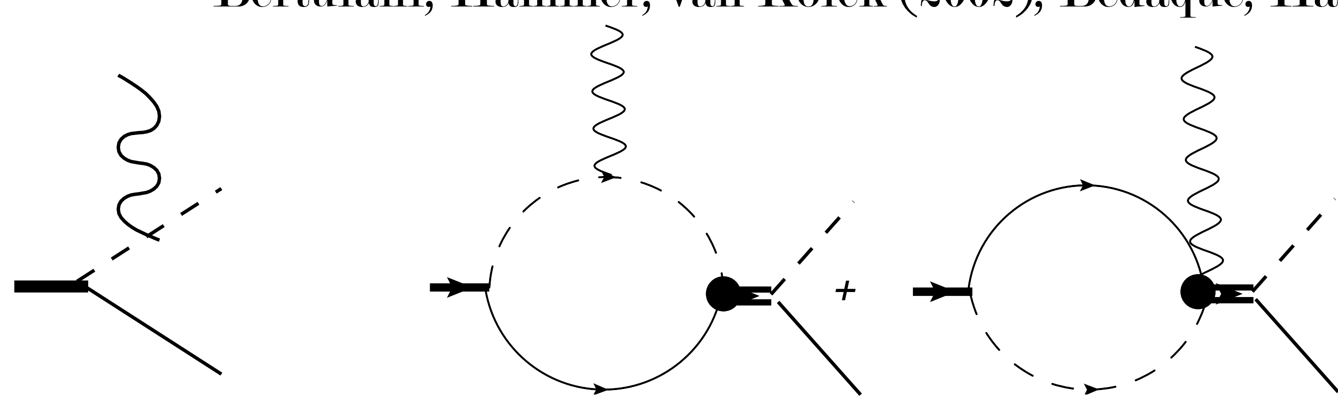
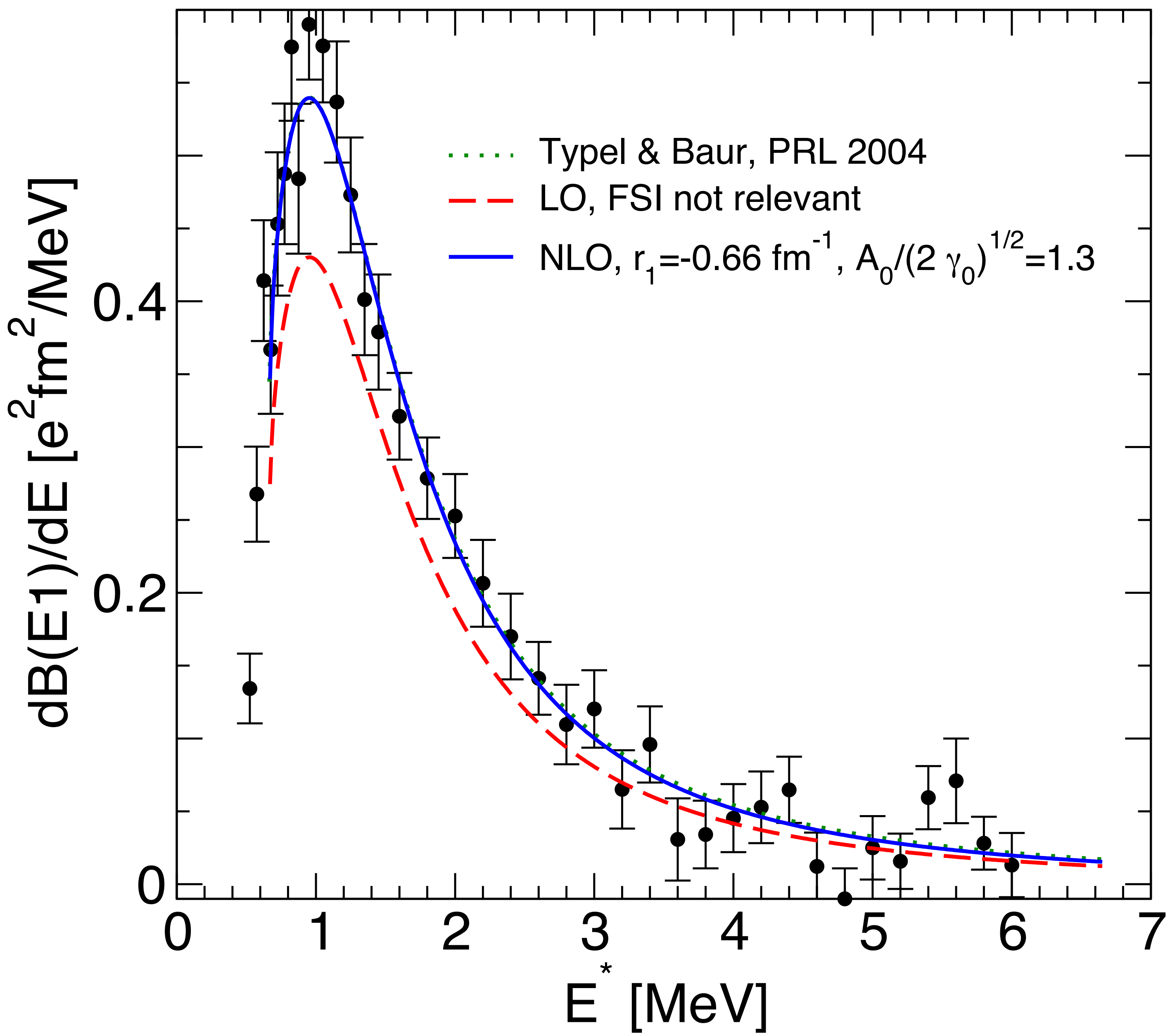


LO NLO

- Need both γ_1 and $r_1 \equiv A_1$ at NLO in this observable. A_0 also becomes a free parameter at NLO: fit it to Coulomb dissociation data

Coulomb dissociation: result

Data: Palit et al., 2003
Analysis: Hammer, Phillips. NPA, 2011



- Reasonable convergence
- Information on value of r_0 through fitting of A_0:

r_0=2.7 fm

Need P-wave effective range

- Here value of r_1 used to fit $B(E1:1/2^+ \rightarrow 1/2^-)$ works.

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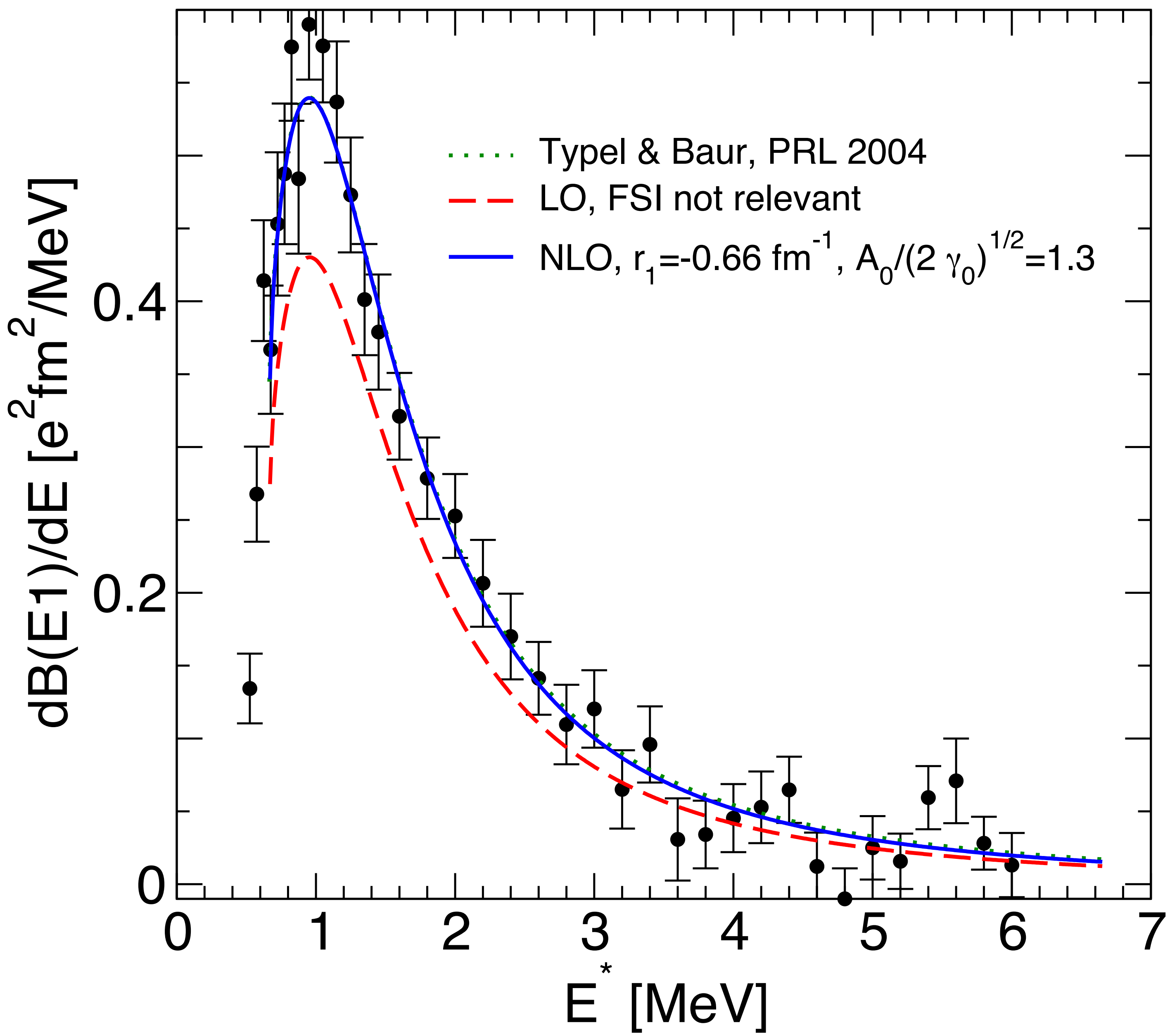

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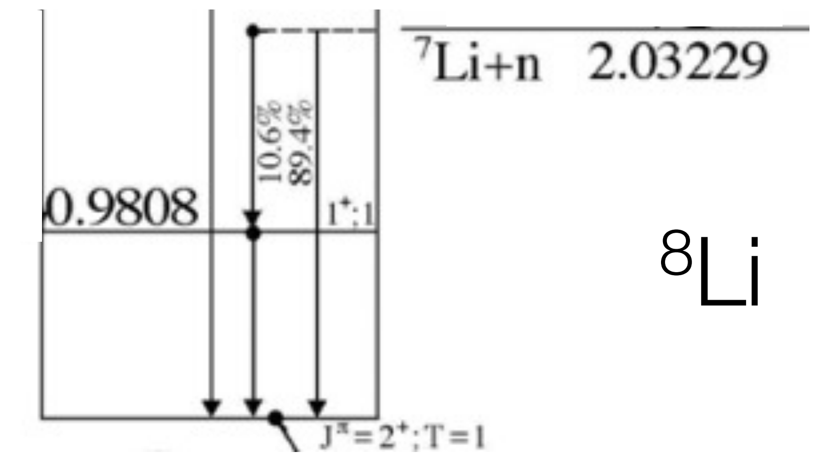
EFT gives universal relations between different observables: valid for all halo nuclei

Turning things around: $^7Li + n \rightarrow {}^8Li + \gamma_{E1}$

- ^{7}Li has spin-3/2: S-wave n scattering in $^5S_2$ and $^5S_1$

$a_{S=2} \sim R_{halo}$; $a_{S=1} \sim R_{core}$



^{8}Li

http://www.tunl.duke.edu

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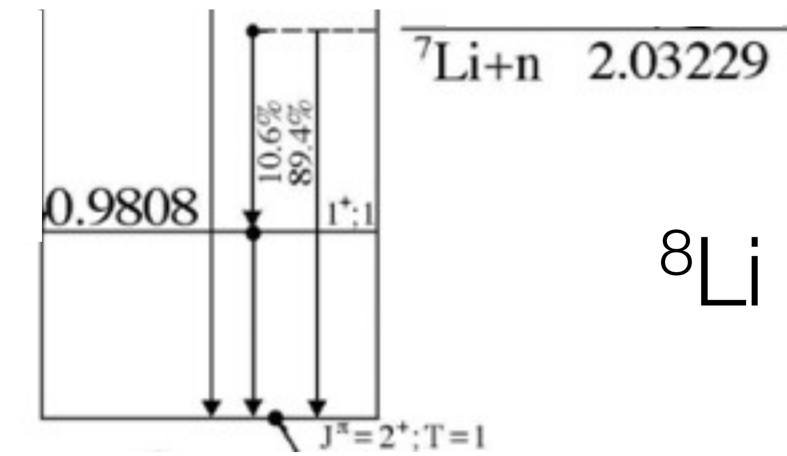

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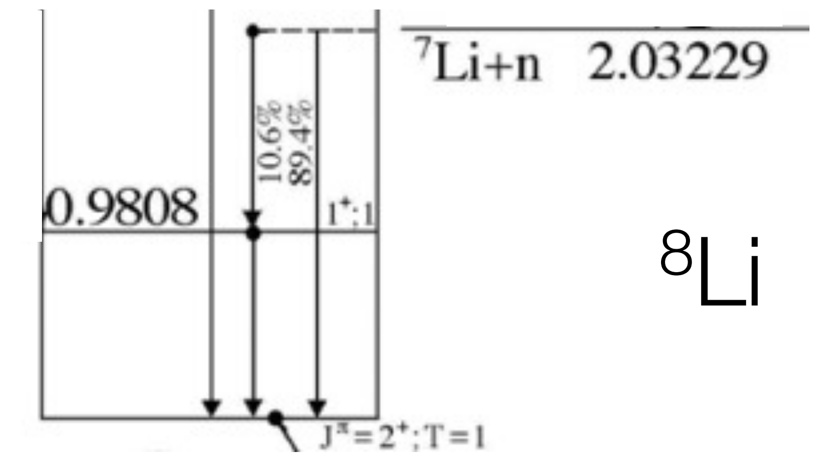

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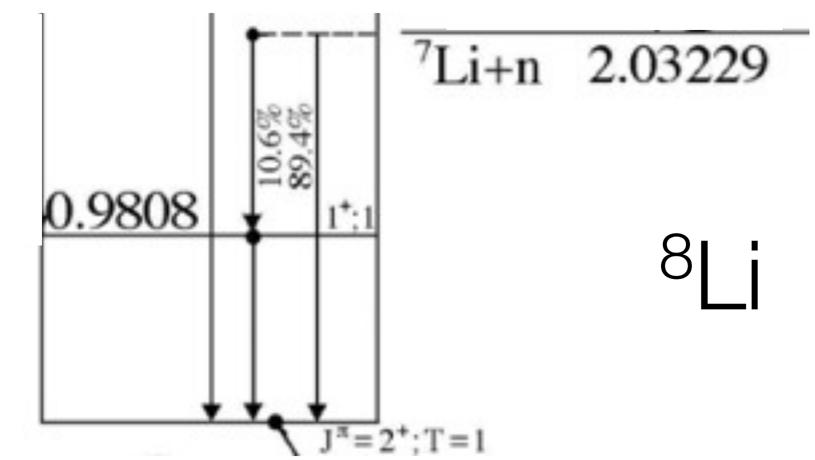

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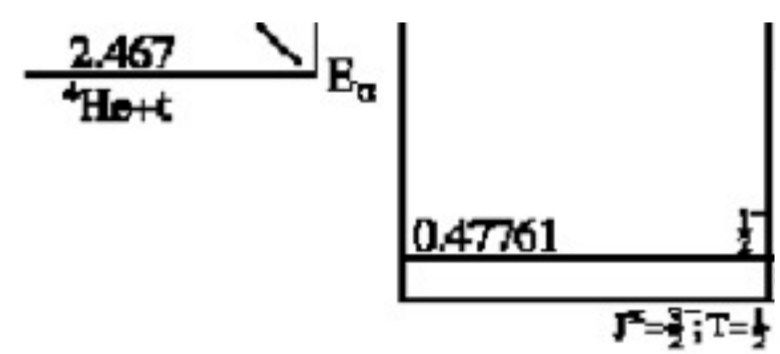

^{7}Li

- ^{8}Li first excited state: 1$^+$, bound by 1.05 MeV

Fixing ^{8}Li parameters

Zhang, Nollett, Phillips, in preparation

- Input at LO: B_1=2.03 MeV; B_1^*=1.05 MeV⇒γ_1=58 MeV; γ_1^*=42 MeV.

$\gamma_1 \sim 1/R_{halo}$

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- Also include 1/2- excited state of ^{7}Li as explicit d.o.f. for consistency. ANC for $|^8\text{Li}\rangle \rightarrow |^7\text{Li}^*\rangle|n\rangle$ from same VMC calculation⇒**(+1+2)** P-wave ANCs needed

c.f. Rupak, Higa, PRL 106, 222501 (2011);
Fernando, Higa, Rupak, EPJA 48, 24 (2012)

Results for $^7\text{Li} + n \rightarrow {}^8\text{Li} + \gamma_{E1}$

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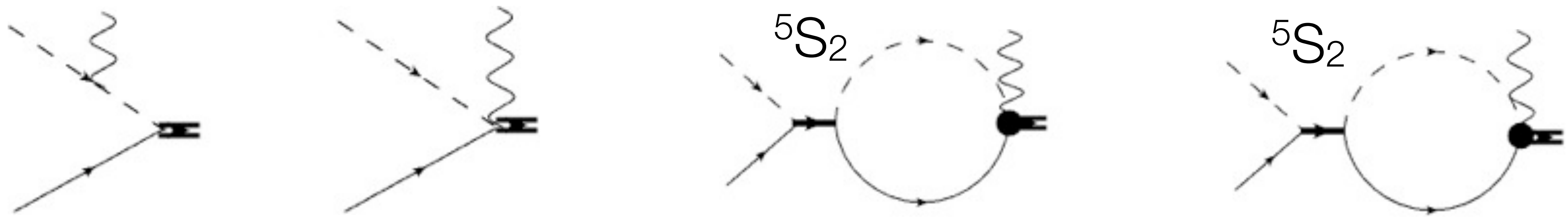
LO



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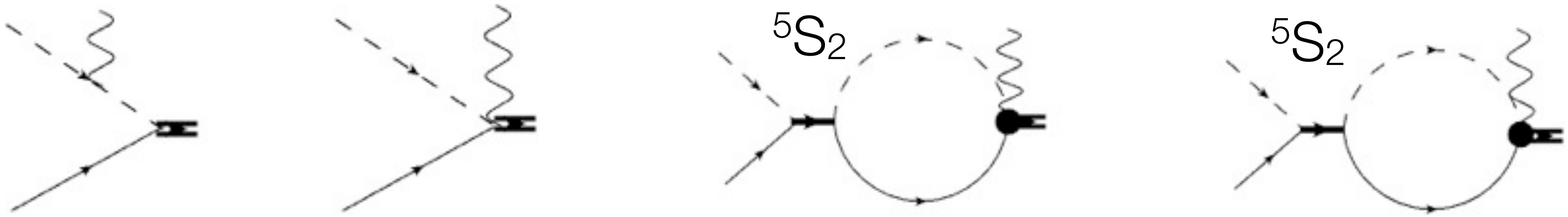
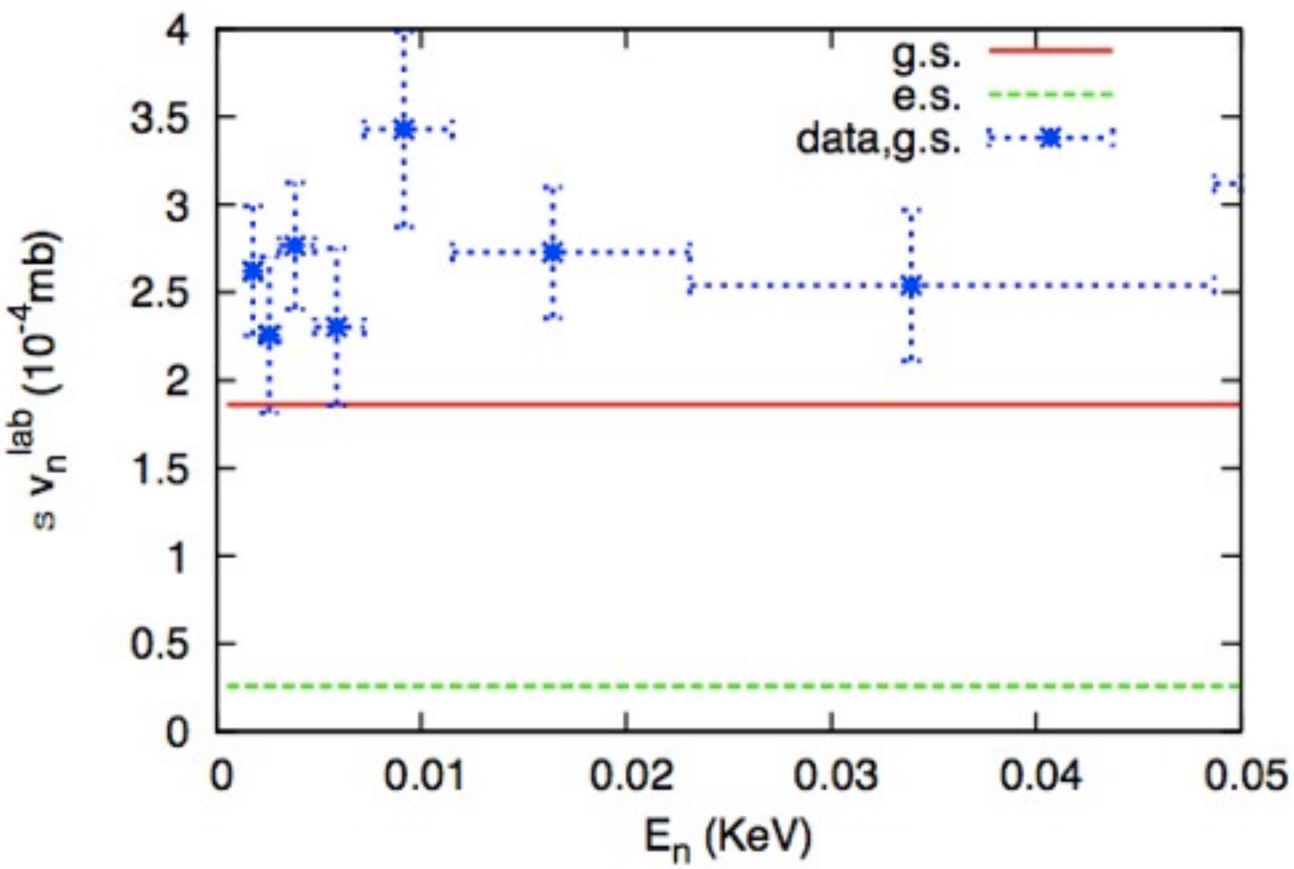




$$\frac{\sigma(\to 2^+)}{\sigma(\to 2^+) + \sigma(\to 1)} = 0.89$$

Experiment=0.88

Lynn et al., 1991

$$\frac{\sigma(^5S_2 \to 2^+)}{\sigma(\to 2^+)} = 0.95$$

Experiment > 0.86

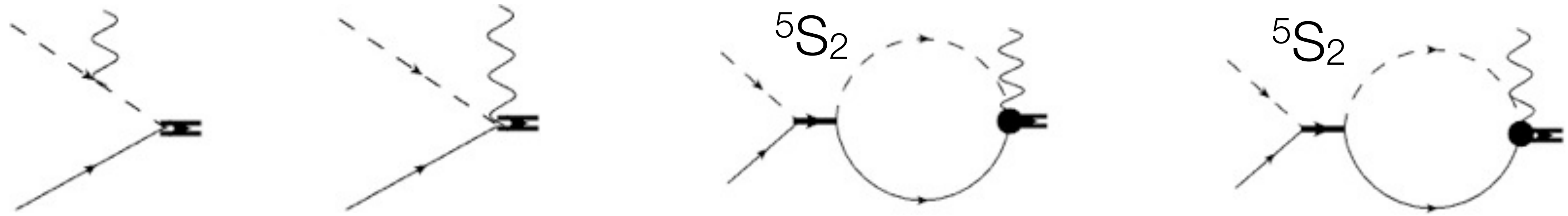
Barker, 1996

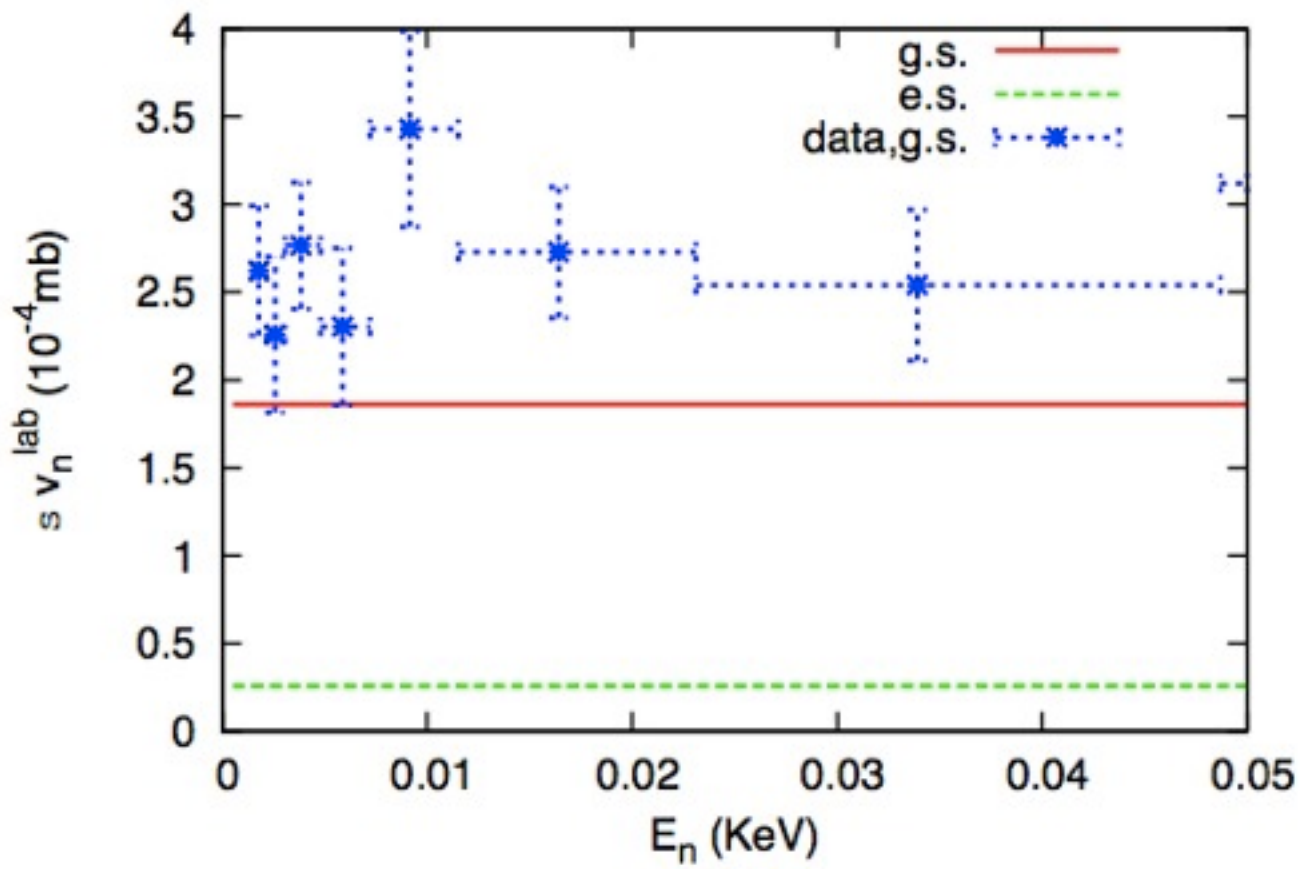
Results for ^{7}Li + n→^{8}Li + γ_{E1}

Data: Barker (1996), c.f. Nagai et al. (2005)

Analysis: Zhang, Nollett, Phillips, in preparation

LO





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Dynamics **predicted** through *ab initio* input

Need to think about how to go to higher order: NLO effects include $a_{S=1}$, ^{7}Li* in loop, change in ANCs, and CT for E1 capture at NLO.

What is known about ^{20}C, ^{21}C, and ^{22}C?

- ^{20}C: 0^+ ground state, S_{1n}= 2.9(3) MeV; $\langle r^2\rangle^{1/2}$=2.97(5) fm

Ozawa et al. (2001)

- ^{21}C: unbound but possibility of low-energy s-wave resonance

c.f. Mosby et al. (2013)

- ^{22}C: S_{2n}=420 ± 940 keV, but known to be bound

Audi, Wapstra, Thibault (2003)

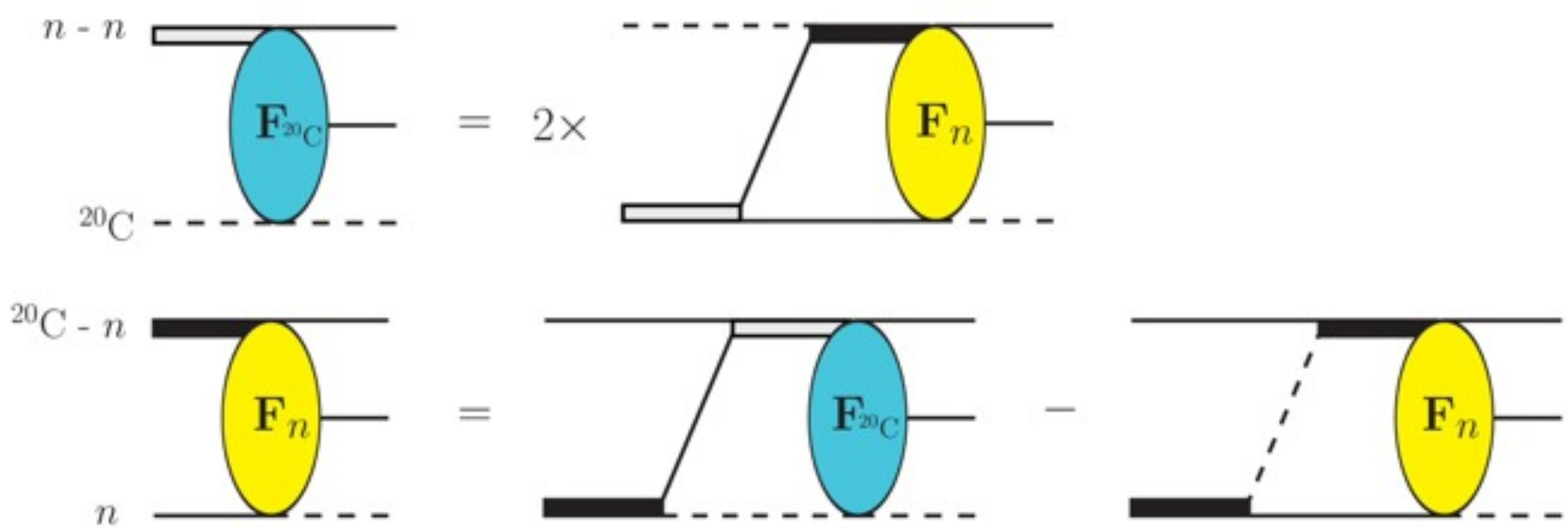
- $\langle r^2\rangle^{1/2}$=5.4(9) fm from a ^{22}C + p reaction, analyzed via Glauber

Tanaka et al. (2010)

- Also indicated a $2s^2$ configuration for last two neutrons

Halo EFT for ^{22}C

- Working hypothesis: ^{22}C is an s-wave 2n Borromean halo with a ^{20}C core
- Halo EFT: ^{20}C-n and n-n contact interactions at leading order



Canham, Hammer (2011)

- ^{21}C-n contact interaction to stabilize three-body system

→Efimov/Thomas effects

- Inputs: $E_{nn}=1/(m\, a_{nn}^2)=120$ keV, E_{nc}, B (=S_{2n})
- Output: everything. At LO accuracy

Universality and matter radii of 2n halos

- Define: $f\left(\frac{E_{nn}}{B}, \frac{E_{nc}}{B}; A\right) \equiv mB\langle r_0^2\rangle$

- "Unitary limit", $E_{nn}=E_{nc}=0$: f becomes a number depending solely on A

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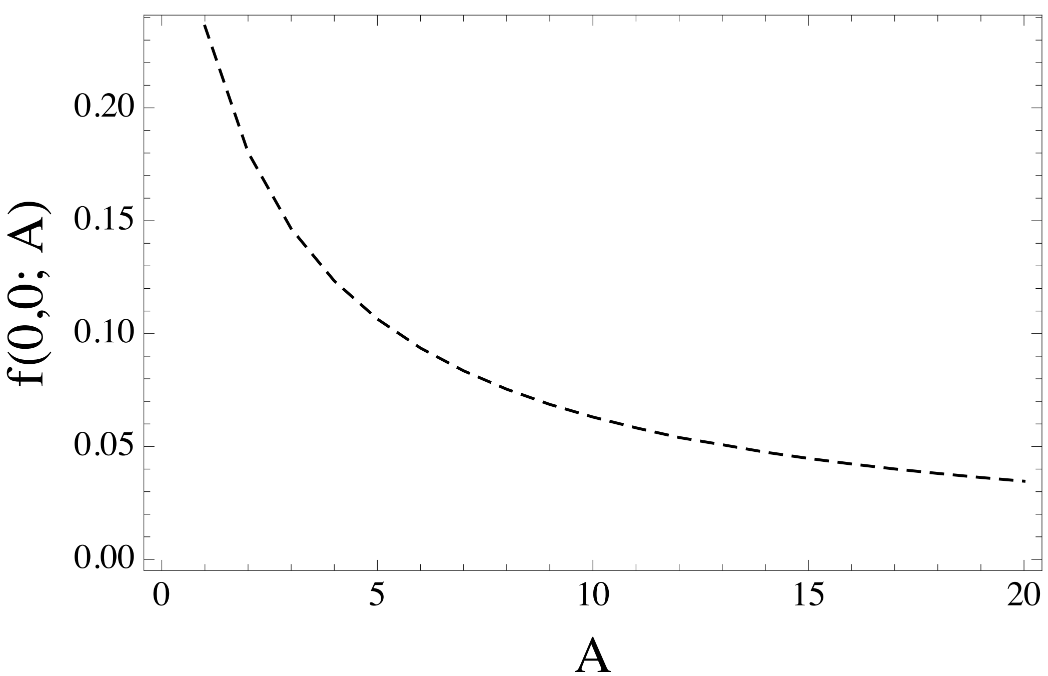

c. f. Yamashita et al. (2004): 15% lower at A=20

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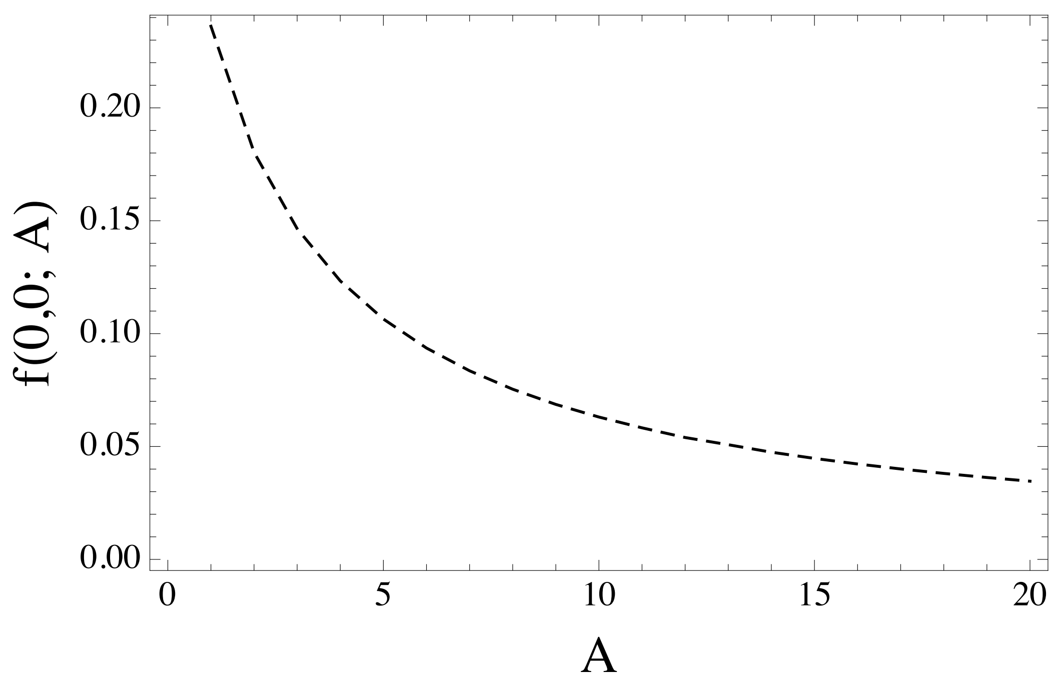
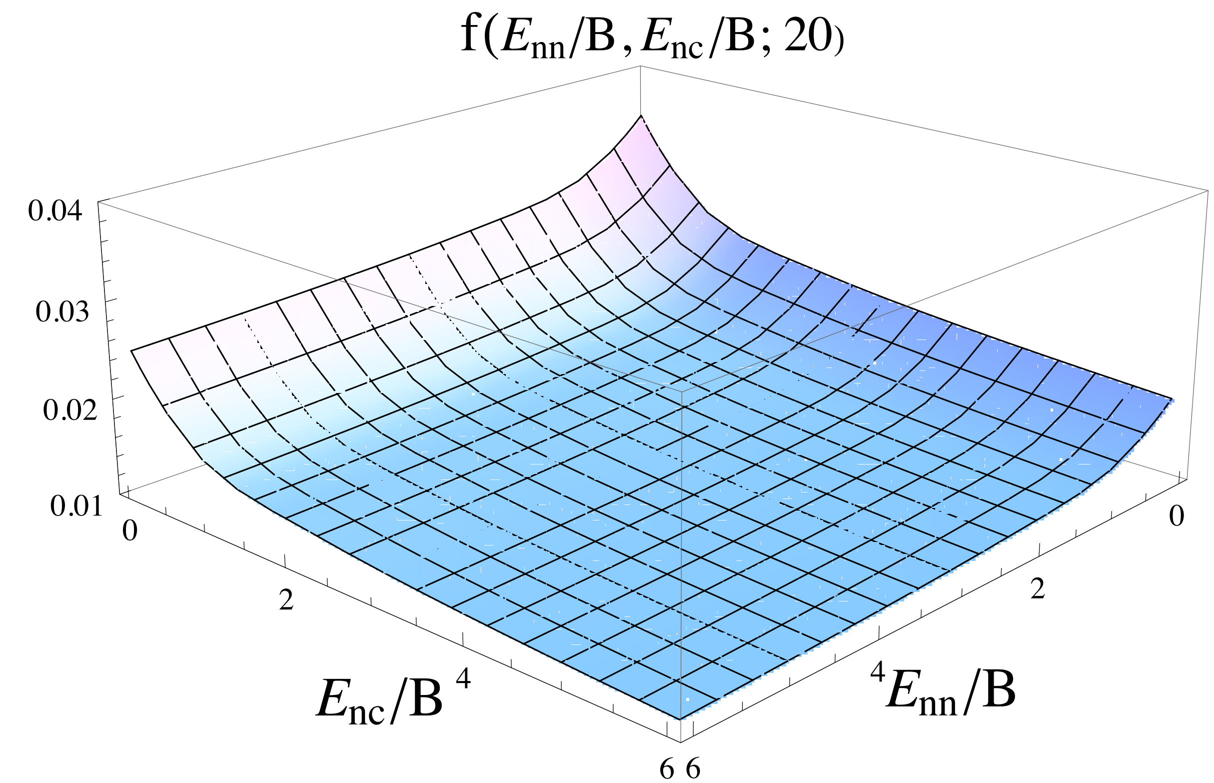


- Fix A=20, plot f as a function of E_{nn} and E_{nc}

c. f. Yamashita et al. (2004): 15% lower at A=20

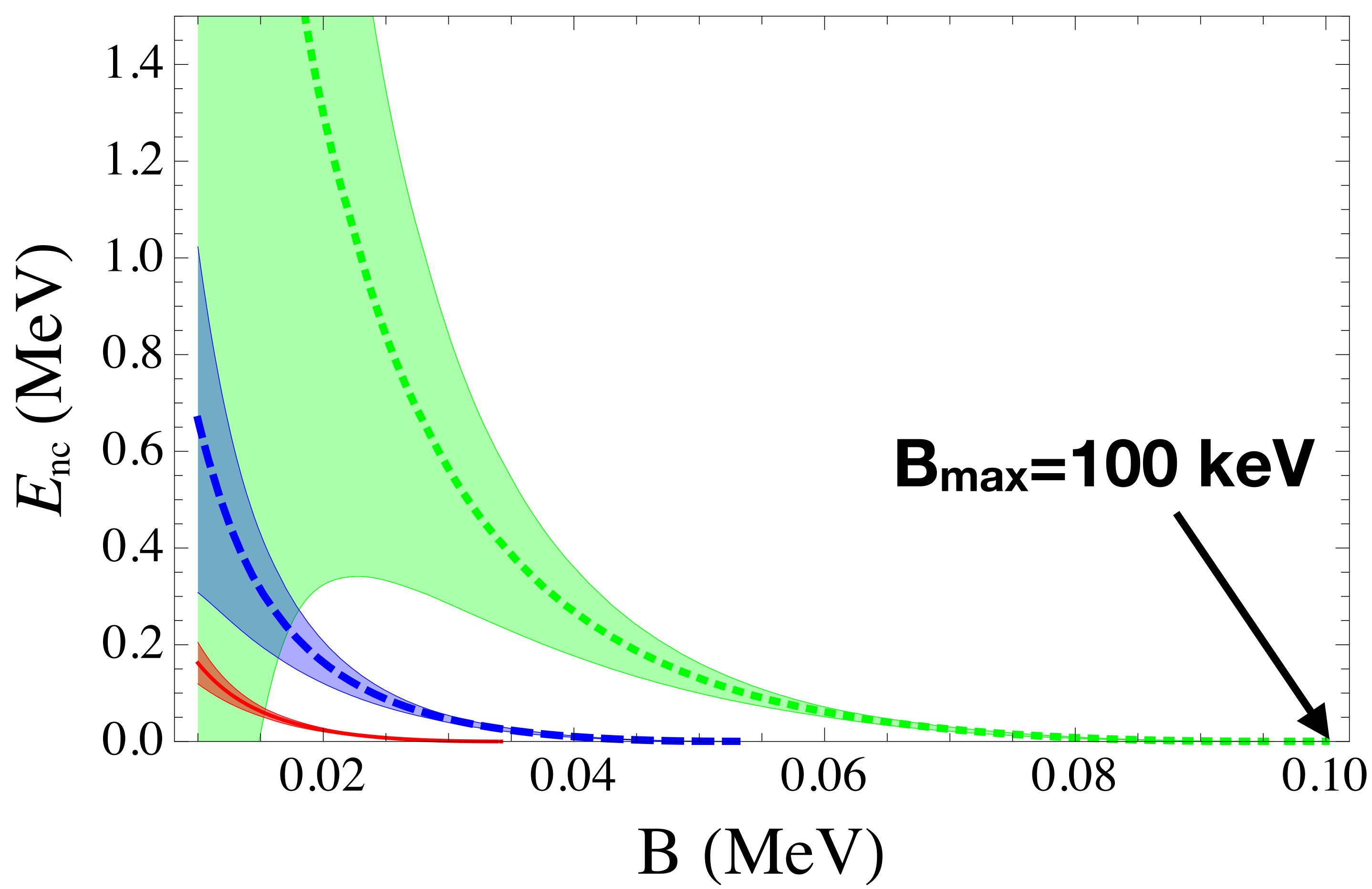


Implications for ^{21}C and ^{22}C

- Include finite size of ^{20}C

- Consider uncertainty due to NLO effects:

Relative size ~ largest of $(mE_{nn})^{1/2}/\Lambda_0$; $(2mE_{nc})^{1/2}/\Lambda_0$; $(2mB)^{1/2}/\Lambda_0$



c. f. Yamashita et al. (2011); Fortune & Sherr (2012)

^{22}C likely too shallow to support Efimov states

Conclusions: photoreactions and one-neutron halos

- Carbon-19: one-neutron halo, shallow S-wave state
- Coulomb dissociation can determine a and r_0 rather accurately. Test?
- Beryllium-11: one-neutron halo, shallow S- and P-wave state
- Has big B(E1) strength, can be hard to calculate in *ab initio* methods because of extended nature of p-wave state. Controlled by r_1.
- Lithium-8: too many parameters to fit to data. Use *ab initio* input.

c.f. Rupak & Higa PRL 106, 222501 (2011);
Fernando, Higa, Rupak, EPJA 48, 24 (2012)

- Successful LO description of a number of capture observables
- Halo EFT reveals correlations between low-energy observables; complementary and supplementary to *ab initio* calculations

Outlook: two-neutron and one-proton halos

- Predictions for ^{22}C radii in terms of input parameters: R_{core}/R_{halo} expansion
- Computation of ^{6}He

  Rotureau, van Kolck (2013); Ji, Phillips, Elster, in progress
- Other 2n halos with s- and p-wave interactions: ^{11}Li, ^{12}Be
- Coulomb dissociation of 2n halos: signatures of Efimovian physics?

  Nakamura et al., Experiment on ^{22}C, data taken
- Proton halos: p halo: big E1 strength, e.g. $^{7}\text{Be} + p \rightarrow {}^{8}\text{B} + \gamma$

  see also Ryberg, Forssen, Hammer, Platter

FOR USE IF NEEDED....

Our approach

- S-wave (and P-wave) states generated by cn contact interactions
- No discussion of nodes, details of n-core interaction, spectroscopic factors

$$u_0(r) = A_0 \exp(-\gamma_0 r)$$

- ^{19}C: input at LO: neutron separation energy of s-wave state.
- A_0 ("wave-function renormalization") can be fit at NLO.
- P-wave states require two inputs already at LO.

Coulomb dissociation: formulae

c.f. Rupak & Higa arXiv:1101.0207

- Straightforward computation of diagrams yields:

$$\frac{d\mathrm{B(E1)}}{dE} = e^2 Z_{eff}^2 \frac{m_R}{2\pi^2} A_0^2 \left(\frac{p'^3[2p'^3 \cot(\delta^{(1/2)}(p')) + \gamma_0^3 + 3\gamma_0 p'^2]^2}{[p'^6 + p'^6 \cot^2(\delta^{(1/2)}(p'))](p'^2+\gamma_0^2)^4} + \frac{8p'^3}{(p'^2+\gamma_0^2)^4} \right)$$

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No FSI

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Spin-1/2 channel

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Wf renormalization

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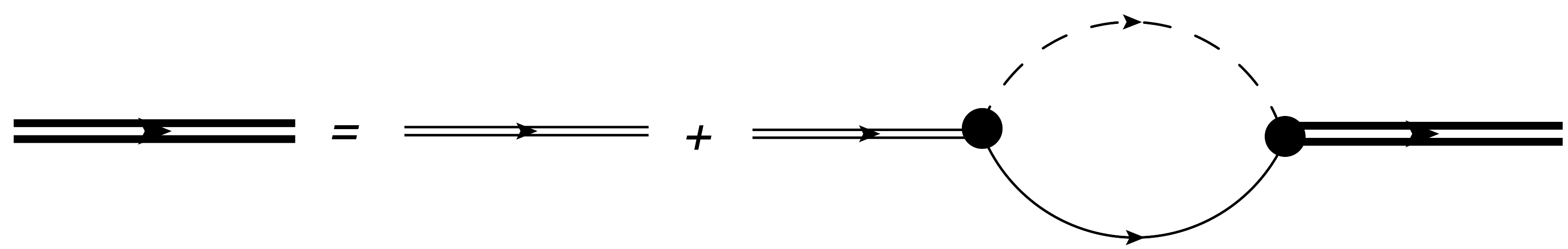
- Higher-order corrections to phase shift at NNLO. Appearance of S-to-$^2P_{1/2}$ E1 counterterm also at that order.

Dressing the P-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Proceed similarly for P-wave state as for S-wave state



$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

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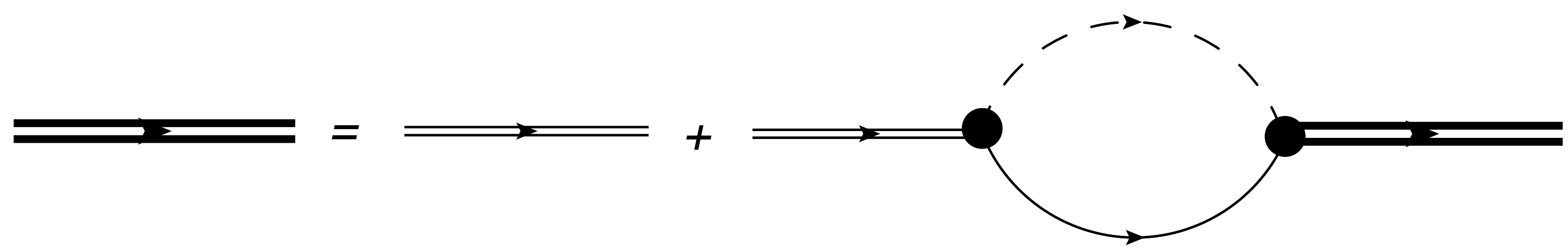

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Counting in P waves:
$R_{halo} \sim 1/\gamma_1$; $r_1 \sim R_{core}$.
$A_1 \longleftrightarrow r_1$ at LO

ANCs from an integral relation

Nollett, Wiringa, PRC (2011)

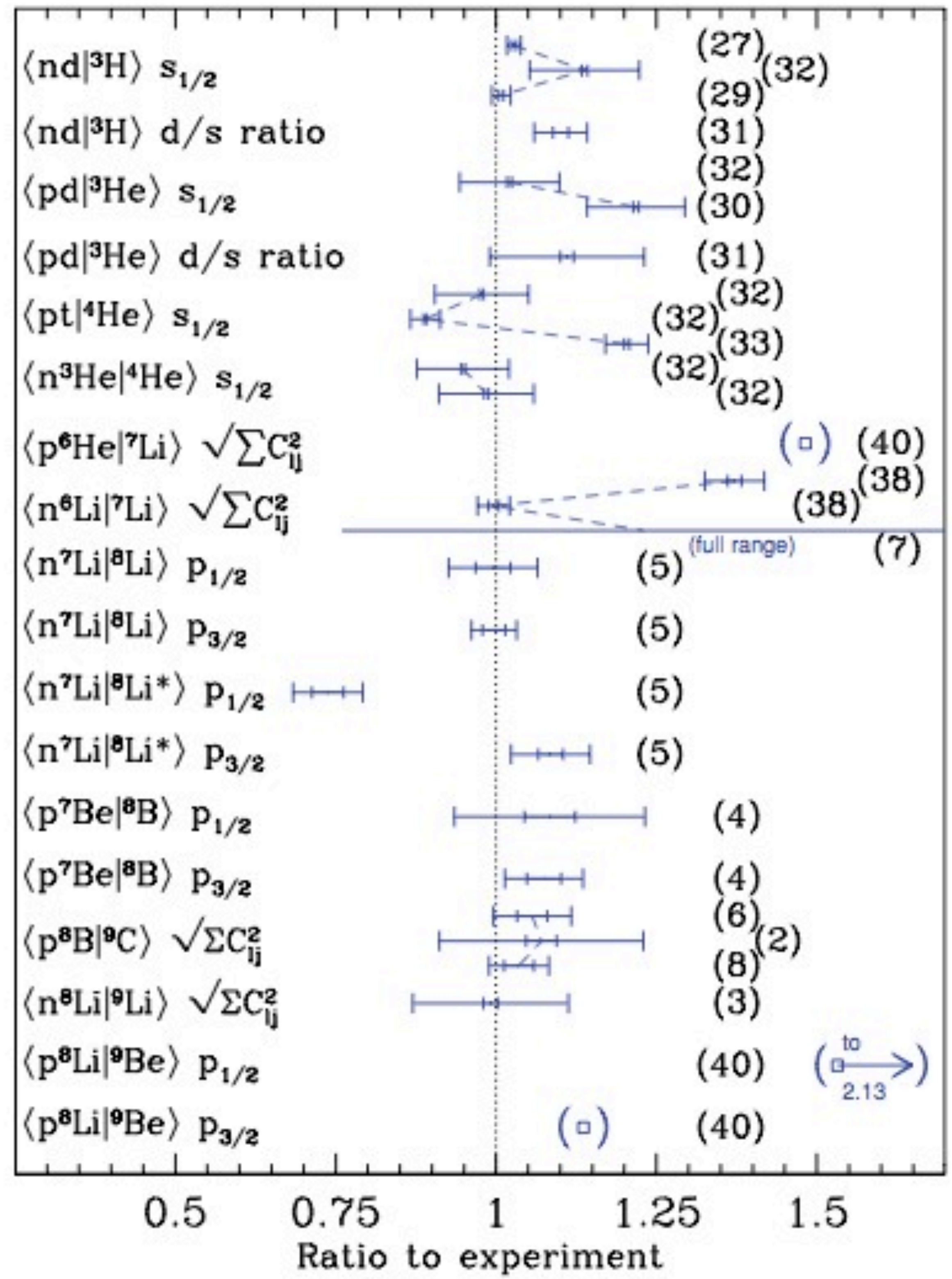
- VMC calculation using AV18 + UIX
- Integral relation:

$$C_{lj} = \frac{2\mu}{k\hbar^2 w}\mathcal{A}\int \frac{M_{-\eta m}(2kr_{cc})}{r_{cc}}\Psi^{\dagger}_{A-1}\chi^{\dagger}Y^{\dagger}_{l}(\hat{\mathbf{r}}_{cc}) \times (U_{rel} - V_C)\Psi_A \, d\mathbf{R}.$$

facilitates extraction using MC sampling

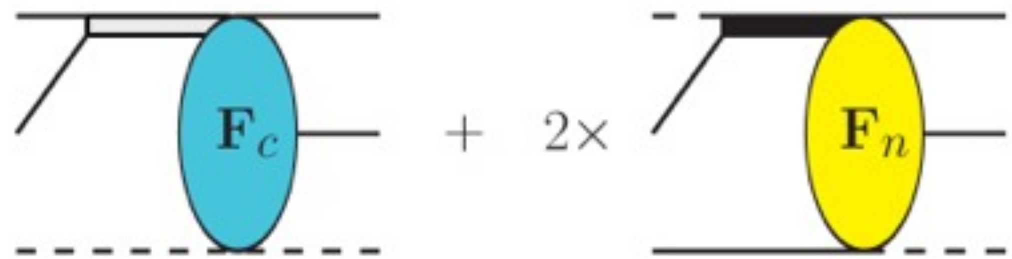
- Also results for resonant states

Nollett, PRC (2012)



Matter radii of 2n s-wave halos

- Wave function: $\Psi_c(p,q) =$



- One-body form factors:

$$\mathcal{F}_x(k^2) = \int_0^\infty \mathrm{d}p\ p^2 \int_0^\infty \mathrm{d}q\ q^2 \int_{-1}^{1} \mathrm{d}\left(\hat{q}\cdot\hat{k}\right) \Psi_x(p,q)\ \Psi_x(p,|\vec{q}-\vec{k}|).$$

x=c,n

Canham, Hammer (2011)

- Radii: $\mathcal{F}_x(k^2) = 1 - \frac{1}{6}\langle r_x^2\rangle k^2 + O(k^4)$

- Matter radius:

$$\langle r_0^2\rangle = \frac{2(A+1)^2}{(A+2)^3}\langle r_n^2\rangle + \frac{4A}{(A+2)^3}\langle r_c^2\rangle$$

- So matter radius can be computed straightforwardly, for a given E_{nc} and B

Efimov states in ^{22}C

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Mazumdar et al. (2000), Frederico et al. (2012)

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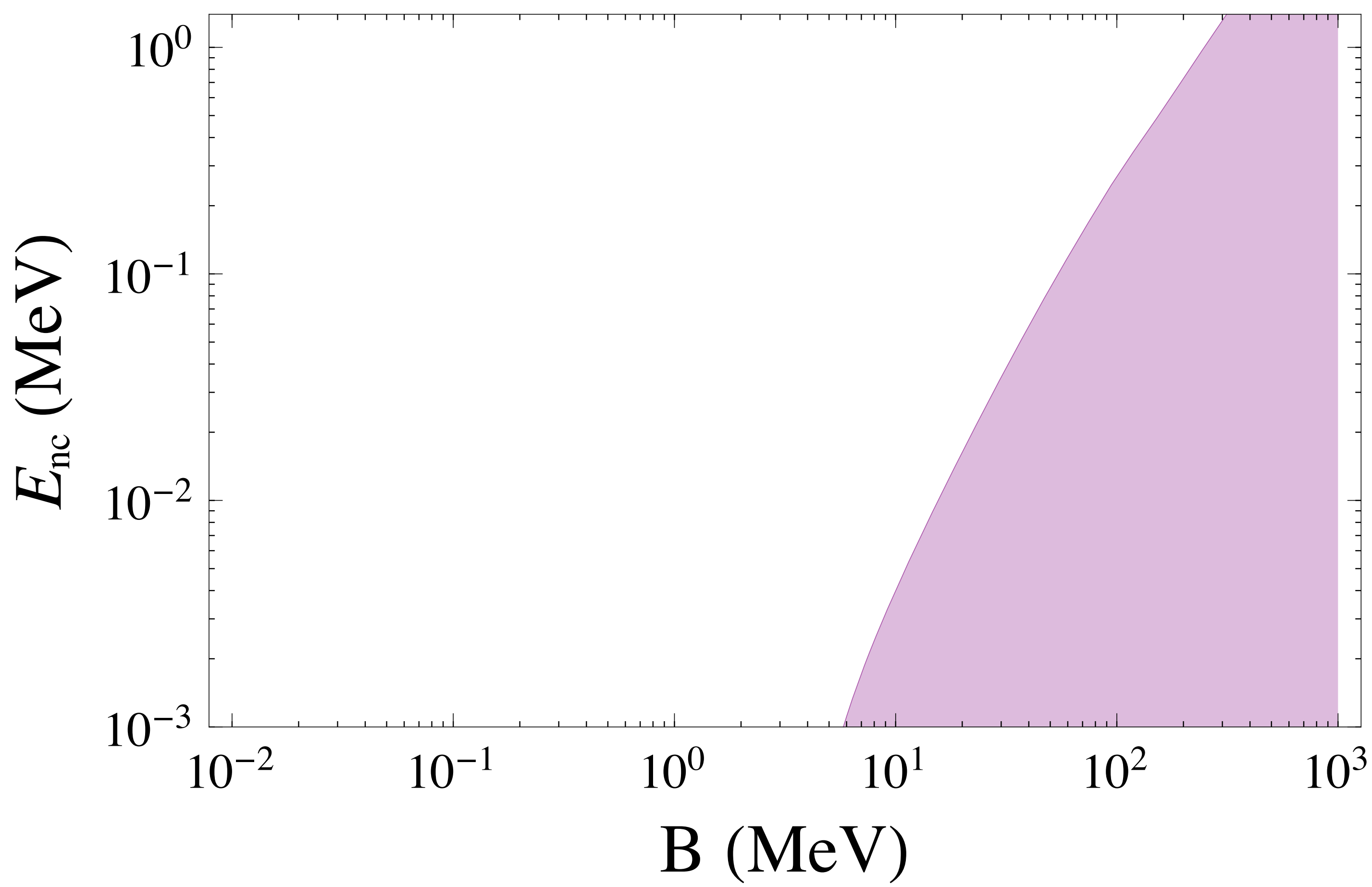

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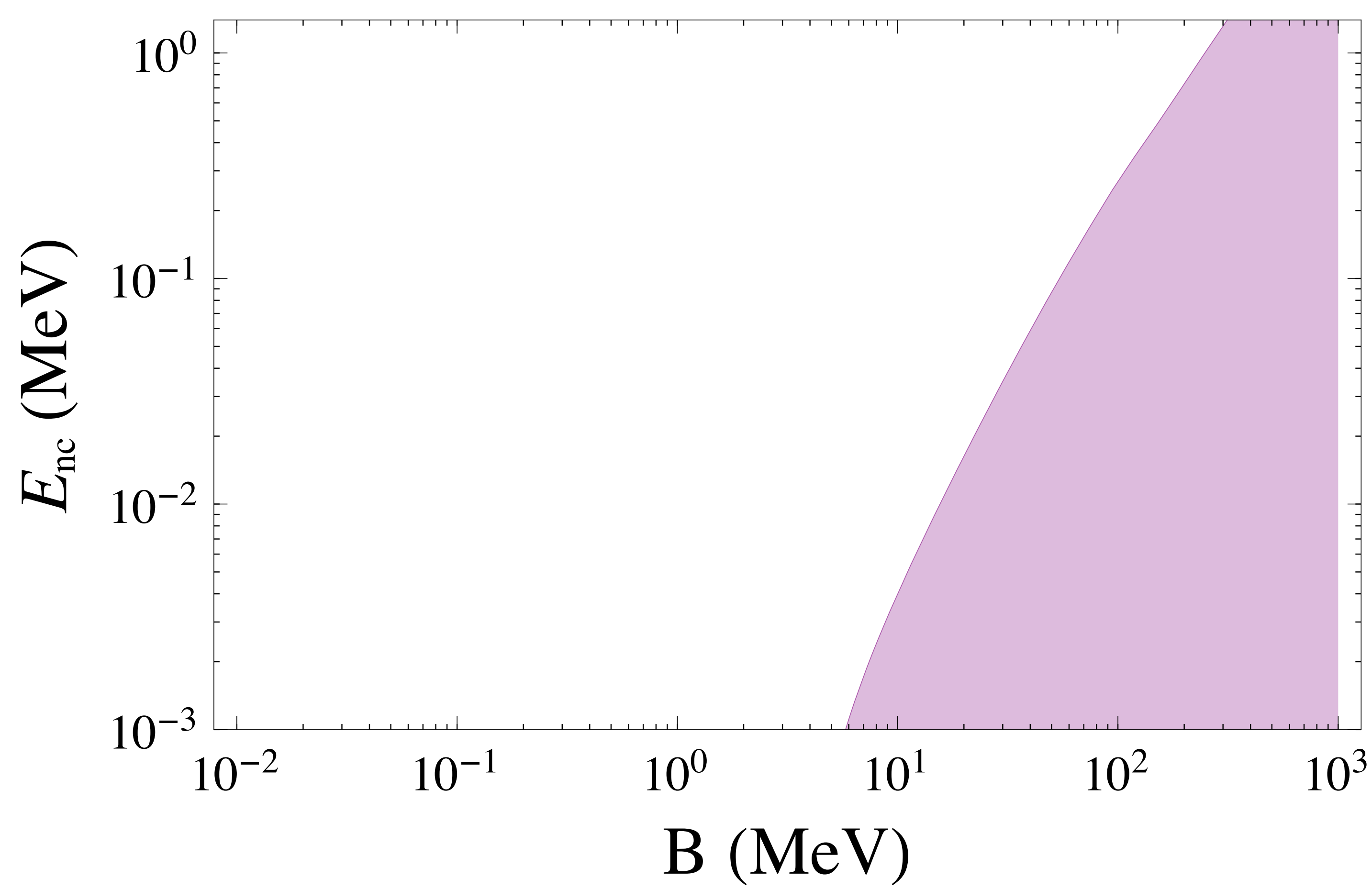

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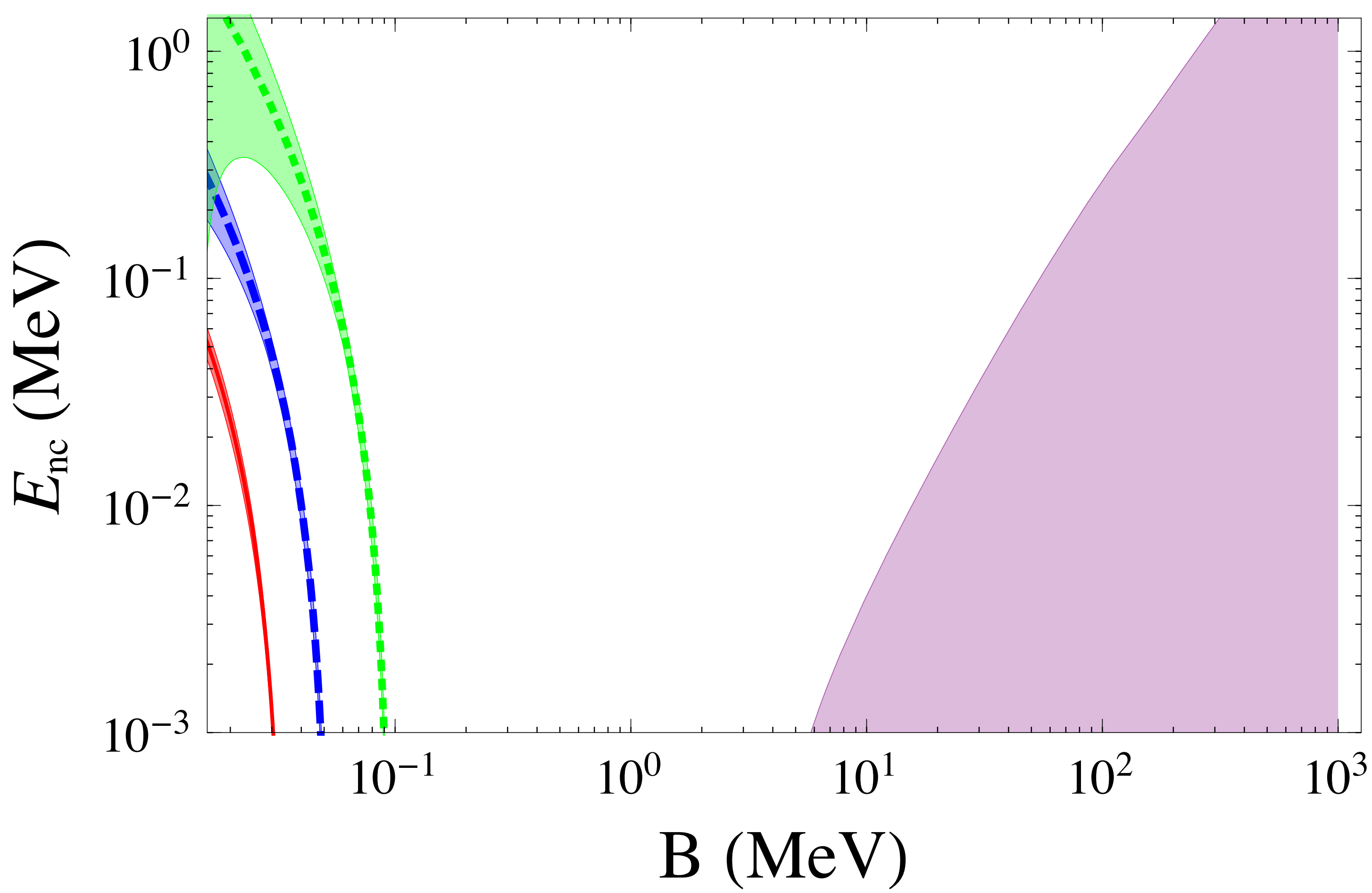

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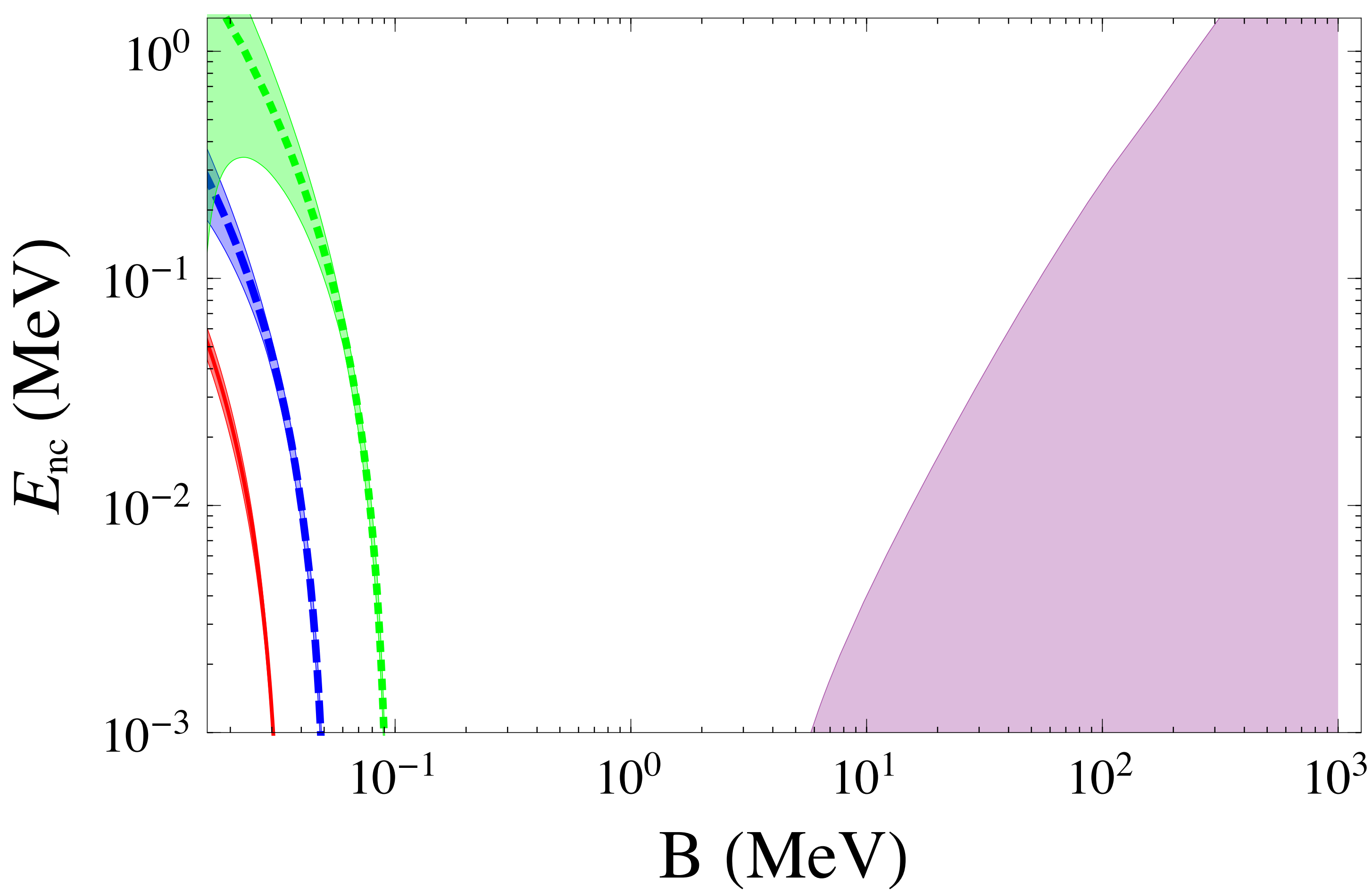


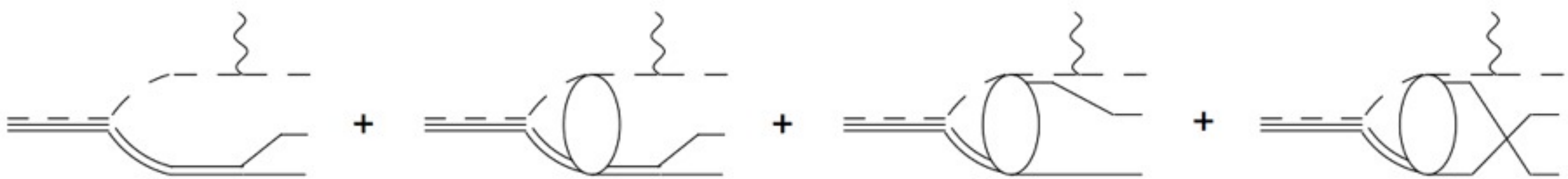
- But ground state of ^{22}C bound by at most 100 keV

- No Efimov state in ^{22}C unless ^{21}C resonance occurs within 1 keV of threshold

Coulomb dissociation of two-neutron halos

Acharya, Hagen, Hammer, Phillips, in progress

- Extend treatment of 1n halos to 2n halos
- So far, calculation “without FSI” :



- Still under development: FSI will be included
- Plan is to predict forthcoming data on ^{22}C, as well as apply to ^{6}He, ^{11}Li, etc.

Nakamura et al., experiment at RIKEN

- Differential distributions also contain information on sub-system interactions