



**Nuclear Halos and Efimov Effect:  
A three-body approach**

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**The 22<sup>nd</sup> European Conference on Few-Body Problems in Physics  
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# Plan of the talk

- ✚ *Introduction to Efimov Effect*
- ✚ *The three-body formalism*
- ✚ *Searching for Efimov states in  $2n$  halo nuclei:*  
 *$^{14}\text{Be}$ ,  $^{19}\text{B}$ ,  $^{22}\text{C}$ ,  $^{20}\text{C}$  some results*
- ✚ *Fano resonances of Efimov states*
- ✚ *Probing few other candidates*
- ✚ *Summary and future scope*

# Collaborators

- V.S. Bhasin *Delhi Univ.*
- V. Arora *Delhi Univ.*
- A.R.P. Rau *Louisiana State Univ.*

- *Phys. Rev. Lett.* 99, 269202
- *Nucl. Phys.* A790, 257
- *Phys. Rev. Lett.* 97, 062503
- *Phys. Rev.* C69, 061301(R)
- *Phys. Rev.* C61, 051303(R)
- *Phys. Rev.* C56, R5
- *Phys. Rev.* C50, R550
- *Few Body Systems*, 2009
- *Pramana*, 2010
- *Phys. Lett. B* (2011)

- Phys. Rep* 212 (1992) J.M. Richard
- Phys. Rep.* 231 (1993) 151(Zhukov et al.)
- Phys. Rep.* 347 (2001) 373 (Nielsen et al.)
- Prog. Part. Nucl. Phys.* 47,517 (2001) (Brown)
- Rev. Mod. Phys.* 76,(2004) 215(Jensen et al.)
- Phys. Rep.* 428, (2006) 259(Braaten & Hammer)
- Ann Rev. Nucl. Part. Sci.* 45, 591(Hansen et al.)
- Rev. Mod. Phys.* 66 (1105)(K. Riisager)
- Rev. Mod. Phys.* 82 (2910) 1225 (C. Chin et al.)

## Efimov Effect

“ A three-body system can support infinite bound states when none of the three pairs are bound, or one or two pairs are barely bound.”

V. Efimov, *Phys. Lett. B* 33, 563 (1970); *Comments Nucl. Part. Phys.* 19, 271 (1990), Amado & Noble, *Phys. Lett. B* 35, 25 (1971)

### Universality:

Independent of the details of the 2-body interaction

Adjacent energy levels are related by

$$\frac{E_{N+1}}{E_N} = e^{-2\pi}$$

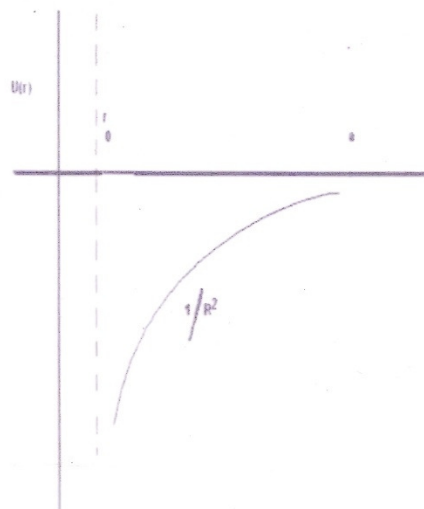
Size of the Nth state is

$$R_{\text{size}} \sim r_0 e^{N\pi}$$

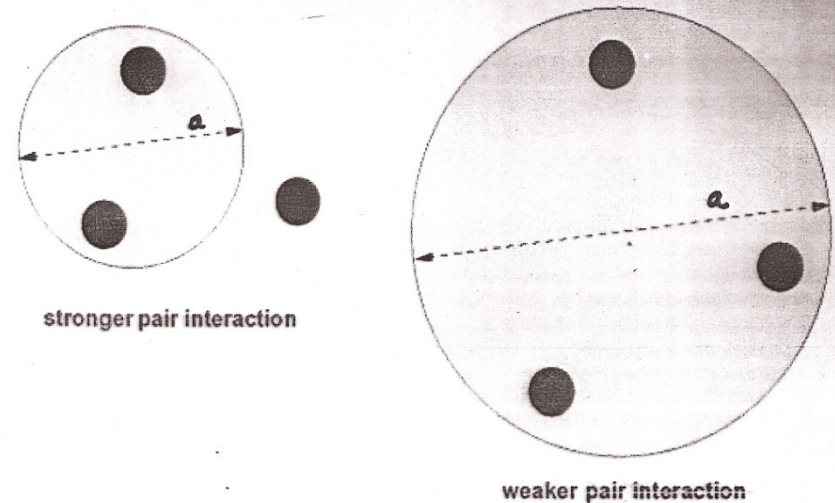
The number of states decreases with increasing 2-body strength

### Necessary Conditions:

- Low energy requirement
- Large scattering length



## The Efimov Effect (A Simple Visualisation)



This scenario was predicted for three-body systems with

$$a \gg r_0 \quad \text{where} \quad \begin{array}{l} a = \text{two-body scattering length} \\ r_0 = \text{two-body effective range} \end{array}$$

Note: modern helium pair potentials have  $a \approx 104 \text{ \AA}$   
 $r_0 \approx 11 \text{ \AA}$

Artificially weakening the pair interaction introduces up to infinitely many three-body bound states.

V. Efimov; *Phys. Lett.* 33B, 563 (1970)

V. Efimov; *Comments Nucl. Part. Phys.* 19, 271 (1990)

## Efimov effect:

*To  
Efimov Physics*

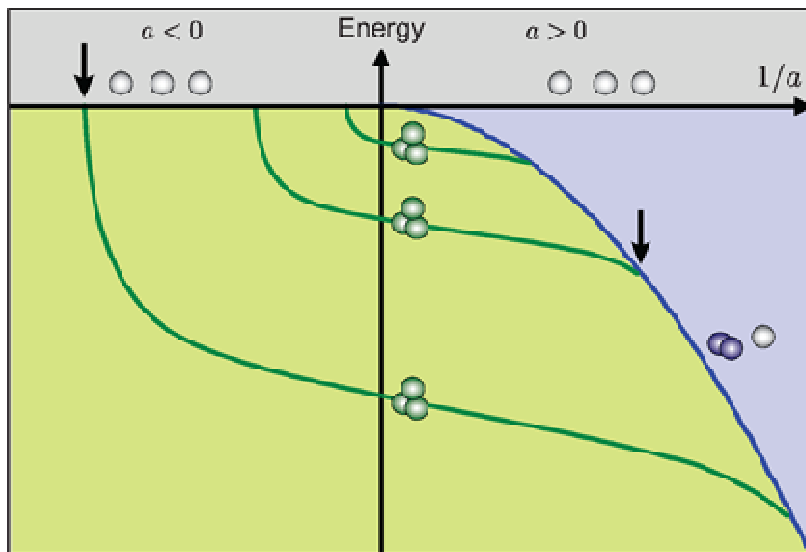


*“ From questionable to pathological to exotic to a hot topic ... ”*

**Nature Physics 5, 533 (2009)**

**Vitaly Efimov**

*Univ. of Washington, Seattle*



**Efimov, 1990**

**Ferlaino & Grimm 2010**

## **V. Efimov:**

Sov. J. Nucl. Phys 12, 589 (1971)  
Phys. Lett. 33B (1970)  
Nucl. Phys A 210 (1973)  
Comments Nucl. Part. Phys.19 (1990)

## **Amado & Noble:**

*Phys. Lett. 33B (1971)*  
*Phys. Rev. D5 (1972)*

## **Fonseca et al.**

*Nucl. PhysA320, (1979)*

## **Adhikari & Fonseca**

*Phys. Rev D24 (1981)*

**L.H. Thomas,**  
**Phys.Rev.47,903(1935)**

## Theoretical searches in Atomic Systems

T.K. Lim *et al.* PRL38 (1977)

Cornelius & Glockle, J. Chem Phys. 85 (1986)

T. Gonzalez-Lezana *et al.* PRL 82 (1999),

*The case of  
He trimer*

## Diffraction experiments with transmission gratings

**Carnal & Mlynek, PRL 66 (1991)**

**Hegerfeldt & Kohler, PRL 84, (2000)**

## Three-body recombination in ultra cold atoms

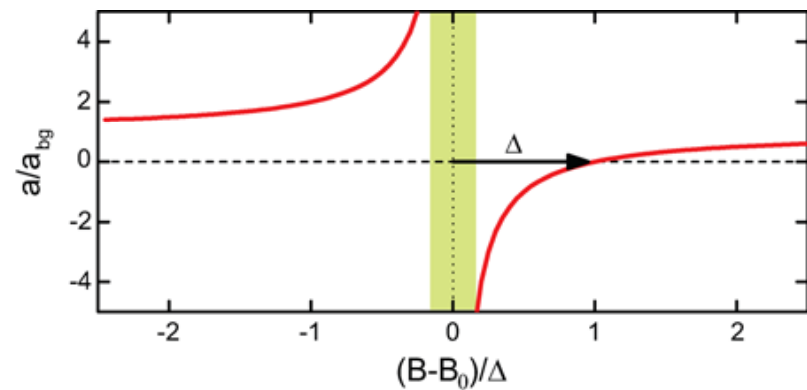
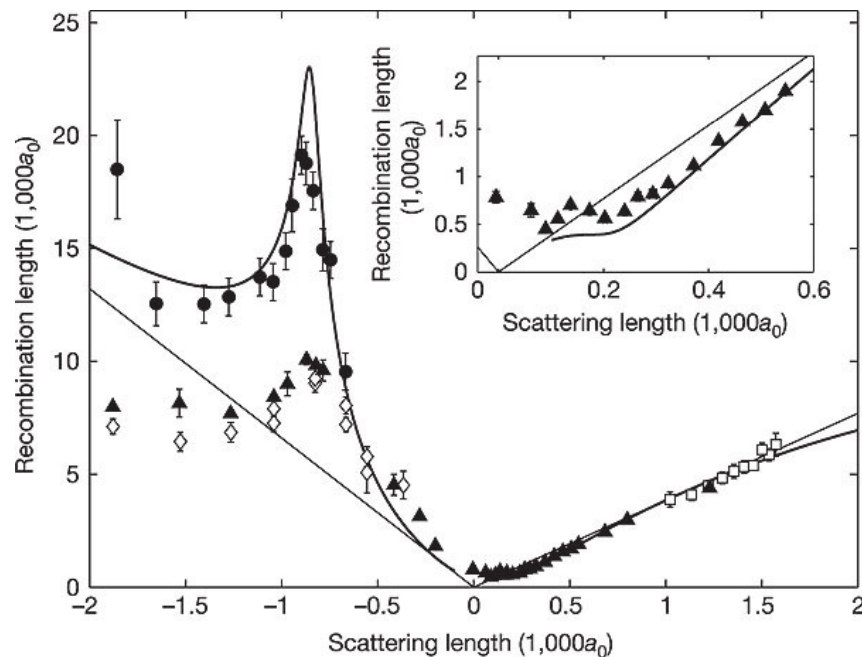
# First Observation of Efimov States

## Letter

*Nature* **440**, 315-318 (16 March 2006) |

## Evidence for Efimov quantum states in an ultracold gas of caesium atoms

*T. Kraemer, M. Mark, P. Waldburger, J. G. Danzl, C. Chin, B. Engeser, A. D. Lange, K. Pilch, A. Jaakkola, H.-C. Nägerl and R. Grimm*



**Observation of an Efimov spectrum in an atomic system.**

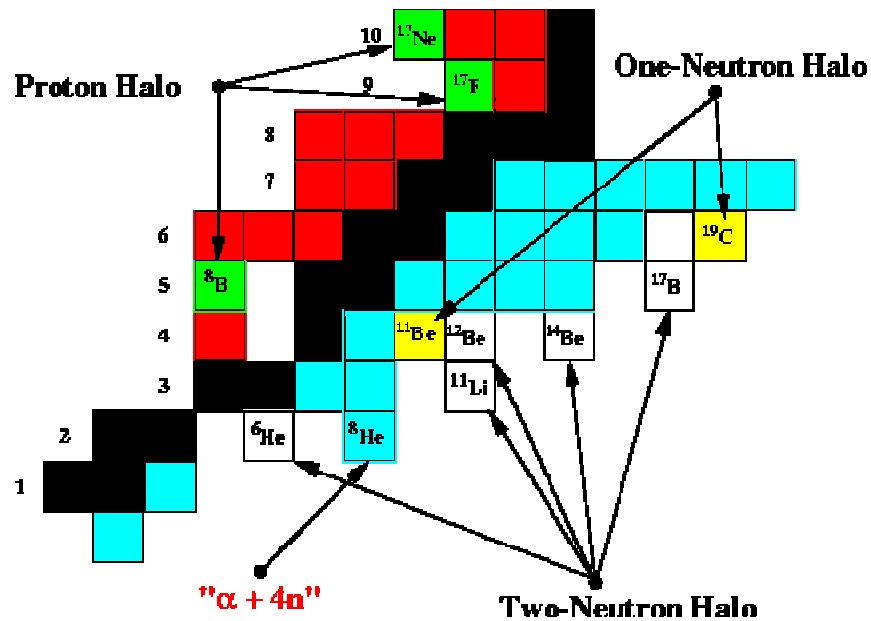
*M. Zaccanti et al. Nature Physics 5, 586 (2009)*

*First two states of the Efimov spectrum in ultra cold 39K*

Can we find Efimov Effect in the atomic nucleus?

Unlike cold atom experiments we have no control over the scattering lengths.

*The discovery of 2-neutron halo nuclei, characterized by very low separation energy and large spatial extension are ideally suited for studying Efimov effect in atomic nuclei.*



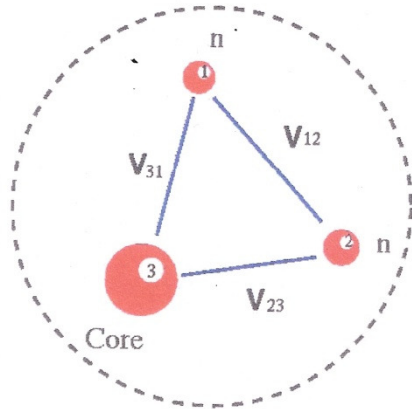
## The Formalism

The 2-neutron halo nucleus  $^{11}\text{Li}$  is modeled as a three-body system consisting of a compact core of  $^9\text{Li}$  and two valence neutrons forming a halo around the core. We label the two neutrons and the core as 1,2,3 with momenta  $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$  respectively. Assuming the core to be a structureless and spinless object, we write the Schrodinger equation in momentum space as

$$(\mathbf{T} - E)\psi = -(\mathbf{V}_{12} + \mathbf{V}_{23} + \mathbf{V}_{31})\psi$$

Where  $E$  is the total energy (= - binding energy, B.E.) and  $\mathbf{T}$  represents the kinetic energy such that

$$\begin{aligned} \mathbf{T} - E &= \mathbf{p}_1^2/2m + \mathbf{p}_2^2/2m + \mathbf{p}_3^2/2m_3 - E \\ &= \mathbf{p}_{ij}^2/2\mu_{ij} + \mathbf{p}_{k}^2/2\mu_{ij-k} - E \end{aligned}$$



For the two-body interactions we consider non-local, separable potentials of the Yamaguchi form and assume s-state interactions both for n-n and n- $^9\text{Li}$  systems.

*Dasgupta, Mazumdar, Bhasin,  
Phys. Rev C50,550*

## Structural properties of $^{11}\text{Li}$

The three body bound state wave function in momentum space using the binary separable potentials

$$V_{12} = -\frac{\lambda_n}{2\mu_{12}}g(p_{12})g(p'_{12}),$$

$$V_{23} = -\frac{\lambda_c}{2\mu_{23}}g(p_{23})g(p'_{23}),$$

$$V_{31} = -\frac{\lambda_c}{2\mu_{31}}g(p_{31})g(p'_{31}) \text{ is}$$

$$\psi(\vec{p}_{12}, \vec{p}_3; E) = D^{-1}(\vec{p}_{12}, \vec{p}_3; E)[g(p_{12})F(\vec{p}_3) + f(p_{23})G(\vec{p}_1) + f(p_{31})G(\vec{p}_2)] \quad (1)$$

$$g(p) = 1/(p^2 + \beta_n^2), \quad f(p) = 1/(p^2 + \beta_c^2), \quad \lambda_{n,c}, \beta_{n,c}$$

reproduce spin singlet scattering length and effective range.

The spectator functions  $F(p)$  and  $G(p)$  satisfy the homogeneous coupled integral equations

$$[\Lambda_n^{-1} - h_n(p)]F(\vec{p}) = 2 \int d\vec{q} K_1(\vec{p}, \vec{q})G(\vec{q}) \quad (2)$$

$$[\Lambda_c^{-1} - h_c(p)]G(\vec{p}) = \int d\vec{q} K_2(\vec{p}, \vec{q})F(\vec{q}) + \int d\vec{q} K_3(\vec{p}, \vec{q})G(\vec{q}) \quad (3)$$

After the angular integration, the two couple equation reduce to an integral equation in one variable. These equations are numerically computed as an eigenvalue problem.

$$\lambda_n = 23.43 \alpha^3, \quad \beta = 6.255\alpha$$
$$a_{nn} = -23.69 \text{ fm}, \quad r_{nn} = 2.32 \text{ fm}$$

**D.E. Gonzalez Trotter et al.**  
*Physical Rev. Letters. 83*

Assuming virtual state of 0.8 Mev for n-core

$$\lambda_c = 14.0\alpha^3, \quad \beta_1 = 5.5\alpha$$
$$a_{nc} = -7.7 \text{ fm}, \quad r_{nc} = 2.67 \text{ fm}$$

$$a_{nn} = -18.7 (0.6) \text{ fm}$$

$^2\text{H}(n, nnp)$  reaction at TUNL

## We Calculate

- 2-n separation energy
- Momentum distribution of n & core
- Root mean square radius

*Inclusion of p-state in n-core interaction*

*$\beta$ -decay of  $^{11}\text{Li}$*

$\tau_n^{-1}(p)F(p) \equiv \phi(p)$  and  $\tau_c^{-1}(p)G(p) \equiv \chi(p)$   
**Where**

$$\tau_n^{-1}(p) = \mu_n^{-1} - [\beta_r (\beta_r + \sqrt{p^2/2a + \epsilon_3})^2]^{-1}$$

$$\tau_c^{-1}(p) = \mu_c^{-1} - 2a[1 + \sqrt{2a(p^2/4c + \epsilon_3)}]^{-2}$$

where  $\mu_n = \pi^2 \lambda_n / \beta_1^2$  and  $\mu_c = \pi^2 \lambda_c / 2a \beta_1^3$

are the dimensionless strength parameters.  
**Variables  $p$  and  $q$  in the final integral equation are also now dimensionless,**

$$p/\beta_1 \rightarrow p \text{ \& } q/\beta_1 \rightarrow q$$

and

$$-mE/\beta_1^3 = \epsilon_3, \quad \beta_r = \beta/\beta_1$$

*Factors  $\tau_n^{-1}$  and  $\tau_c^{-1}$  appear on the left hand side of the spectator functions  $F(p)$  and  $G(p)$  and are quite sensitive. They blow up as  $p \rightarrow 0$  and  $\epsilon_3$  approaches extremely small value.*

*The basic structure of the equations in terms of the spectator functions  $F(p)$  and  $G(p)$  remains same.*

*But for the sensitive computational details of the Efimov effect we recast the equations in dimensionless quantities.*

For our purpose of studying the sensitive computational details for the Efimov effect, the equations are suitably transformed in terms of  $\phi(p)$  and  $\chi(p)$ , involving only the dimensionless quantities.

$$\tau_n^{-1}(p)F(p) \equiv \phi(p) \quad \text{and} \quad \tau_c^{-1}(p)G(p) \equiv \chi(p) \quad (1)$$

where

$$\tau_n^{-1}(p) = \mu_n^{-1} - [\beta_r(\beta_r + \sqrt{\frac{p^2}{2a} + \epsilon_3})^2]^{-1},$$

and

$$\tau_c^{-1}(p) = \mu_c^{-1} - 2a[1 + \sqrt{2a(\frac{p^2}{4c} + \epsilon_3)}]^{-2},$$

with  $\beta_r = \frac{\beta}{\beta_1}$ , and  $\mu_n = \pi^2 \lambda_n / \beta_1^3$  and  $\mu_c = \pi^2 \lambda_c / 2a\beta_1^3$  are the dimensionless strength parameters.

The two coupled equations are now reduced to one integral equation for  $\chi(p)$

$$\Lambda_i \chi(\vec{p}) = \int d\vec{q} K_3(\vec{p}, \vec{q}, \epsilon_3) \tau_c(\vec{q}) \chi(\vec{q}) + 2 \int d\vec{q} d\vec{q}' K_2(\vec{p}, \vec{q}, \epsilon_3) \tau_n(\vec{q}) \times K_1(\vec{q}, \vec{q}', \epsilon_3) \tau_c(\vec{q}') \chi(\vec{q}'). \quad (2)$$

Here the kernels  $K_1$ ,  $K_2$ ,  $K_3$  are essentially the same except that the variables  $p, q$  etc. are now dimensionless quantities:  $\frac{p}{\beta_1} \rightarrow p$ ,  $\frac{q}{\beta_1} \rightarrow q$ , and  $\frac{-mE}{\beta_1^2} \equiv \epsilon_3$ .

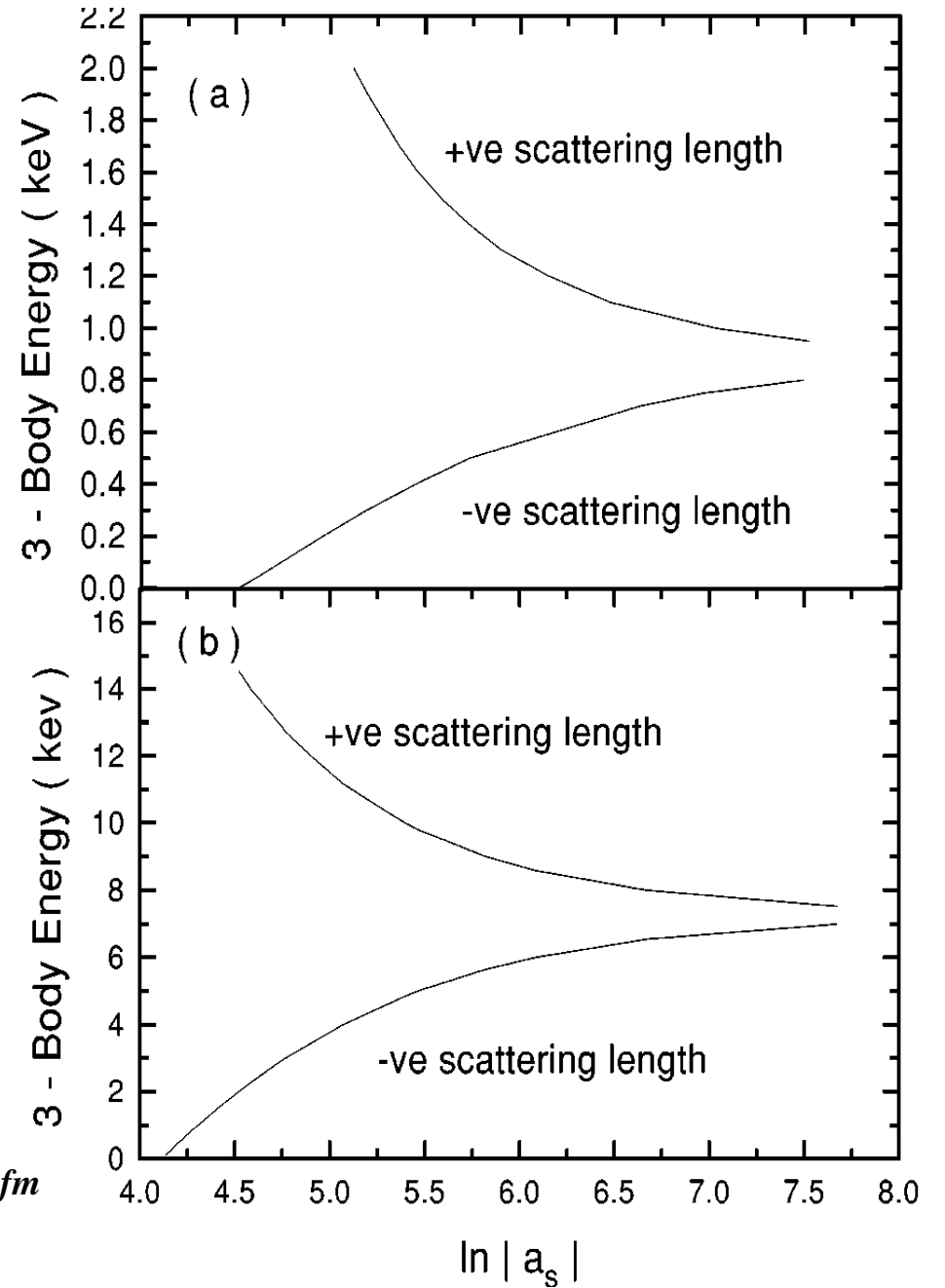
*The integral equation is basically an eigenvalue equation in  $\Lambda_i$  is computed numerically to determine the three-body (core- $n$ - $n$ ) ground-state energy as well as the Efimov states. It is to be noted that the factors  $\tau_n$  and  $\tau_c$  are quite sensitive particularly when the scattering lengths of the binary systems get large values.*

TABLE I.  $^{14}\text{Be}$  ground and excited states three-body energy for different two-body input parameters.

$n$ - $^{12}\text{Be}$ Energy keV	$\lambda_1$	$a_s$ fm	$\epsilon_0$ keV	$\epsilon_1$ keV	$\epsilon_2$ keV
50	11.71	-21	1350		
5.8	12.32	-61.6	1408	.053	
2	12.46	-105	1450	2.56	0.061
1	12.52	-149	1456	3.8	0.22
0.1	12.62	-483	1488	6.1	0.62
0.05	12.63	-658	1490	6.4	0.68
0.01	12.65	-1491	1490	6.9	0.72

### Mazumdar and Bhasin, PRC 56, R5

First Evidence for low lying s-wave strength in  $^{13}\text{Be}$   
 Fragmentation of  $^{18}\text{O}$ , virtual state with scattering length  $< 10$  fm  
 Thoennessen, Yokoyama, Hansen  
 Phys. Rev. C 63, 014308

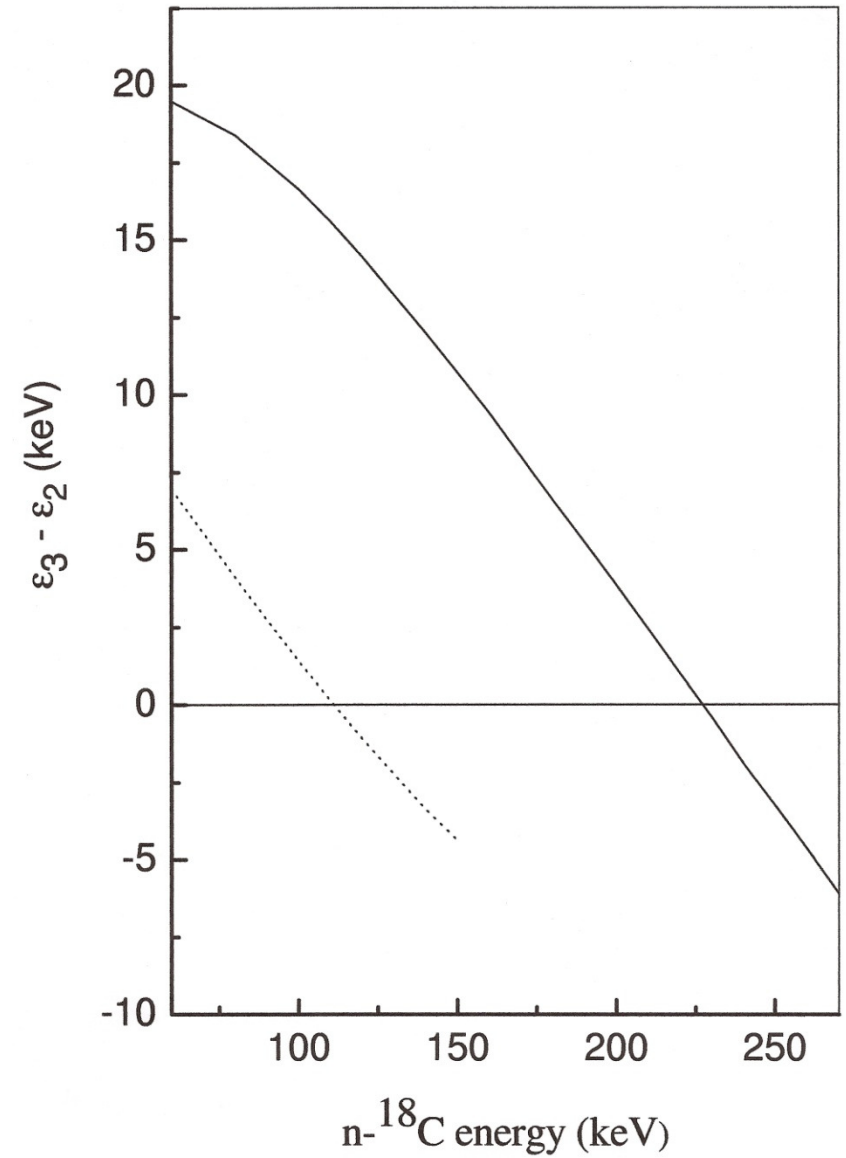


## Search for Efimov states in $^{19}\text{B}$ , $^{22}\text{C}$ , and $^{20}\text{C}$

$n\text{-}^{17}\text{B}$ energy (keV)	$\lambda/\alpha^3$	$a_s$ (fm)	$\epsilon_0$ (keV)	$\epsilon_1$ (keV)	$\epsilon_2$ (keV)
514.8	8.49	-6.515	500		
135.3	9.5	-12.71	728		
48	10.0	-21.16	851		
7.7	10.5	-53.2	978	0.16	
0.67	10.75	-179.6	1042	5.4	0.36

$n\text{-}^{20}\text{C}$ energy (keV)	$\lambda/\alpha^3$	$a_s$ (fm)	$\epsilon_0$ (keV)	$\epsilon_1$ (keV)	$\epsilon_2$ (keV)
319	9.82	-8.23	1120		
127	10.5	-13.02	1287		
48.8	11.0	-21.0	1410		
9.3	11.5	-48.2	1540	0.122	
1.46	11.75	-121.5	1608	4.74	0.198

$n\text{-}^{18}\text{C}$ energy (keV)	$\lambda/\alpha^3$	$a_s$ (fm)	$\epsilon_0$ (keV)	$\epsilon_1$ (keV)	$\epsilon_2$ (keV)
60	15.51	20.38	3188.03	78.87	65.8
100	15.89	16.05	3291.54	115.72	100.09
113.2	16.0	15.15	3317.35	127.41	111.76
139.60	16.2	13.77	3371.24	150.32	135.29
168.59	16.4	12.64	3426.03	175.34	163.48
200	16.6	11.71	3482.95	202.15	194.15



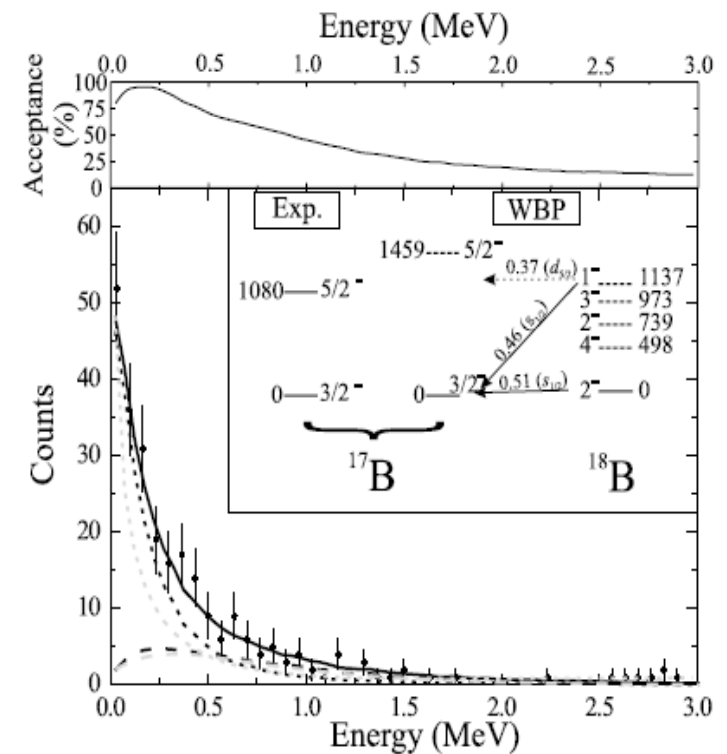
**Mazumdar, Arora Bhasin  
Phys. Rev. C 61, 051303(R)**

## First evidence for a virtual $^{18}\text{B}$ ground state

A. Spyrou<sup>a,b,\*</sup>, T. Baumann<sup>a</sup>, D. Bazin<sup>a</sup>, G. Blanchon<sup>c</sup>, A. Bonaccorso<sup>d</sup>, E. Breitbach<sup>e</sup>, J. Brown<sup>f</sup>,  
 G. Christian<sup>a,b</sup>, A. DeLine<sup>g</sup>, P.A. DeYoung<sup>h</sup>, J.E. Finck<sup>g</sup>, N. Frank<sup>i</sup>, S. Mosby<sup>a,b</sup>, W.A. Peters<sup>j</sup>, A. Russel<sup>g</sup>,  
 A. Schiller<sup>k</sup>, M.J. Strongman<sup>a,b</sup>, M. Thoennessen<sup>a,b</sup>

dimensions, possibly of the order of 100 fm [1,10]. As shown by Mazumdar et al. [9], Efimov states would be expected in  $^{19}\text{B}$  only if the unbound subsystem  $^{18}\text{B}$  ( $n-^{17}\text{B}$ ) would have a virtual ground state corresponding to a scattering length of a few hundred fm.

setup. An  $s$ -wave line shape was used to describe the experimental spectrum resulting in an upper limit for the scattering length of  $-50$  fm which corresponds to a decay energy  $<10$  keV. Observing an  $s$ -wave decay of  $^{18}\text{B}$  provides an experimental verification that the ground state of  $^{19}\text{C}$  includes a large  $s$ -wave component. The presence of this  $s$ -wave component shows that  $s$ - $d$  mixing is still present in  $^{18}\text{B}$  and that the  $s_{1/2}$  orbital has not moved significantly below the  $d_{5/2}$  orbital.



- *The feature observed can be attributed to the singularity in the two body propagator  $[\Lambda_C^{-1} - h_c(\mathbf{p})]^{-1}$ .*
- *There is a subtle interplay between the two and three body energies.*
- *The effect of this singularity on the behaviour of the scattering amplitude has to be studied.*

In order to analyze the effect of this singularity on the behavior of the scattering amplitude for n-<sup>19</sup>C elastic scattering the function G(p) describing the dynamics of the neutron in the presence of (n-<sup>18</sup>C) system, must be subject to the boundary condition, viz,

$$G(\vec{p}) = (2\pi)^3 \delta(\vec{p} - \vec{k}) + \frac{4\pi a_k(\vec{p})}{p^2 - k^2 - i\epsilon} \quad (5)$$

The scattering amplitude is normalized such that, for the s-wave scattering,

$$a_k(\vec{p})|_{|\vec{p}|=|\vec{k}|} \equiv f_k = \frac{e^{i\delta} \sin\delta}{k} \quad (6)$$

Before applying the boundary condition, we rewrite Eq.(3) substituting Eq.(2) for F(p) and finally get the equation for the off-shell scattering amplitude as

$$\begin{aligned} & 4\pi \left(\frac{a}{q}\right) h(p^2, k^2; \alpha_2^2) a_k(\vec{p}) = \\ & (2\pi)^3 K_3(\vec{p}, \vec{k}) + 4\pi \int \frac{d\vec{q} K_3(\vec{p}, \vec{q}) a_k(\vec{q})}{q^2 - k^2 - i\epsilon} + (2\pi)^3 \int d\vec{q} K_3(\vec{p}, \vec{q}) K_1(\vec{q}, \vec{k}) \tau_n(q) + \\ & (8\pi) \int d\vec{q} K_3(\vec{p}, \vec{q}) \tau_n(q) \int d\vec{q}' \frac{K_1(\vec{q}, \vec{q}') a_k(\vec{q}')}{q'^2 - k^2 - i\epsilon} \end{aligned} \quad (7)$$

This integral equation needs to be solved numerically for the scattering amplitude.

*For  $k \rightarrow 0$ , the singularity in the two body cut Does not cause any problem. The amplitude has only real part. The off-shell amplitude is computed By inverting the resultant matrix , which in the limit  $a_o(p)_{p \rightarrow 0} \rightarrow -a$ , the n-<sup>19</sup>C scattering length.*

*For non-zero incident energies the singularity in the two body propagator is tackled by the CSM.*

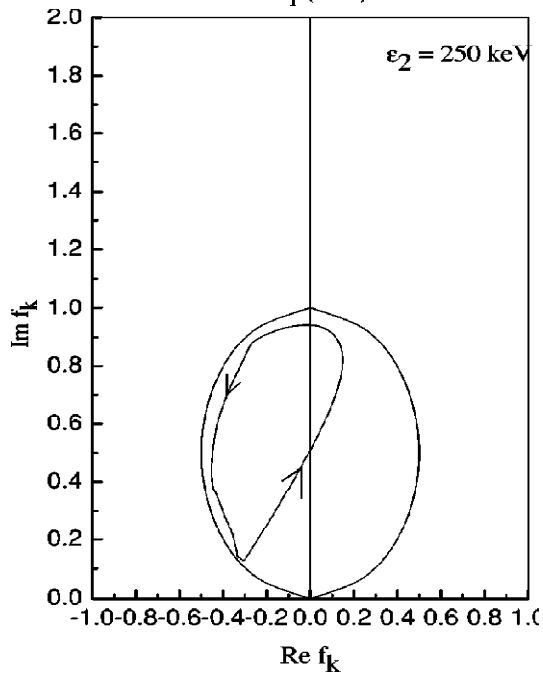
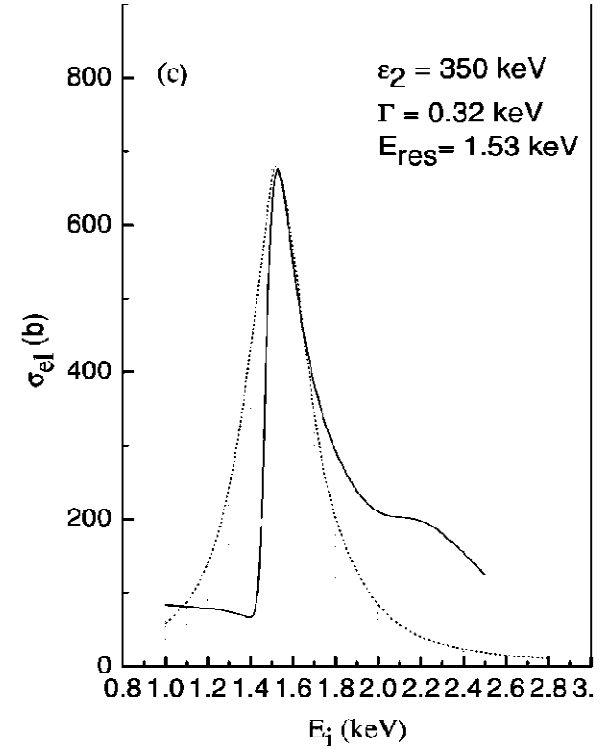
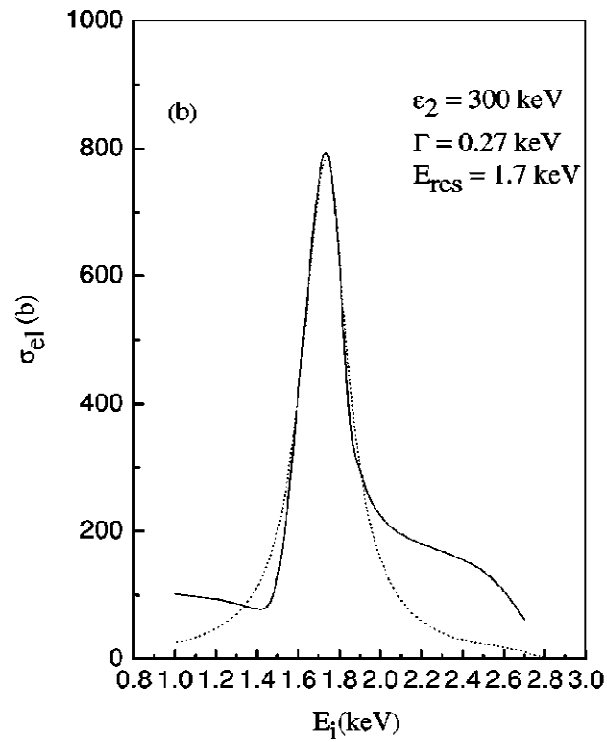
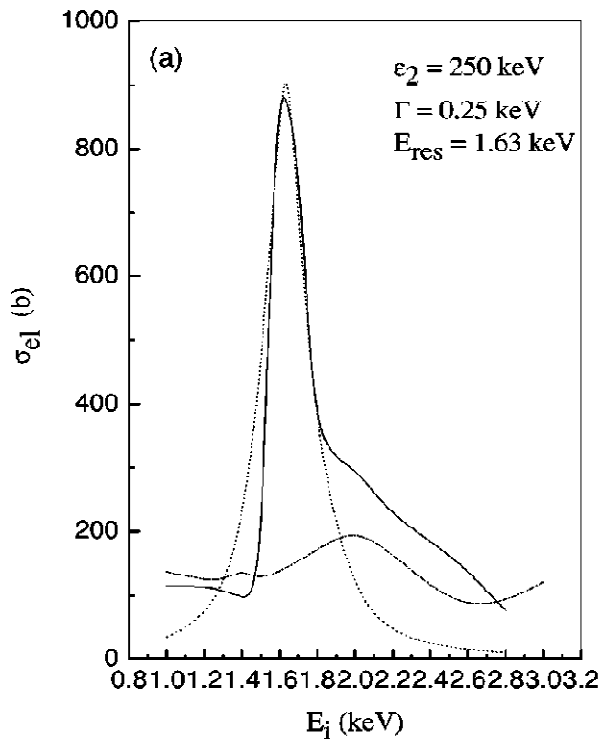
*$P \rightarrow p_1 e^{-i\phi}$  and  $q \rightarrow q e^{-i\phi}$*

*The unitary requirement is the  $Im(f^1_k) = -k$*

***Balslev & Combes (1971)***

***Matsui (1980)***

***Volkov et al.***



n- <sup>18</sup> C Energy (keV)	$\epsilon_3(0)$ (MeV)	$\epsilon_3(1)$ (keV)	$\epsilon_3(2)$ (keV)
60	3.00	79.5	66.95
100	3.10	116.6	101.4
140	3.18	152.0	137.5
180	3.25	186.6	-----
220	3.32	221.0	-----
240	3.35	238.1	-----
250	3.37	-----	-----
300	3.44	-----	-----



**Ugo Fano**  
(1912 – 2001)

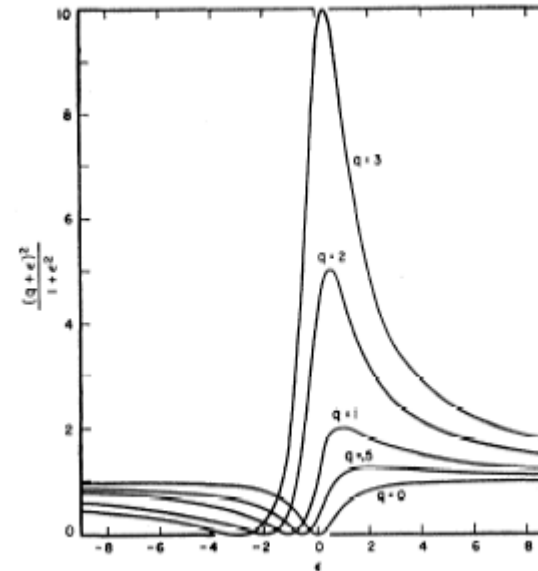


FIG. 1. Natural line shapes for different values of  $q$ . (Reverse the scale of abscissas for negative  $q$ .)

PHYSICAL REVIEW

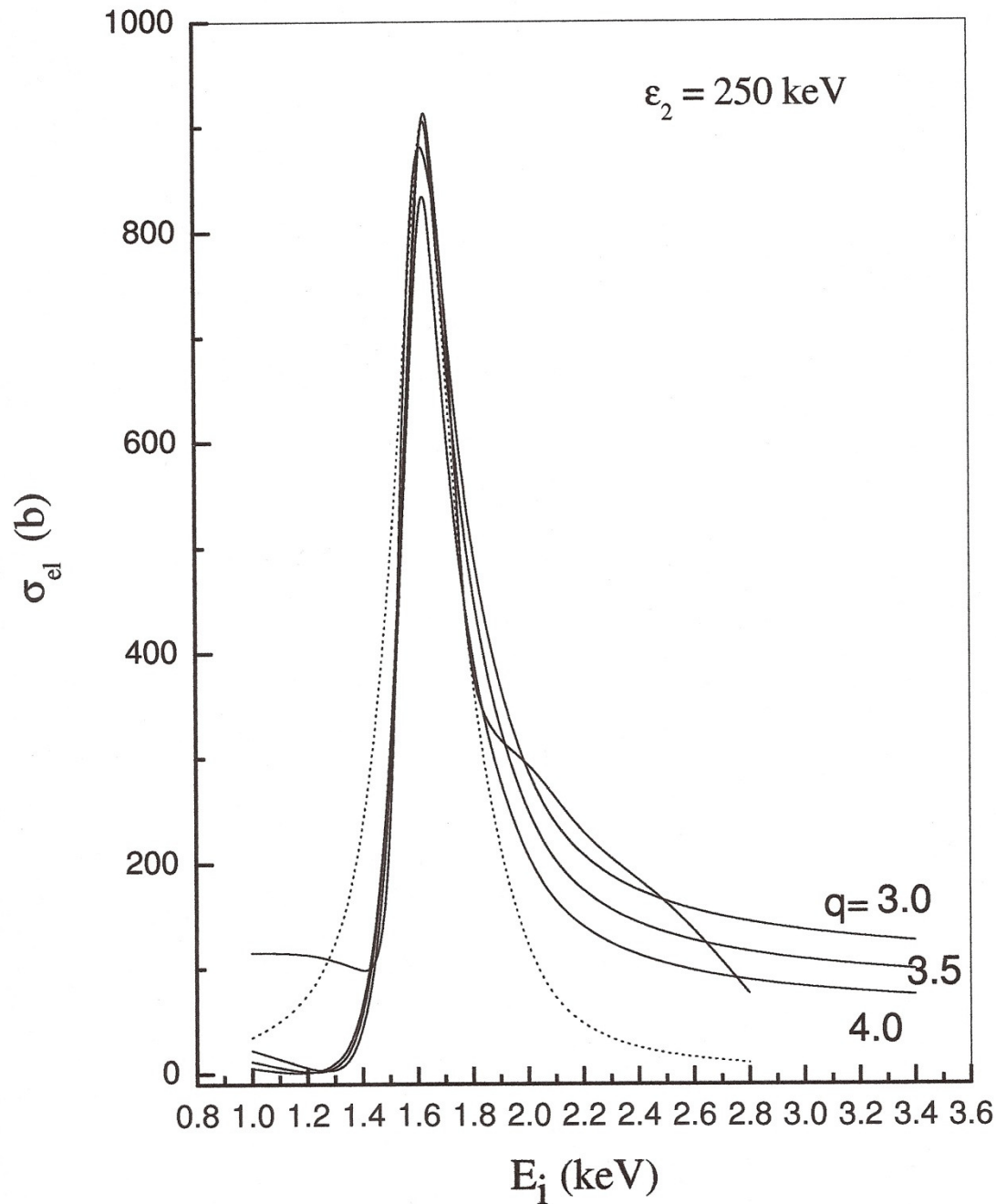
VOLUME 124, NUMBER 6

DECEMBER 15, 1961

**Effects of Configuration Interaction on Intensities and Phase Shifts\***

U. FANO  
National Bureau of Standards, Washington, D. C.  
(Received July 14, 1961)

*Third highest in citation impact of all papers published in the entire Physical Review series.*

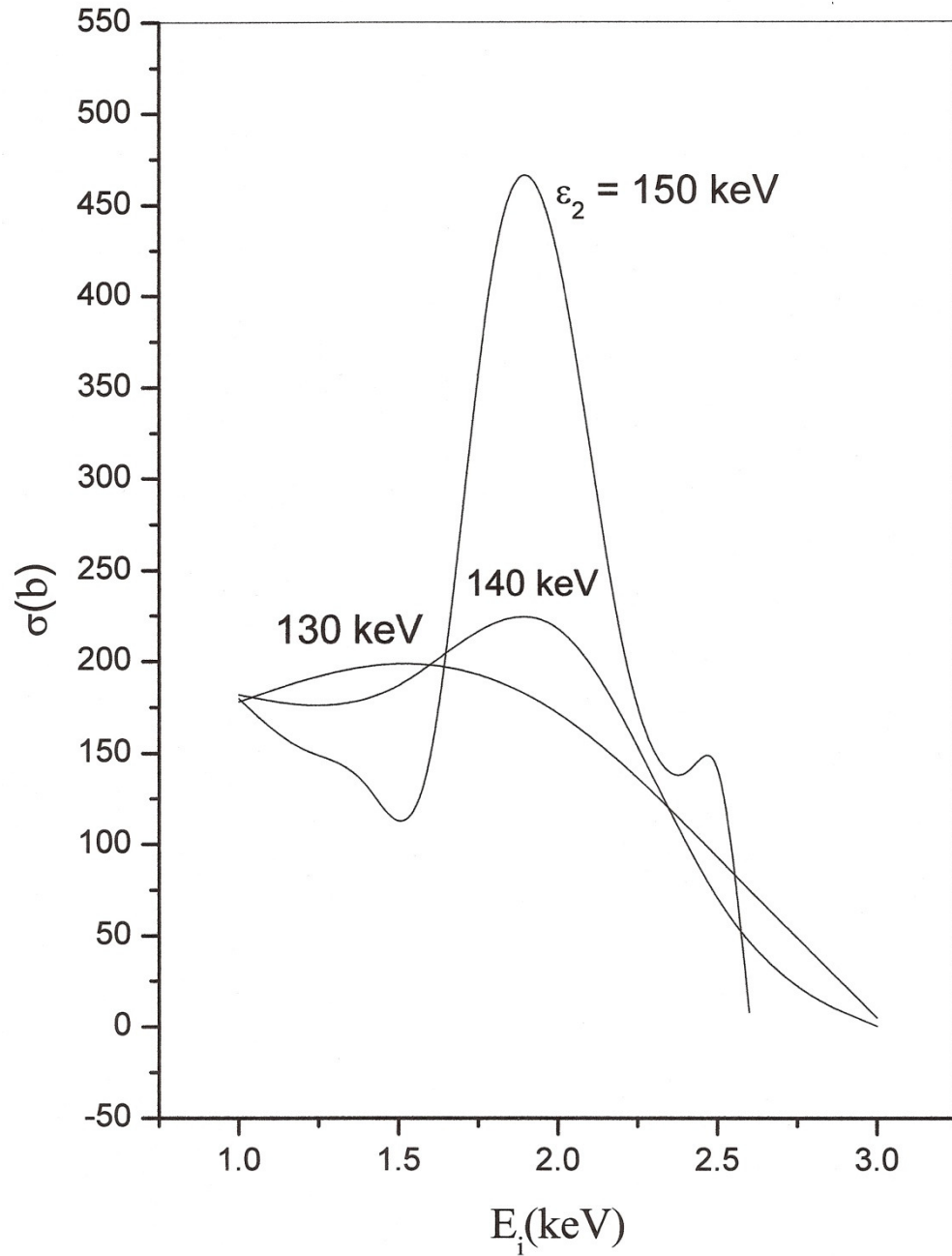


*Fitting the Fano profile to the  
N-<sup>19</sup>C elastic cross section for  
n-<sup>18</sup>C BE of 250 keV*

$$\sigma = \sigma_0 \left[ \frac{(q + \epsilon)^2}{1 + \epsilon^2} \right]$$

**Mazumdar, Rau, Bhasin**  
**Phys. Rev. Lett. 97 (2006)**

*an ancient pond  
a frog jumps in  
a deep resonance*



**The resonance due to the second excited Efimov state for  $n$ - $^{18}\text{C}$  BE 150 keV. The profile is fitted by same value of  $q$  as for the 250 keV curve.**

## *The calculation have been extended to*

1) **Two hypothetical cases:** *very heavy core of mass  $A = 100$  (+  $2n$ )*  
*three equal masses  $m_1=m_2=m_3$*

2) **Two realistic cases of  $^{38}\text{Mg}$  &  $^{32}\text{Ne}$**

Mazumdar, Bhasin, Rau  
*Phys. Lett. B 704, (2011)*

$^{38}\text{Mg}$   $S_{2n} = 2570$  keV  $n + \text{core}$  ( $^{37}\text{Mg}$ ) 250 keV (bound)  
 $^{32}\text{Ne}$   $S_{2n} = 1970$  keV  $n + \text{core}$  ( $^{31}\text{Ne}$ ) 330 keV (bound)

$^{38}\text{Mg}$  ( $S_{2n}$ ) Audi & Wapstra (2003)

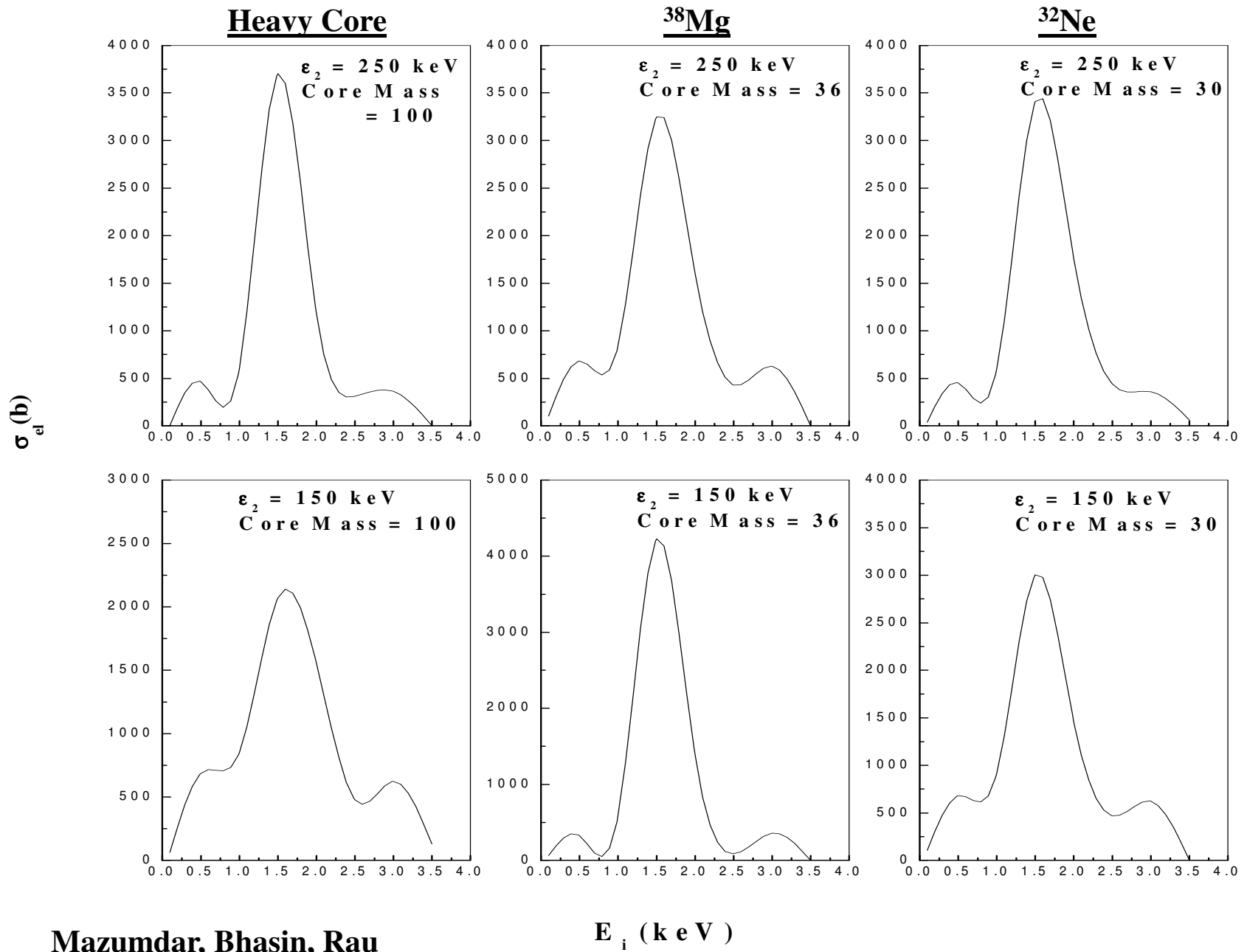
$^{37}\text{Mg}$  &  $^{31}\text{Ne}$  ( $S_n$ ) Sakurai *et al.*, PRC 54 (1996),  
Jurado *et al.* PLB (2007), Nakamura *et al.* PRL (2009),

Hamamoto PRC (2010),  
Urata, Hagino, Sagawa PRC (2011)

*We have reproduced the ground state energies and have found at least two Efimov states that vanish into the continuum with increasing  $n$ -core interaction. They again show up as asymmetric resonances at around 1.6 keV neutron incident energy in the scattering sector.*

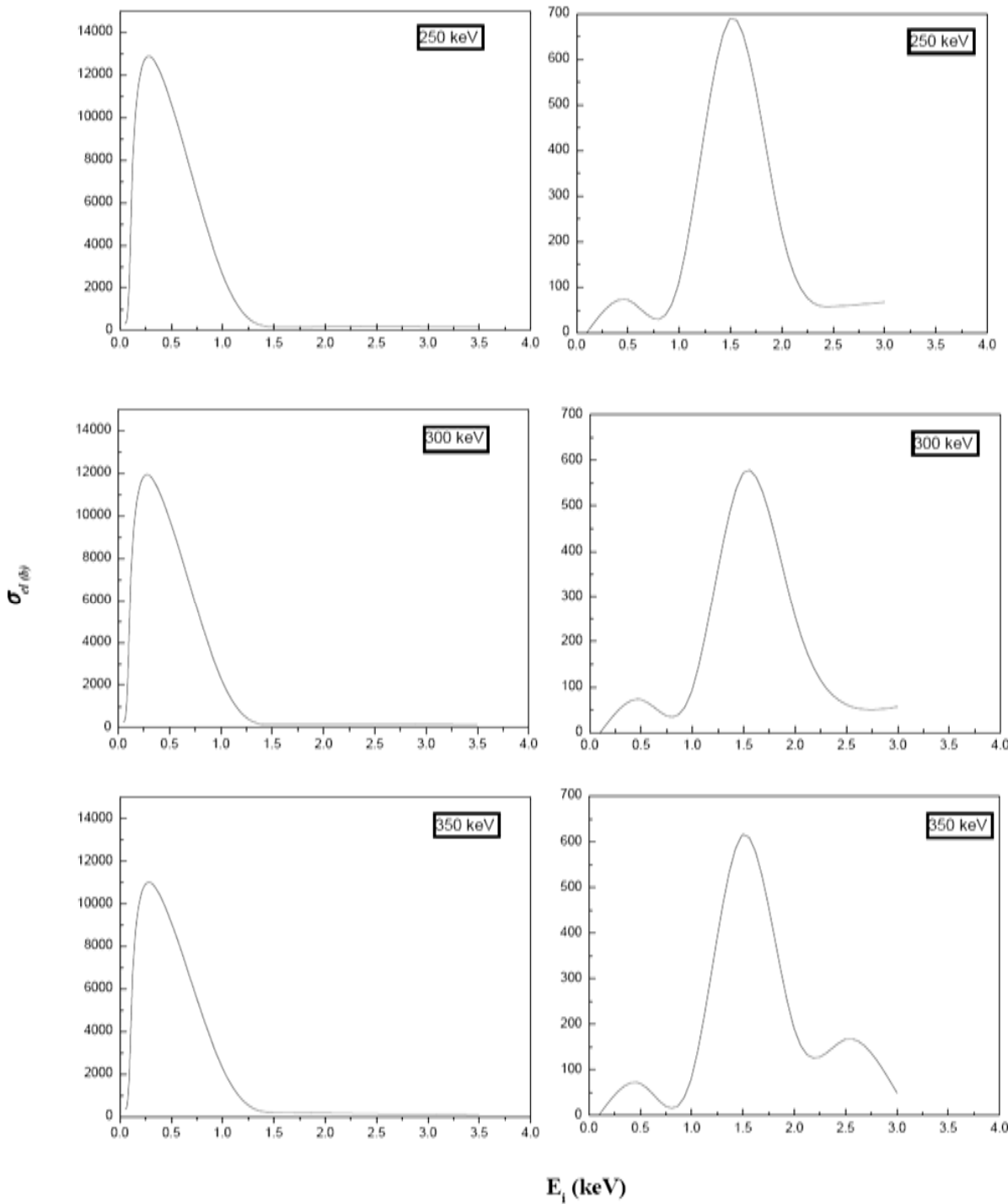
**TABLE: Ground and excited states for three cases studied, namely, mass 102 (columns 2, 3, 4),  $^{38}\text{Mg}$  (column 5, 6, 7), and  $^{32}\text{Ne}$  (columns 8, 9, 10) for different two body input parameters.**

	$^{102}\text{A}$			$^{38}\text{Mg}$			$^{32}\text{Ne}$		
n-Core Energy $\epsilon_2$ keV	$\epsilon_3(0)$ keV	$\epsilon_3(1)$ keV	$\epsilon_3(2)$ keV	$\epsilon_3(0)$ keV	$\epsilon_3(1)$ keV	$\epsilon_3(2)$ keV	$\epsilon_3(0)$ keV	$\epsilon_3(1)$ keV	$\epsilon_3(2)$ keV
40	4020	53.6	44.4	3550	61.3	49.9	3420	61.5	50
60	4080	70.4	61.7	3610	80.8	67.1	3480	81.0	67.2
80	4130	86.9	(78.4)	3670	99.2	84.16	3530	99.8	84.3
100	4170	103.1		3710	117	101.4	3570	117.5	101.5
120	4220	(119.3)		3750	134.5	(118.9)	3620	135	(118.9)
140	4259			3790	151.6		3650	152.5	
180	4345			3860	185.6		3730	186.5	
250	4460			3980			3852		
300	4530			4040			3910		
350	4590			4120			3980		



**Mazumdar, Bhasin, Rau**  
*Phys. Lett. B 704 (2011)*

$E_i$  (keV)



$\epsilon_0$  \_\_\_\_\_ Equal \_\_\_\_\_ Heavy Core

(keV) (keV) (keV)  
 250 455 4400

300 546 4470

350 637 4550

*Ground states for the two cases*

• *Equal mass case strikingly different from unequal ( heavy core ) case.*

• *Evolution of Efimov states in heavy core of 100 fully consistent with  $^{20}\text{C}$  results.*

**Mazumdar**  
*Few Body Systems, 2009*

## Summary

- A three body model with s-state interactions is exploited to search for Efimov states in 2n-halo nuclei.
- A virtual state of a few keV (2 to 4) energy corresponding to scattering length from -50 to -100 fm for the n-<sup>12</sup>Be predicts the ground state and excited states of <sup>14</sup>Be.
- <sup>19</sup>B, <sup>22</sup>C and <sup>20</sup>C are investigated and it is shown that Borromean type nuclei are much less vulnerable to respond to Efimov effect
- <sup>20</sup>C is a promising candidate for Efimov states at energies below the n-(nc) breakup threshold.
- *The bound Efimov states in <sup>20</sup>C move into the continuum and reappear as Resonances with increasing strength of the binary interaction.*
- *Asymmetric resonances in elastic n+<sup>19</sup>C scattering are attributed to Efimov states and are identified with the Fano profile. The conjunction of Efimov and Fano phenomena may lead to the experimental realization in nuclei.*
- <sup>32</sup>Ne & <sup>38</sup>Mg exhibit very similar dynamical structure and are also candidates for probing Efimov states.

## Present Activities:

- ✚ Resonant states above the three body breakup threshold in  $^{20}\text{C}$ .
- ✚ Structure calculations for  $^{12}\text{Be}$  &  $^{14}\text{Be}$
- ✚ Role of Efimov states in Bose-Einstein condensation.
- ✚ Studying the proton halo ( $^{17}\text{Ne}$ ) nucleus.
- ✚ **Planning for possible experiments with  $^{20}\text{C}$  beam**

## Epilogue

**“ the richness of understanding reveals even greater richness of ignorance”**

*D.H. Wilkinson*

*Thank You*



- We emphasize the cardinal role of channel coupling.

*There is also a definite role of mass ratios as observed numerically.*

- However, channel coupling is an elegant and physically plausible scenario.

- Difference can arrive between zero range and realistic finite range potentials in non-Borromean cases.

*Note, that for  $n$ - $^{18}\text{C}$  binding energy of 200 keV, the scattering length is about 10 fm while the interaction range is about 1 fm.*

- The extension of zero range to finer details of Efimov states in non-Borromean cases may not be valid.

- *The discrepancy observed in the resonance vs virtual states in  $^{20}\text{C}$  clearly underlines the sensitive structure of the three-body scattering amplitude derived from the binary interactions.*