Chiral expansion of nuclear forces
explicit Δ(1232) scenario

Hermann Krebs
Ruhr-Universität-Bochum

EFB22, September 10, 2013, Krakow, Poland

With V. Bernard, E. Epelbaum, A. Gasparyan, U.-G. Meißner

RUB

LENPIC

Low Energy Nuclear Physics International Collaboration

TECHNISCHE UNIVERSITÄT DARMSTADT Sven Binder, Angelo Calci, Kai Hebeler, Joachim Langhammer, Robert Roth

IOWA STATE UNIVERSITY Pieter Maris, Hugh Potter, James Vary

THE OHIO STATE UNIVERSITY Richard J. Furnstahl

RUB Evgeny Epelbaum, Hermann Krebs

universität bonn Ulf-G. Meißner

IPN INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY Veronique Bernard

JÜLICH FORSCHUNGSZENTRUM Andreas Nogga

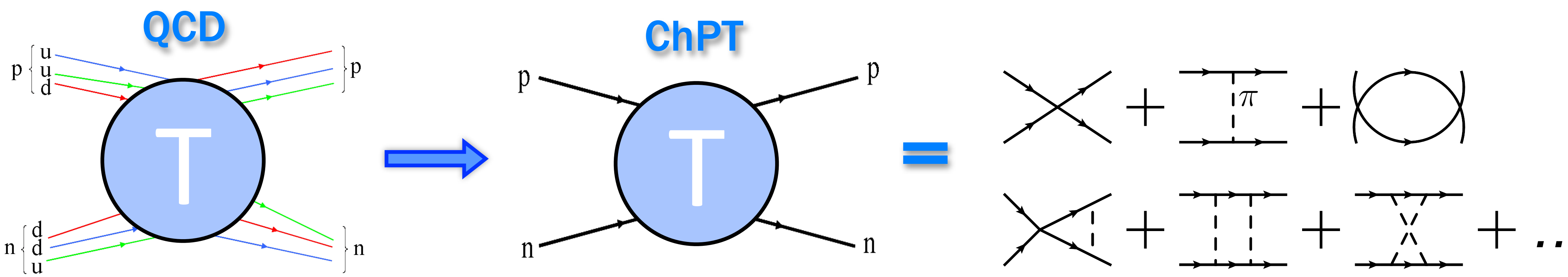
JAGIELLONIAN UNIVERSITY IN KRAKOW Jacek Golak, Roman Skibinski, Kacper Topolniki, Henryk Witala

Kyutech Kyushu Institute of Technology Hiroyuki Kamada

Outline

- Nuclear forces in chiral EFT ⟹ talk by Evgeny Epelbaum
- Role of $\Delta(1232)$ resonance
- Long-range part of three-nucleon forces
- N^3LO-Δ vs. N^4LO Δ-less
- Summary & Outlook

From QCD to nuclear physics

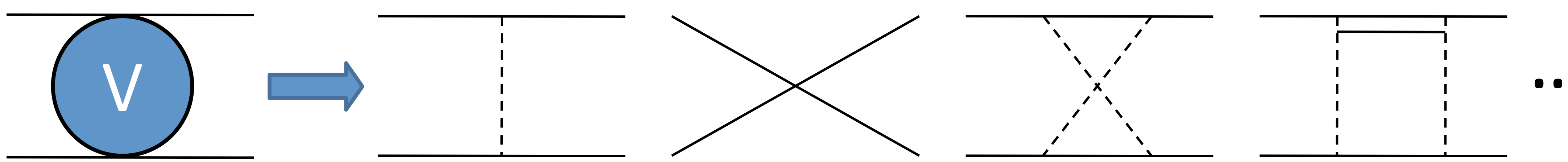


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⟹ the QM A-body problem

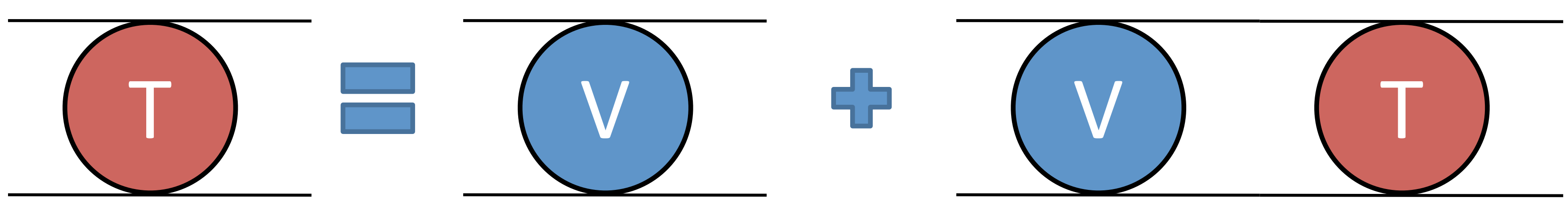
$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle=E|\Psi\rangle$$

Weinberg '91

- Construct effective potential perturbatively

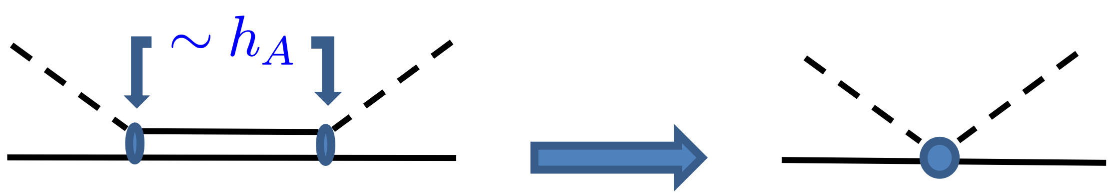


- Solve Lippmann-Schwinger equation nonperturbatively



EFT with explicit Δ(1232)

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293$ MeV
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$ *(Hemmert, Holstein & Kambor '98)*
- Delta contributions encoded in LECs
  (Bernard, Kaiser & Meißner ´97)

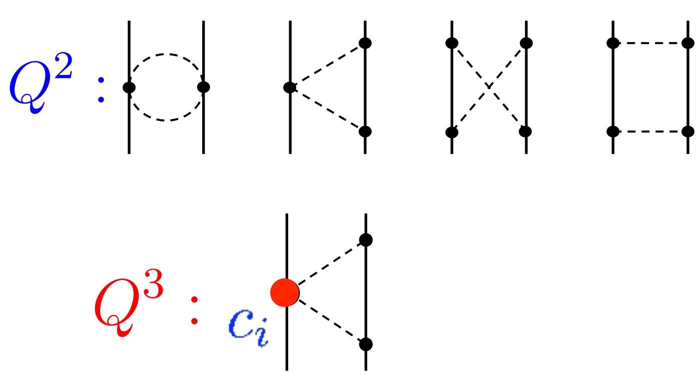


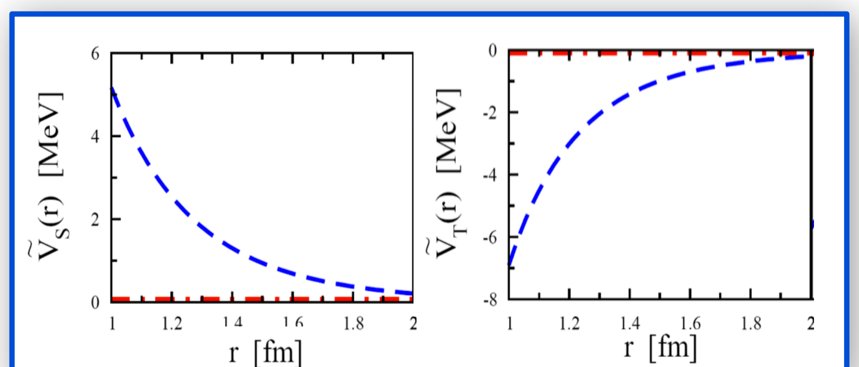
Delta-resonance saturation

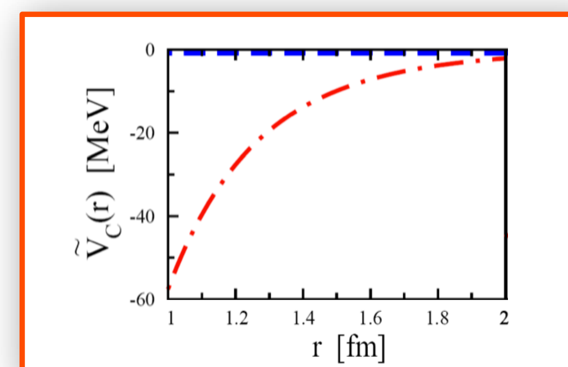
$$c_3 = -2c_4 = c_3(\not\Delta) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

- Convergence of EFT potential







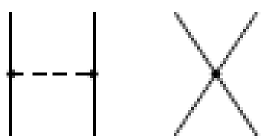
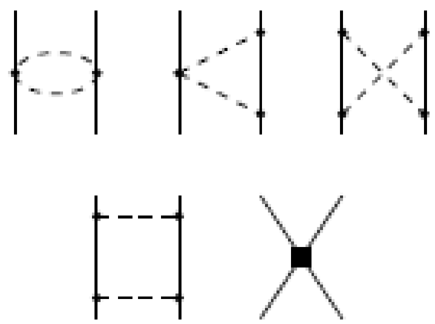
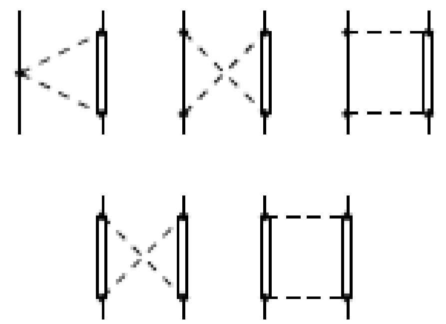
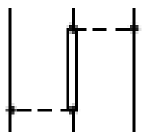
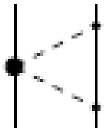
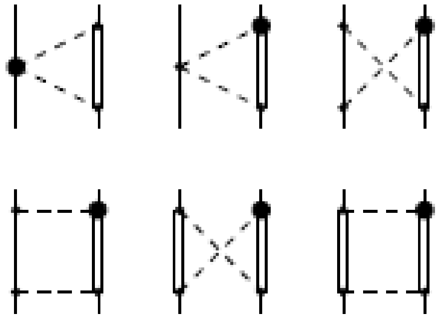
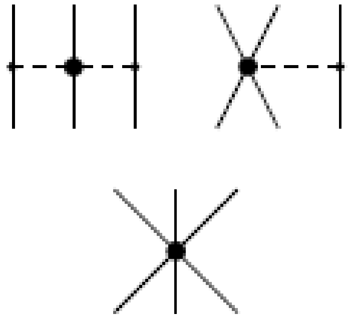
The subleading contributions are larger than the leading one!

Expectation from inclusion of Δ explicitely

- more natural size of LECs
- better convergence
- applicability at higher energies

Few-nucleon forces with the Delta

Isospin-symmetric contributions

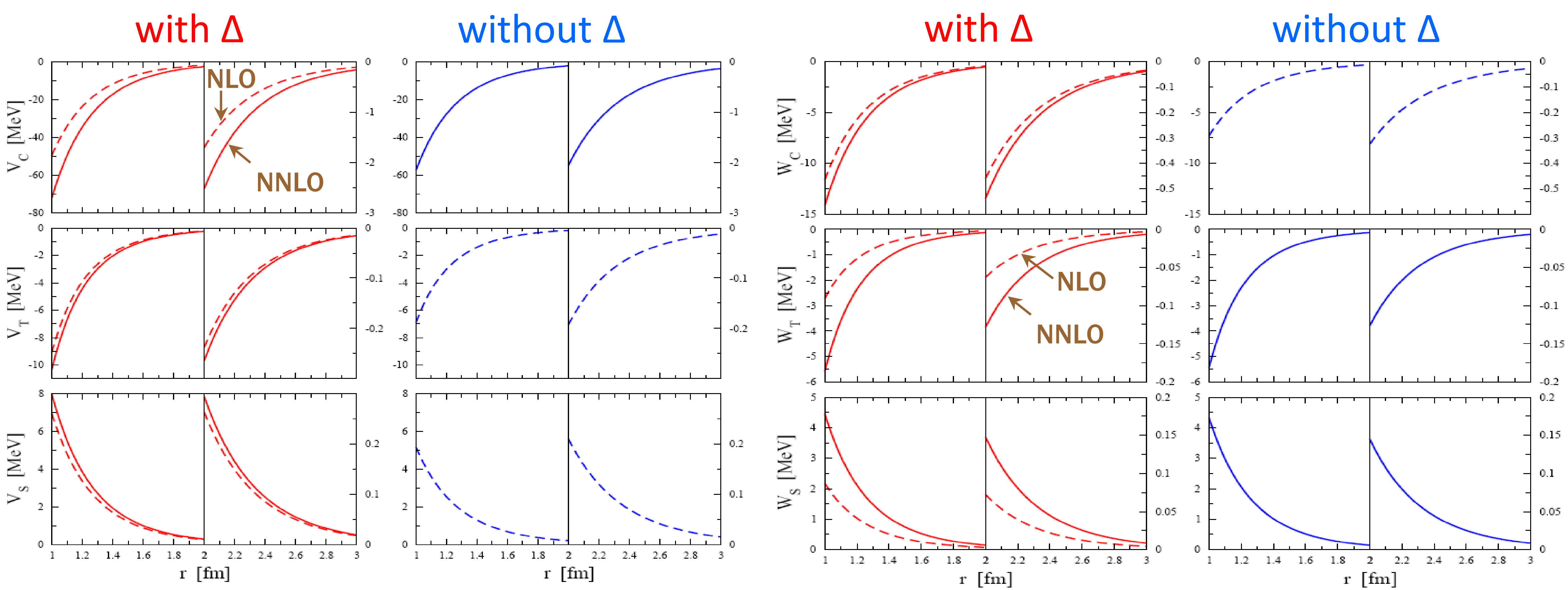
	Two-nucleon force		Three-nucleon force	
	Δ–less EFT	Δ–contributions	Δ–less EFT	Δ–contributions
LO		—	—	—
NLO		 Ordonez et al.'96, Kaiser et al. '98	—	
NNLO		 H.K., Epelbaum & Meißner '07		—

NN potential with explicit Δ

Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127

$$V_{\rm eff} = V_C + W_C \vec{\tau}_1 \cdot \vec{\tau}_2 + [V_S + W_S \vec{\tau}_1 \cdot \vec{\tau}_2] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1 \cdot \vec{\tau}_2] (3\, \vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

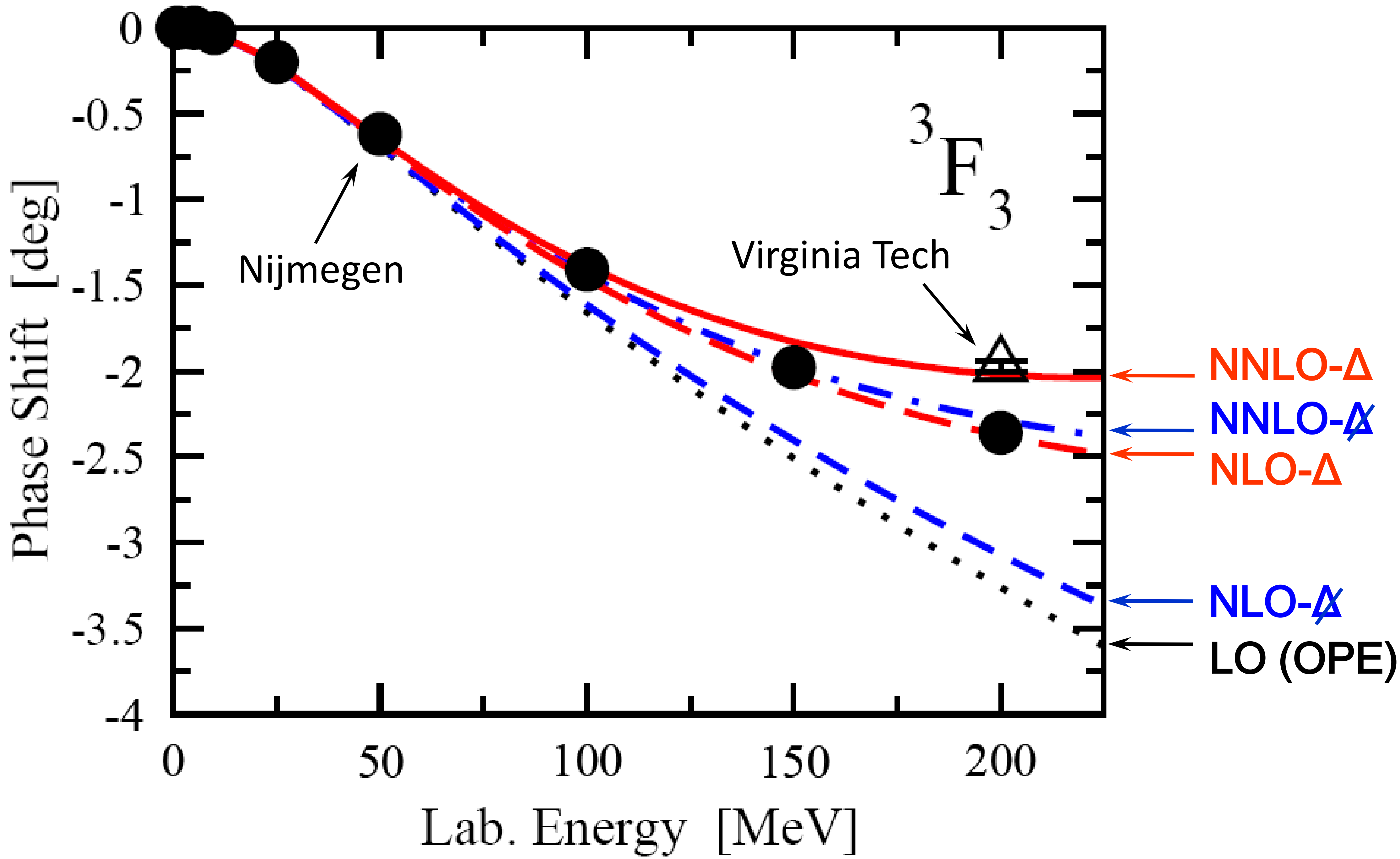
Chiral 2π- exchange potential up to NNLO



Advantages when Δ is included explicitly

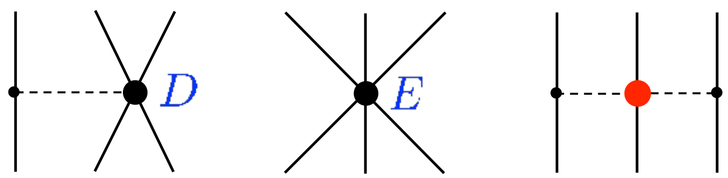
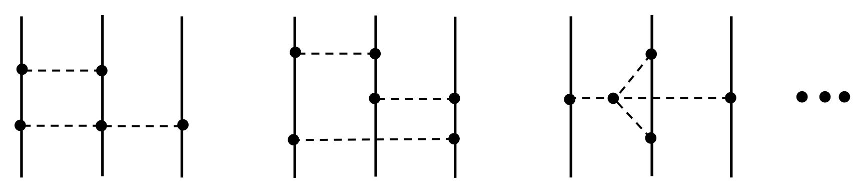
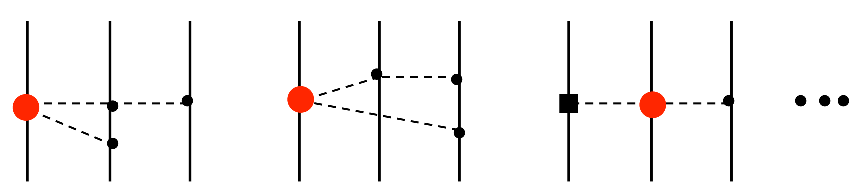
- Dominant contributions already at NLO
- Much better convergence in all potentials

$^3F_3$ partial waves up to NNLO with and without Δ

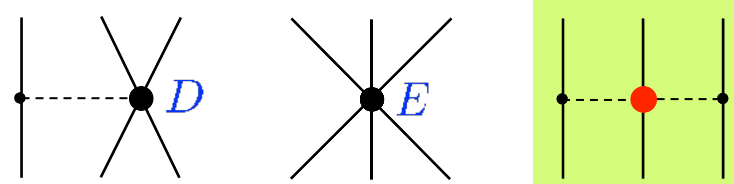
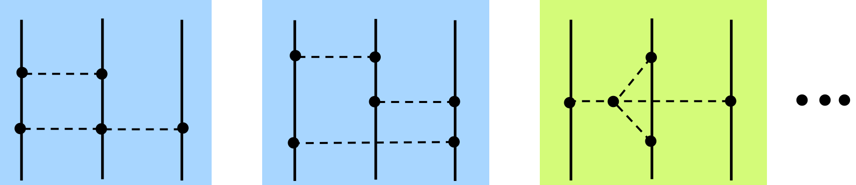
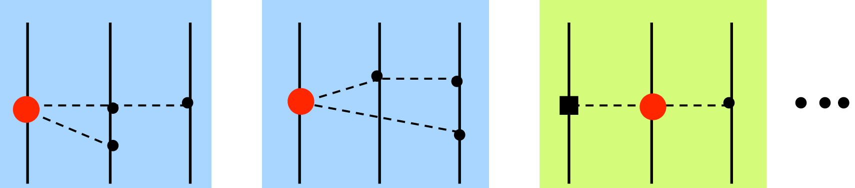


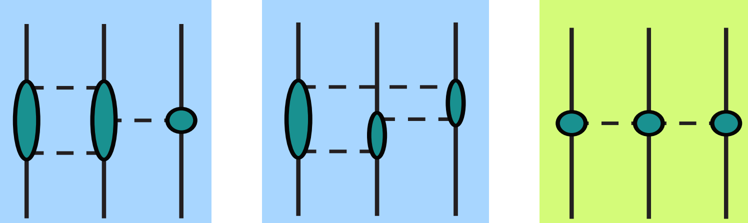
(calculated in the first Born approximation)

Small scale expansion of 3NF

	Δ-less theory	Δ-full theory: additional graphs
NLO	—	
N^2LO	 van Kolck ’94, Epelbaum et al. ’02	
N^3LO	 Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)	
N^4LO	 HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)	

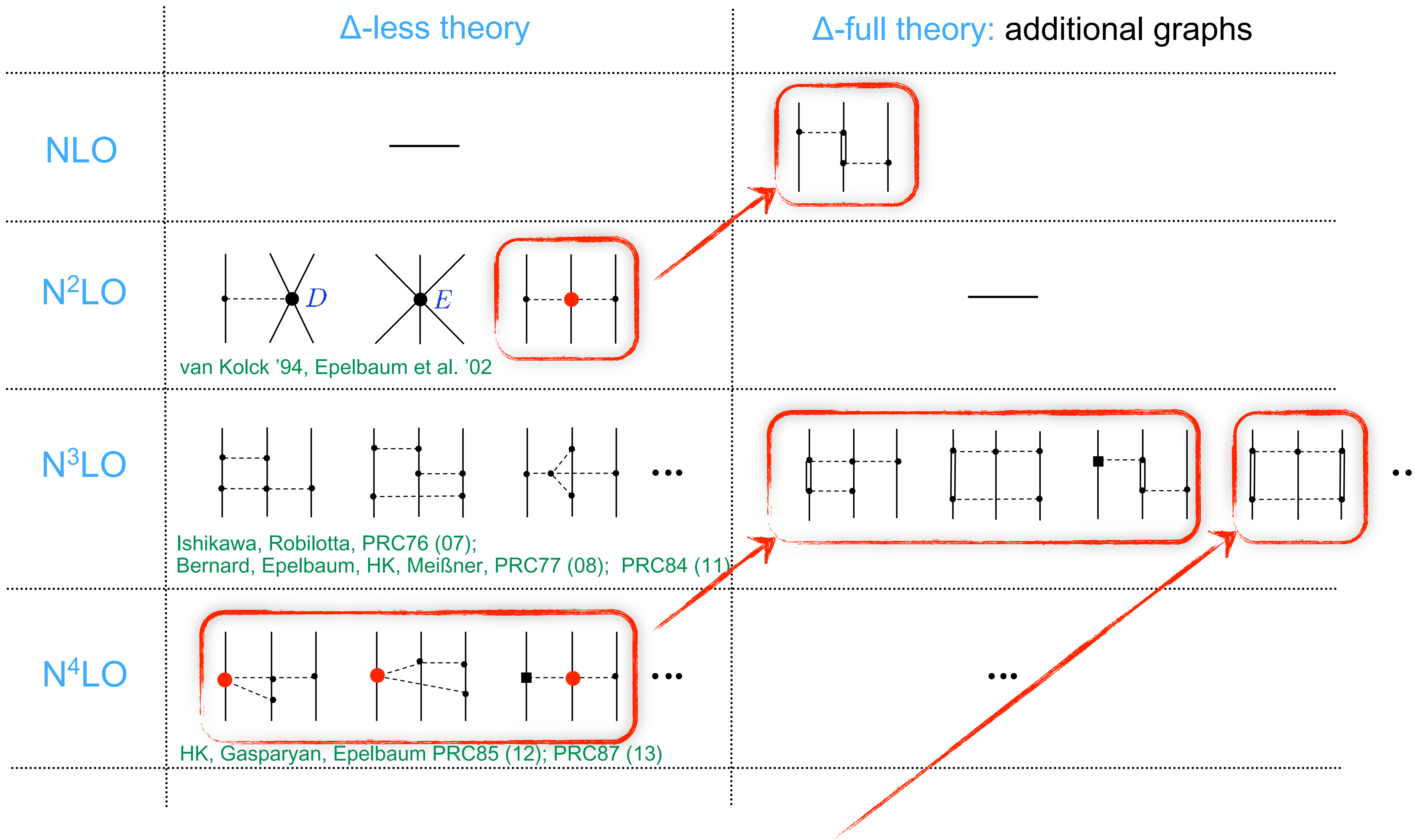
Small scale expansion of 3NF

	Δ-less theory	Δ-full theory: additional graphs
NLO	—	
N^2LO	 van Kolck ’94, Epelbaum et al. ’02	
N^3LO	 Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)	
N^4LO	 HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)	

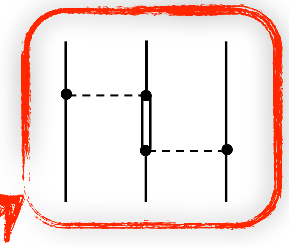
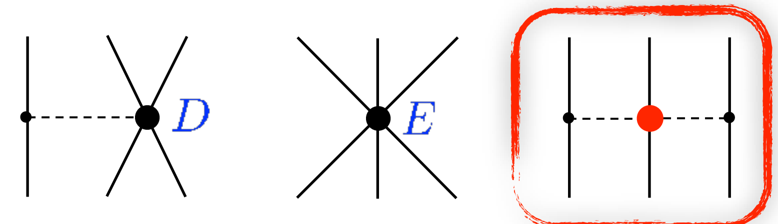
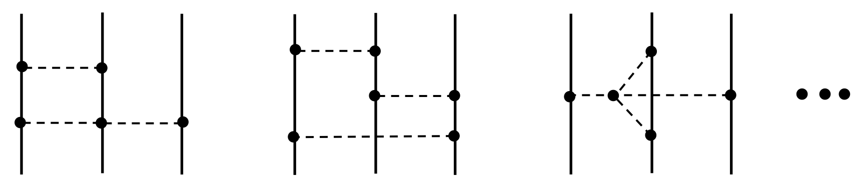
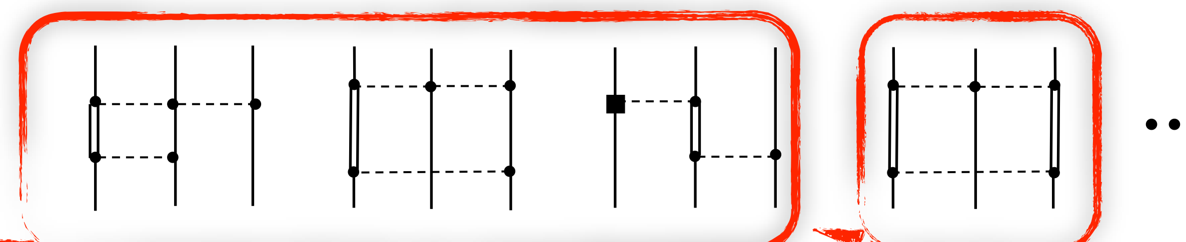
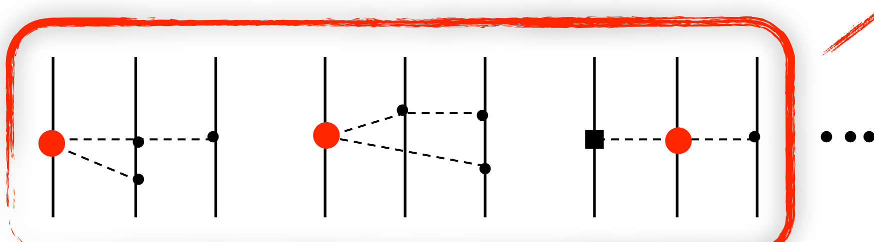


2π-1π | ring | 2π

Small scale expansion of 3NF

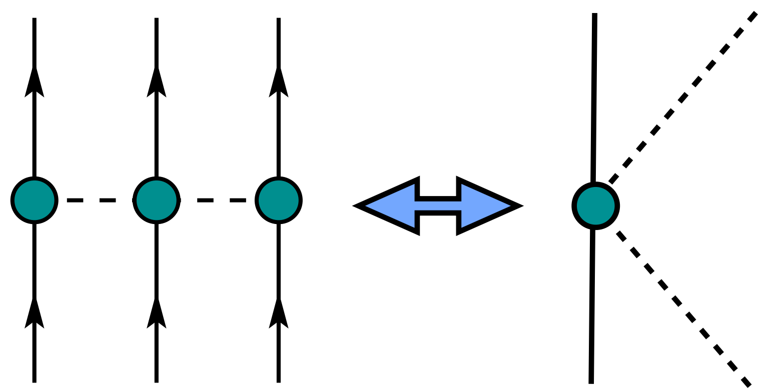


Small scale expansion of 3NF

	Δ-less theory	Δ-full theory: additional graphs
NLO	—	
N^2LO	 van Kolck ’94, Epelbaum et al. ’02	—
N^3LO	 Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)	
N^4LO	 HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)	

- no effect up to N^2LO (modulo reshuffling)
- expect large contributions to the ring & 2π-1π-topologies saturating some of the $N^{4,5,6}LO$ graphs in the Δ-less theory
- What is more efficient: Δ-less N^4LO (and beyond?) vs Δ-full N^3LO ??

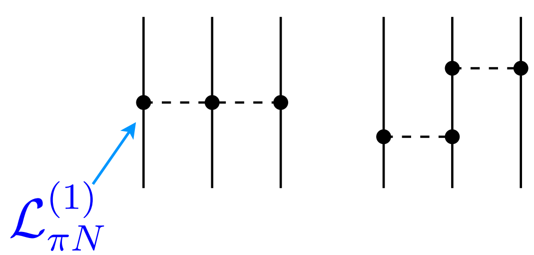
Two-pion-exchange 3NF



- Two-pion-exchange 3NF is connected to pion-nucleon scattering amplitude
  Ishikawa, Robilotta ´07
  - The same linear combinations of LECs
  - The same renormalization

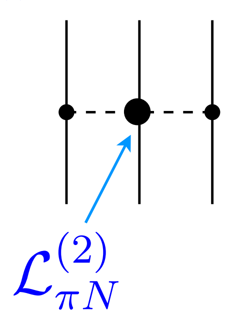
$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

NLO - contr.

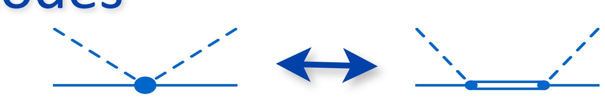


⟵ yield vanishing 3NF contributions

N^{2}LO - contr.



⟵ first nonvanishing 3NF, encodes information about the Δ:

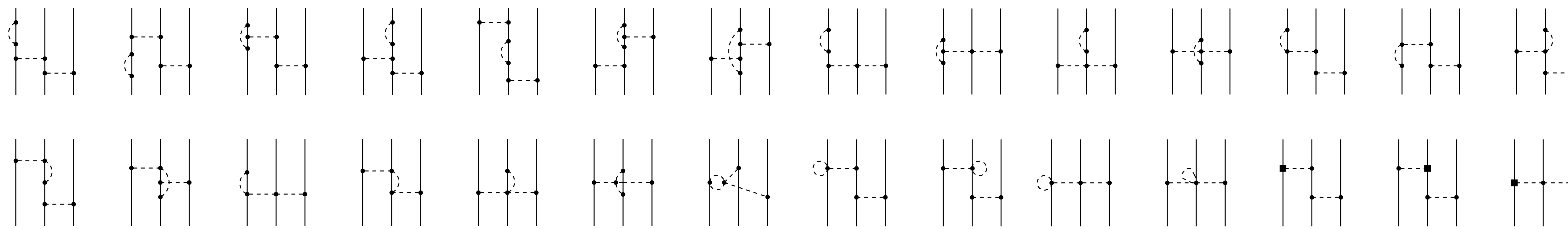


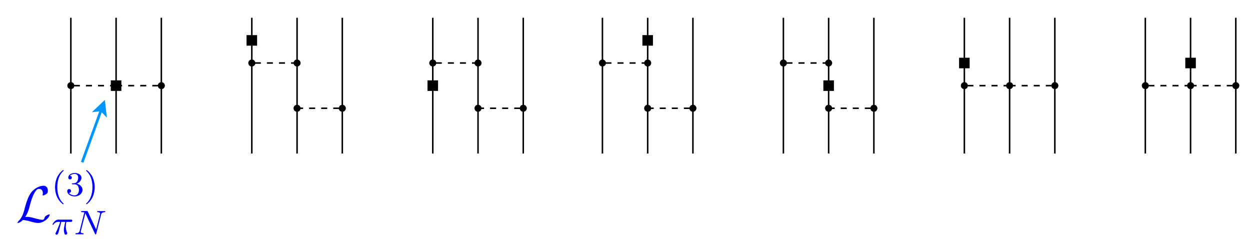
$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4} \Big((2c_3 - 4c_1) M_\pi^2 + c_3 q_2^2 \Big) \,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

U. van Kolck ´94

Two-pion-exchange 3NF

N^{3}LO - contr. (leading 1 loop)





renormalization nontrivial !

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2+1\right)M_\pi^3 + 2\left(g_A^2+1\right)M_\pi q_2^2\right],$$

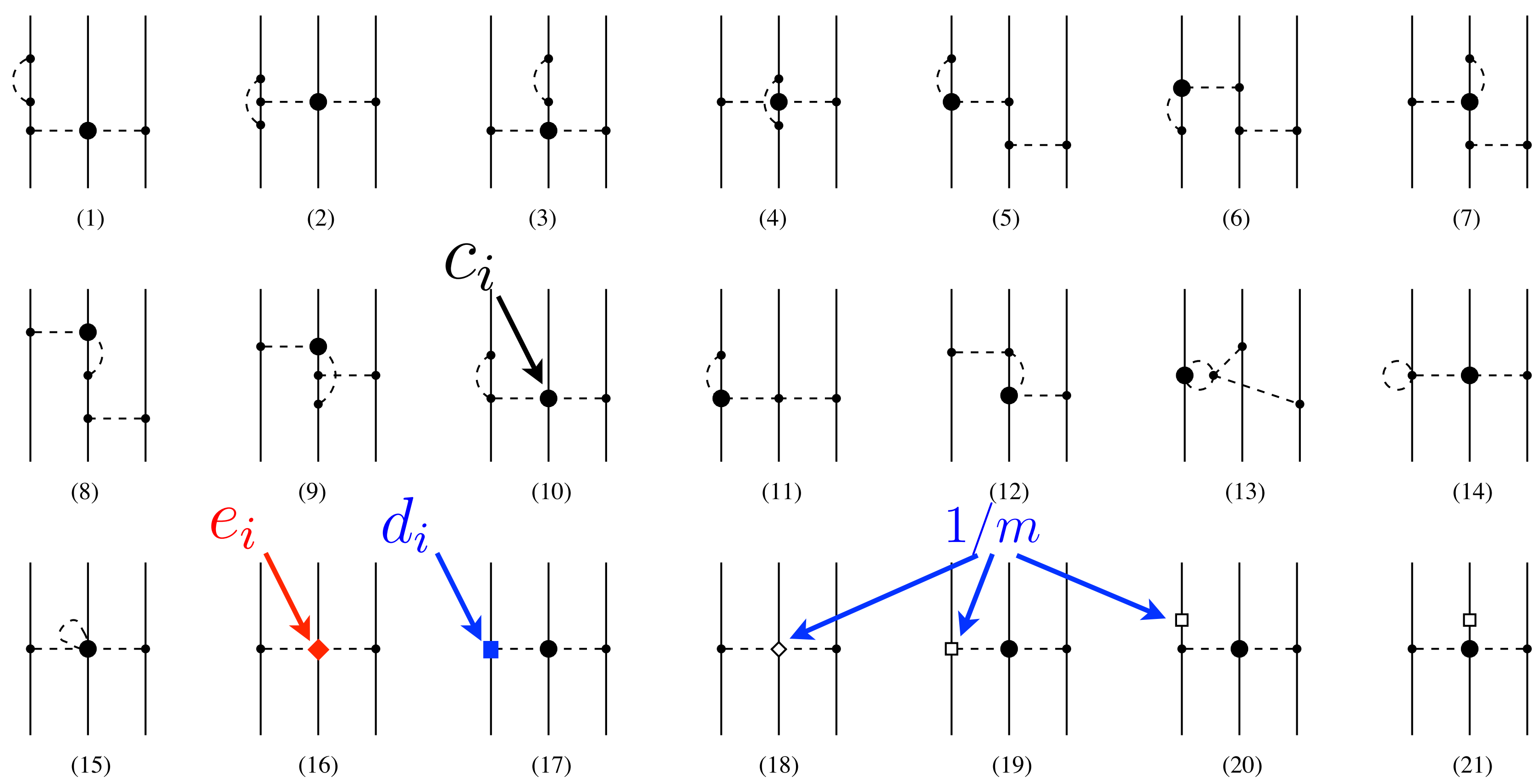
$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2+1)M_\pi\right]$$

Ishikawa, Robilotta ´07, Bernard, Epelbaum, HK, Meißner ´07

- No unknown parameters at this order
- Everything is expressed in terms of loop function $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$
- Additional unitarity transformations required for proper renormalization

Two-pion-exchange 3NF

N^4LO - contr. (subleading 1 loop) *Epelbaum, Gasparyan, HK, ´12*

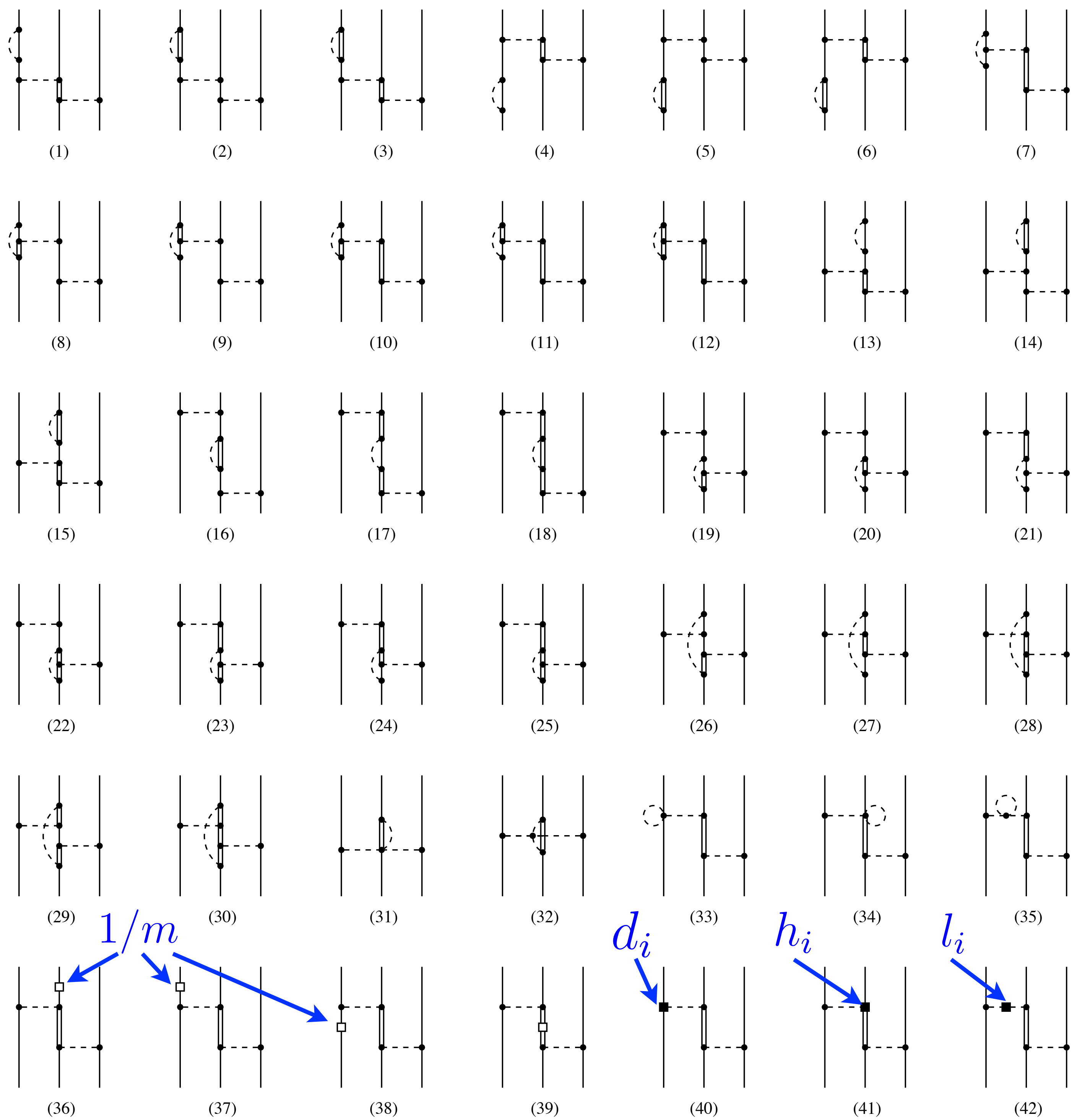


c_i´s LECs from $\mathcal{L}_{\pi N}^{(2)}$, d_i´s LECs from $\mathcal{L}_{\pi N}^{(3)}$, e_i´s LECs from $\mathcal{L}_{\pi N}^{(4)}$: fitted to πN - scattering data

- Leading Δ - contributions are taken into account through c_i´s
- Vanishing $1/m$ - contributions at this order

Two-pion-exchange 3NF

N³LO - delta- contr. (subleading 1 loop) *Epelbaum, Gasparyan, HK, forthcoming*



Additional LECs in the diagrams

$d_i \in \mathcal{L}^{(3)}_{\pi N}, \quad h_i \in \mathcal{L}^{(3)}_{\pi N \Delta}, \quad l_i \in \mathcal{L}^{(4)}_{\pi\pi}$

After renormalization the only additional LECs are

- Leading order πNΔ-constant

$$h_A \simeq \frac{3\,g_A}{2\sqrt{2}} \leftarrow \text{Large-N}_c$$

- Leading order πΔΔ-constant

$$g_1 \simeq \frac{9\,g_A}{5} \leftarrow \text{Large-N}_c$$

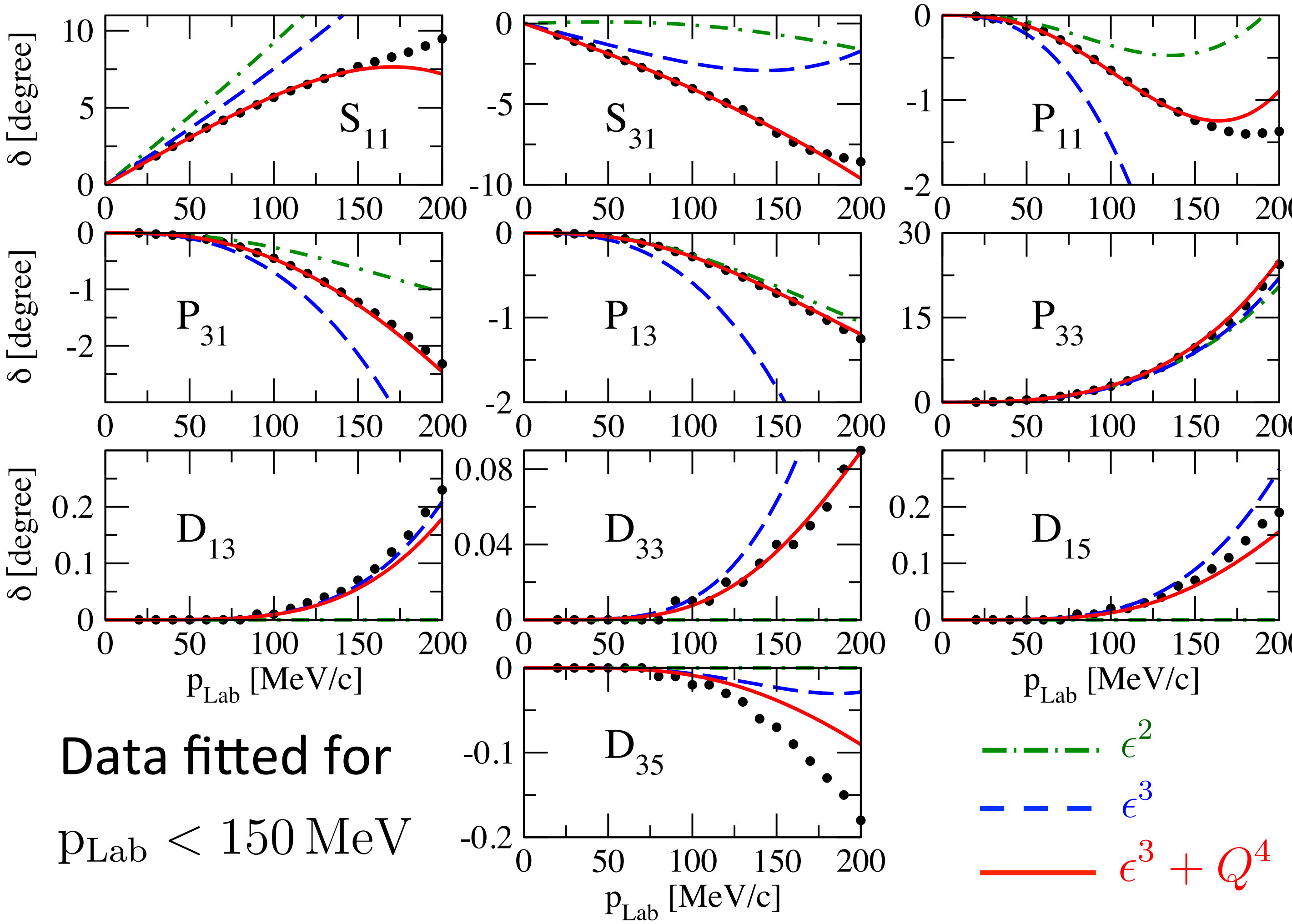
- Δ-resonance saturation of N⁴LO 3NF checked, explicitly

Preliminary

Pion-nucleon scattering

Heavy baryon SSE calculation up to ε^3 : *Fettes & Meißner ´01; Epelbaum, Gasparyan, HK, in preparation*

Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707



Data fitted for

$p_{\rm Lab} < 150\,{\rm MeV}$

- Size of the LECs are consistent with resonance saturation

$$c_1(\Delta) = 0,\ c_2(\Delta) = -c_3(\Delta) = 2\,c_4(\Delta) = \frac{4h_A^2}{9\Delta}$$

$$(\bar{d}_1 + \bar{d}_2)(\Delta) = -\bar{d}_3(\Delta) = -\frac{1}{2}(\bar{d}_{14} - \bar{d}_{15})(\Delta) = \frac{h_A^2}{9\Delta^2}$$

$$\bar{e}_{14}(\Delta) = \frac{h_A^2}{864\,F_\pi^2\,\pi^2\,\Delta}\left(7 + 10\,\log\left(\frac{2\,\Delta}{M_\pi}\right)\right), \ldots$$

- LECs which appear in 3NF up to N^{4}LO are of natural size

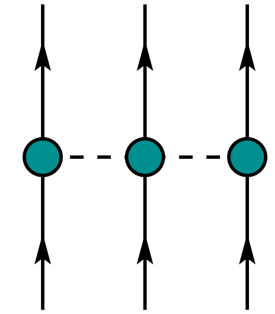
	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
$Q^1 + Q^2 + Q^3 + Q^4$: Fit to KH [60]	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26
$\epsilon^1 + \epsilon^2 + \epsilon^3 + Q^4$: Fit to KH[60]	-0.95	1.90	-1.78	1.50	2.40	-3.87	1.21	-5.25	-0.24	-6.35	2.34	-0.39	2.81
Delta-resonance saturation contribution	0	2.81	-2.81	1.40	2.39	-2.39	0	-4.77	1.87	-4.15	4.15	-0.17	1.32

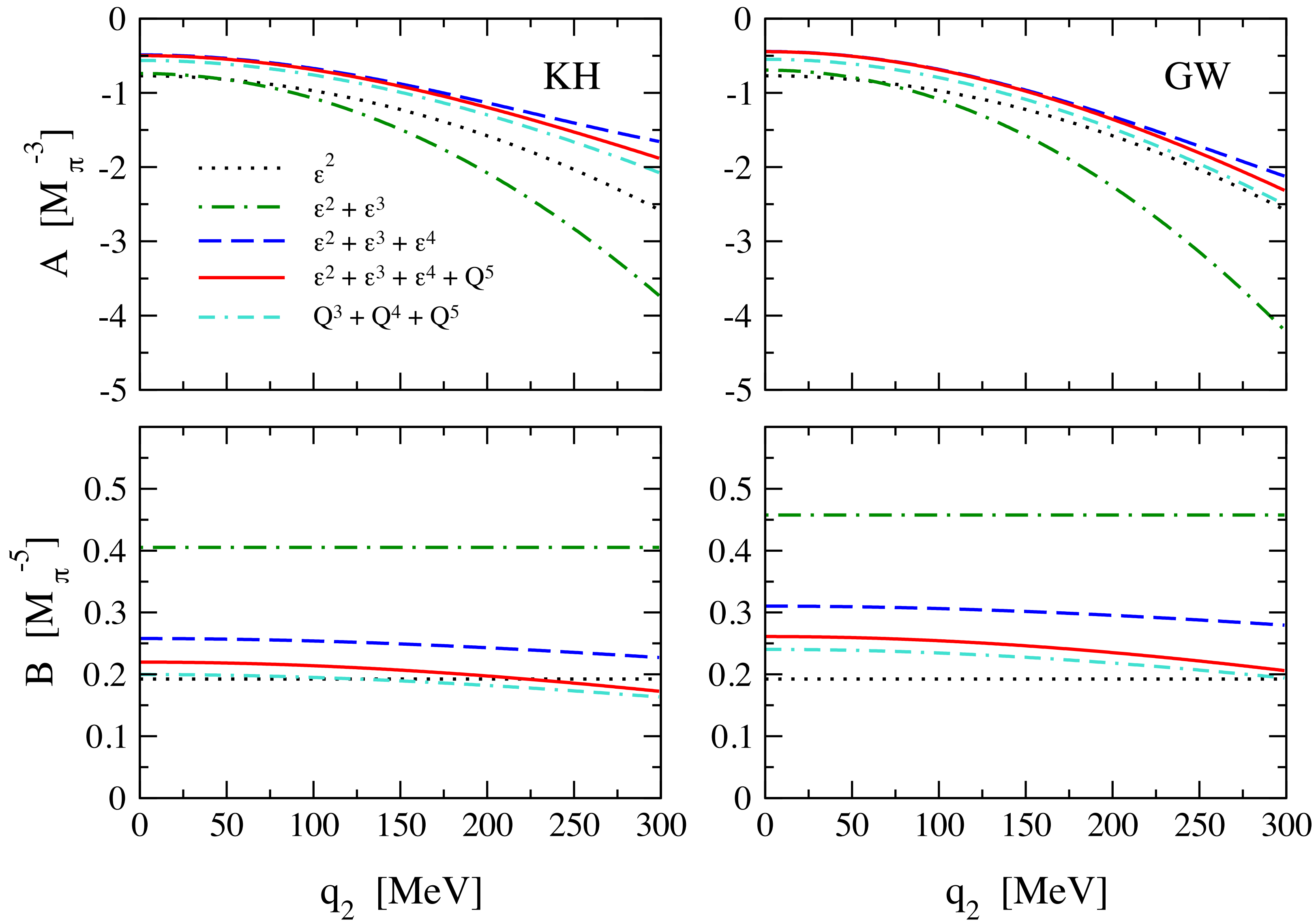
Two-pion-exchange 3NF

Preliminary

Epelbaum, Gasparyan, HK. forthcoming

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$





- Similar results for TPE-3NF in N^3LO-Δ and N^4LO Δ-less approaches
- We expect small explicit-Δ N^4LO contributions to two-pion-exchange 3NF

Most general structure of a local 3NF

Epelbaum, Gasparyan, H.K., PRC87 (2013) 054007

Up to N^{4}LO, the computed contributions are local ⟶ it is natural to switch to r-space.

A meaningful comparison requires a complete set of independent operators

Generators $\mathcal{G}$ of 89 independent operators	S	A	G_{12}	G_{22}	G_{11}	G_{21}
$\mathcal{G}_1 = 1$	O_1	0	0	0	0	0
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	O_2	0	O_3	O_4	0	0
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	O_5	0	O_6	O_7	0	0
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	O_8	0	O_9	O_{10}	0	0
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	O_{11}	O_{12}	O_{13}	O_{14}	O_{15}	O_{16}
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	O_{17}	0	0	0	0	0
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{18}	0	O_{19}	O_{20}	0	0
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	O_{21}	O_{22}	O_{23}	O_{24}	O_{25}	O_{26}
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	O_{27}	0	O_{28}	O_{29}	0	0
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{30}	0	O_{31}	O_{32}	0	0
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	O_{33}	O_{34}	O_{35}	O_{36}	O_{37}	O_{38}
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	O_{39}	O_{40}	O_{41}	O_{42}	O_{43}	O_{44}
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	O_{45}	O_{46}	O_{47}	O_{48}	O_{49}	O_{50}
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	O_{51}	O_{52}	O_{53}	O_{54}	O_{55}	O_{56}
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	O_{57}	0	O_{58}	O_{59}	0	0
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{60}	O_{61}	O_{62}	O_{63}	O_{64}	O_{65}
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	O_{66}	0	O_{67}	O_{68}	0	0
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{69}	0	O_{70}	O_{71}	0	0
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	O_{72}	O_{73}	O_{74}	O_{75}	O_{76}	O_{77}
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{78}	O_{79}	O_{80}	O_{81}	O_{82}	O_{83}
$\mathcal{G}_{21} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_2 \vec{\sigma}_3 \cdot \vec{q}_2 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{84}	0	O_{85}	O_{86}	0	0
$\mathcal{G}_{22} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	O_{87}	0	O_{88}	O_{89}	0	0

Most general, local 3NF involves 89 operators, can be generated (by permutations) from 22 structures:

$$V_{3N}^{\text{loc}} = \sum_{i=1}^{22} \mathcal{G}_i F_i(r_{12}, r_{23}, r_{31}) + 5 \,\text{perm.}$$

80 operators? *Schat, Phillips ´13*

The structures $\mathcal{O}_i$ are defined as:

$$S(\mathcal{G}) := \frac{1}{6} \sum_{P \in S_3} P\mathcal{G}$$

$$A(\mathcal{G}) := \frac{1}{6} \sum_{P \in S_3} (-1)^P P\mathcal{G}$$

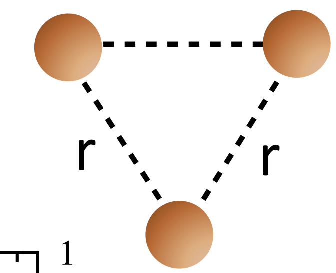
$$G_{ij}(\mathcal{G}) := \frac{1}{3} \sum_{P \in S_3} \mathcal{D}_{ij}(P) P\mathcal{G}, \quad i, j = 1, 2$$

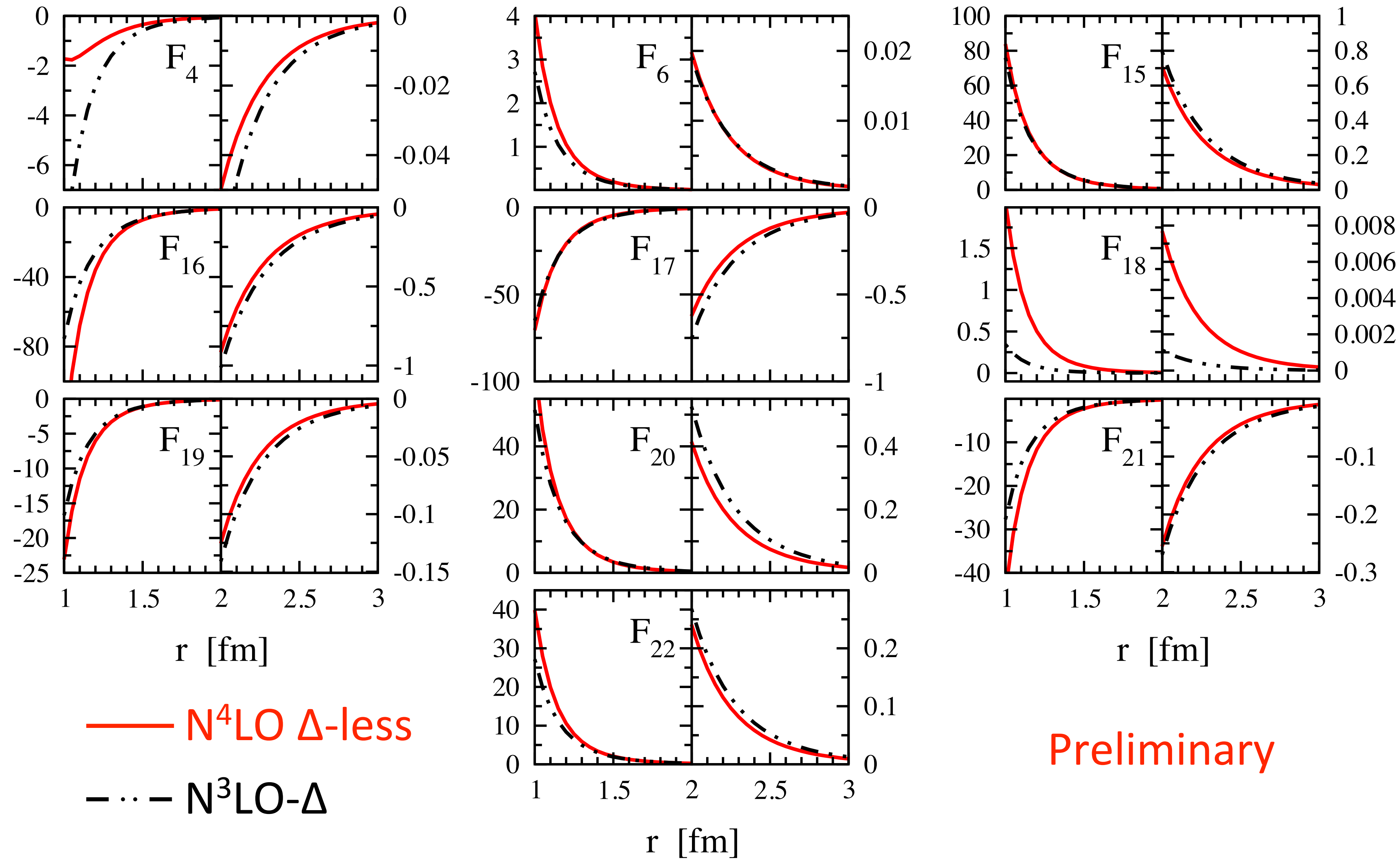
2-dim. irred. repr. of S_3

Two-pion-exchange up to N^4LO

Epelbaum, Gasparyan, HK, in preparation

Chiral expansion of TPE „structure functions" F_i (in MeV) in the equilateral-triangle configuration

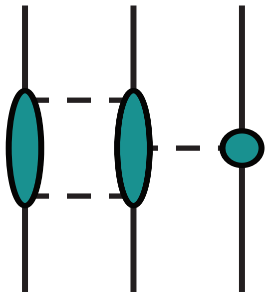




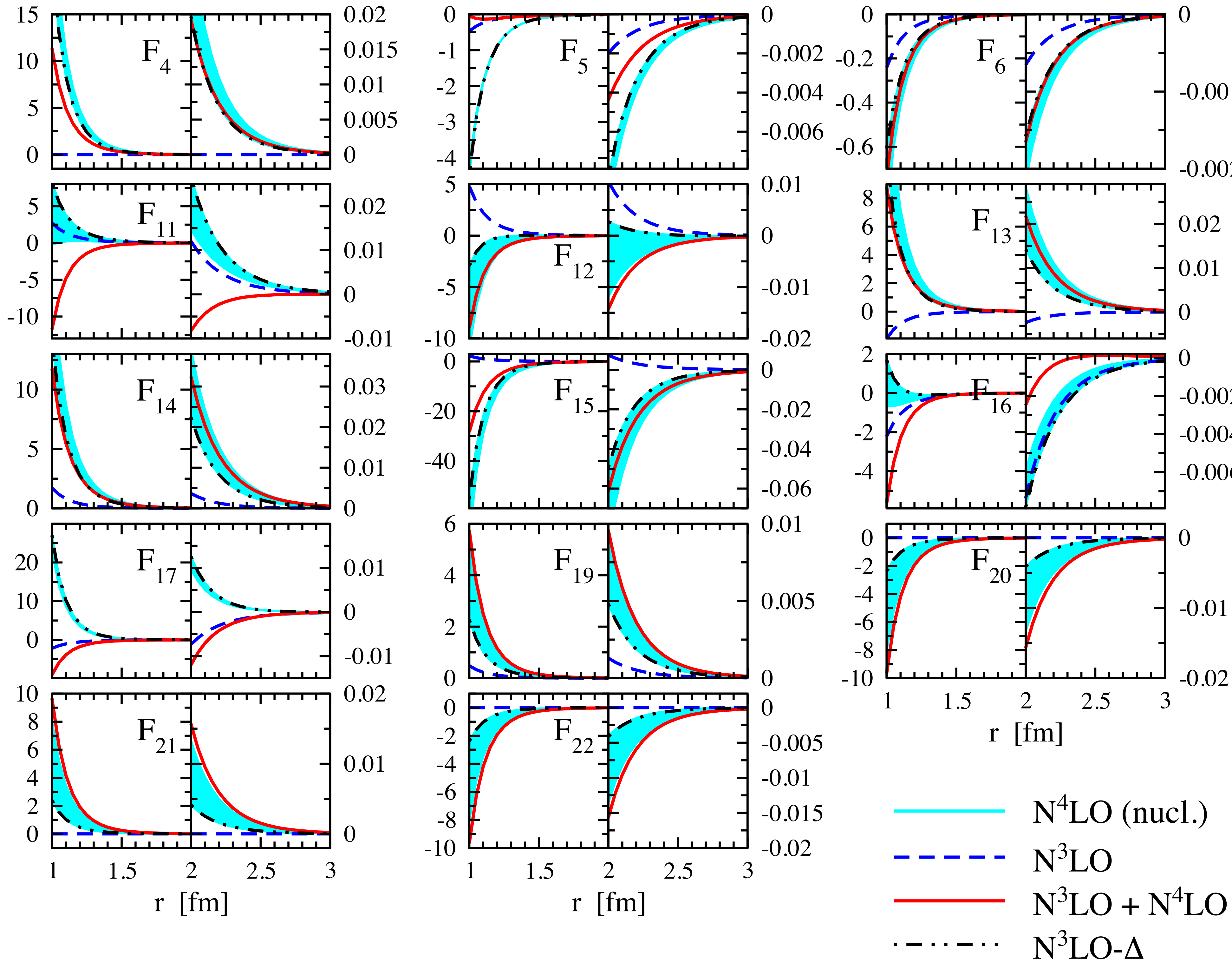
Δ-less and Δ-full approaches for TPE-force compared

- similar results if contributions are sizeable
- slightly different results if contributions are smaller

Two-pion-one-pion-exchange 3NF

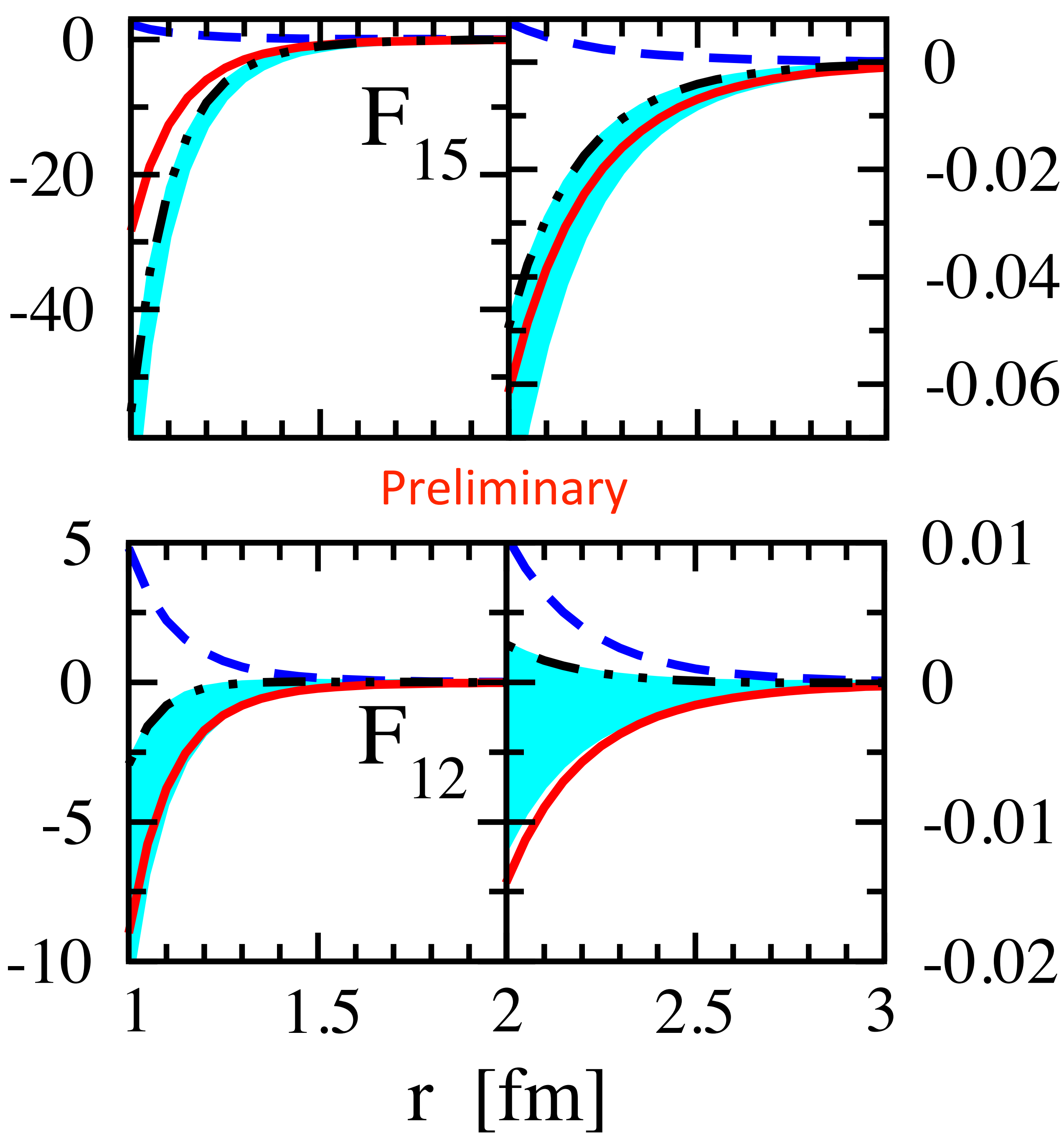


Preliminary



Bands indicate physics which is not described by explicit Δ-contributions

Two-pion-one-pion-exchange 3NF



- Most sizable contribution
- Δ-less/full results are similar
- Band is narrow

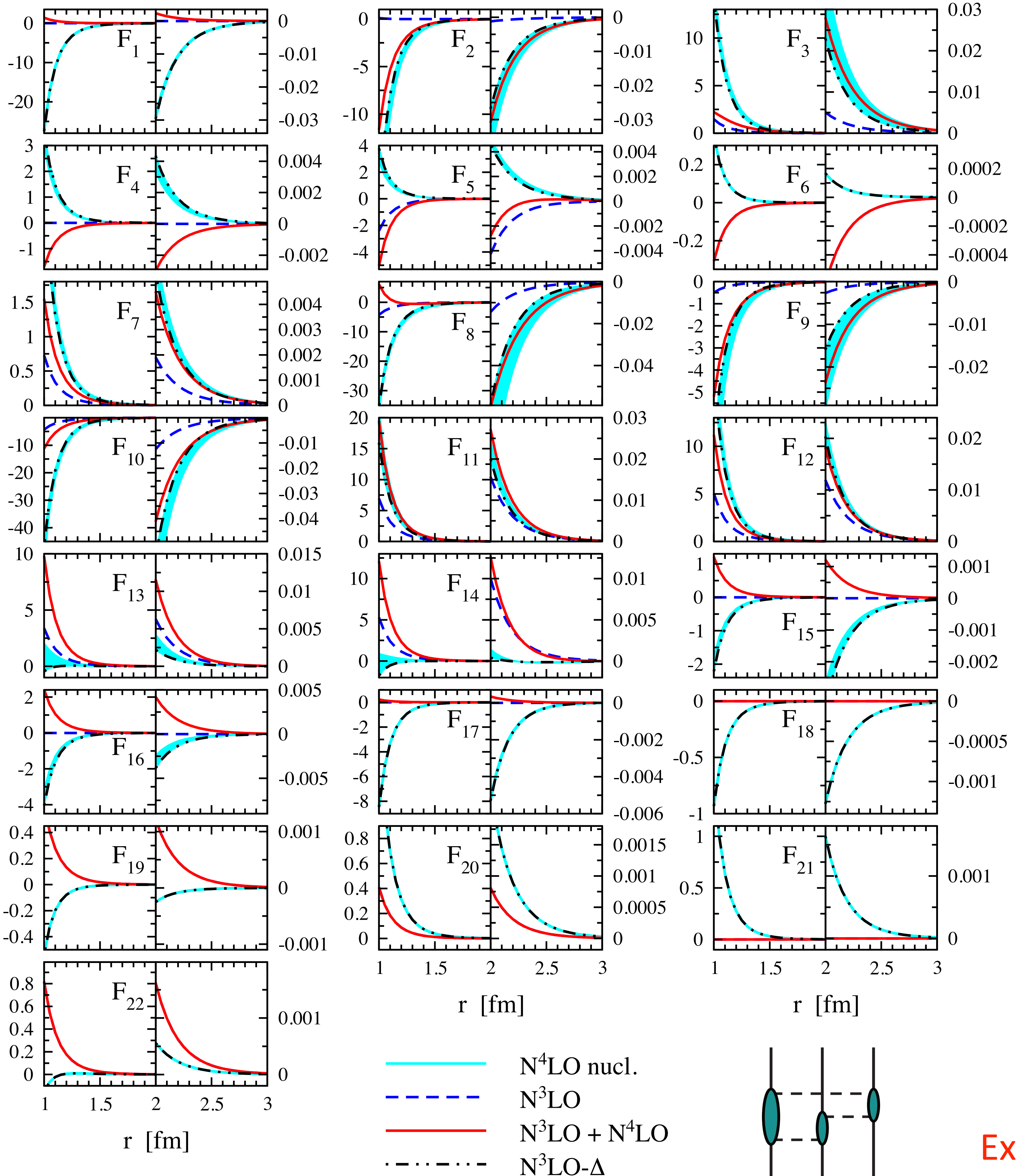
N^{4}LO Effects beyond Δ-contr. are small

- Small contribution
- Δ-less/full results differ
- Band is broad

N^{4}LO Effects beyond Δ-contr. are important

- N^{3}LO nucleon-contributions are of smaller size
- Dominant effects come from N^{3}LO Δ-/N^{4}LO-contr. in Δ-full/Δ-less approach

Ring - 3NFs



Preliminary

- Narrow bands
  - ➡ Higher order contributions beyond Δ are small
- Strong central isoscalar 3NF due to double-Δ excitation

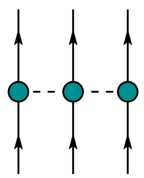
Two different cases:

1) Δ-resonance saturation contribution to a given F_i is sizable
   - ➡ N^3LO-Δ and N^4LO-Δ-less results are similar
2) Δ-resonance saturation contribution to a given F_i is negligible
   - ➡ N^3LO-Δ and N^4LO-Δ-less results deviate

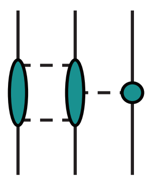
Explicit-Δ approach is more efficient!

Summary

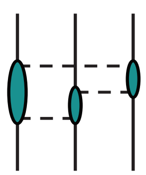
- Long-range part of 3NFs is analyzed up to N^{4}LO Δ-less/N^{3}LO-Δ



- Chiral expansion of TPE-3NF seems to be converged
- TPE-3NF dominates 3NF but does not fill all 22 structures



- Sizeable contr. are similar for 2π-1π-3NF in N^{4}LO Δ-less and N^{3}LO-Δ approach
- Dominant effects come from N^{4}LO-/N^{3}LO Δ-contr. in Δ-less/Δ-full approach



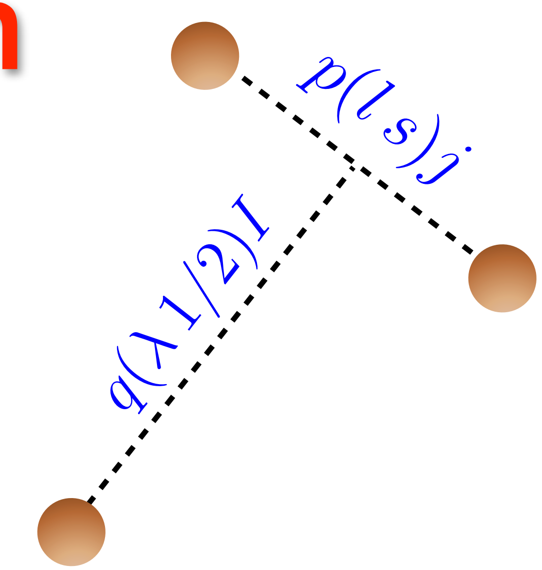
- Ring-3NFs fill all 22 structures
- N^{4}LO-/N^{3}LO Δ-contr. in Δ-less/Δ-full approach dominate N^{3}LO-nucleon contr.
- Some missing sizeable Δ-contr. in N^{4}LO results like central attractive force ~O(1/Δ^2)

Outlook

- Partial wave decomposition of N^{3}LO three-nucleon forces
- N^{4}LO Δ-less/N^{3}LO-Δ calc. of shorter range part of 3NF
- N^{4}LO with explicit-Δ of long range part of 3NF (convergence-test)
- Large-N_c estimate of different contributions to 3NF
  First step: Schat & Phillips ´13

Partial wave decomposition

Golak et al. Eur. Phys. J. A 43 (2010) 241



- Faddeev equation is solved in the partial wave basis

$$|p,q,\alpha\rangle \equiv |pq(ls)j(\lambda\tfrac{1}{2})I(jI)JM_J\rangle\,|(t\tfrac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand ⟹ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\,d\hat{q}'\,d\hat{p}\,d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\dots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\,Y_{l',m_l'}(\hat{p}')\,\dots\Big)\underbrace{\langle m'_{s_1} m'_{s_2} m'_{s_3}|V|m_{s_1} m_{s_2} m_{s_3}\rangle}_{\text{depends on spin \& isospin}}$$

- For local 3NF's PWD can be reduced to a 3-dim. integral

- Ring-diagram-contr. expensive to calculate on the fly

We prestore ring-contr. to 3nf's on a fine momentum grid ⟹ Numerical interpolation of ring terms

- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis
  see talk by A. Calci

Straightforward implementation of high order 3nf's in many-body calc. within No-Core Shell Model

Computational strategy

- d-dim one loop tensor integrals by Passarino-Veltman reduction

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1} \dots l_{\mu_n}}{[l^2 - M^2][(l+p)^2 - M^2]} = T^{(1)}_{\mu_1 \dots \mu_n}(p) f_1(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - M^2][(l+p)^2 - M^2]} + T^{(2)}_{\mu_1 \dots \mu_n}(p) f_2(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - M^2]}$$

Tensors in p

$f_1(p^2)$ and $f_2(p^2)$ include in general non-physical singularities which cancel in final result

- Dimensional-shift reduction Davydychev '91

Combinatorial factors

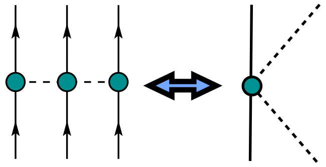
$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1} \dots l_{\mu_n}}{[l^2 - M^2][(l+p)^2 - M^2]} = \sum_{ij} T^{(i)}_{\mu_1 \dots \mu_n}(p) \int \frac{d^{d+2i} l}{(2\pi)^{d+2i}} \frac{c_{ij}}{[l^2 - M^2]^{n_{ij}}[(l+p)^2 - M^2]^{m_{ij}}}$$

- Partial integration techniques provide recursion relations

$$\int \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial l_\mu} f(l) = 0$$ ⟸ Connection betw. Dimensional-shift and Passarino-Veltman red.

Implement Heavy-Baryon extension of these techniques in Mathematica/FORM

Pion-nucleon scattering



Heavy baryon calculation up to order q^4 *Fettes, Meißner Nucl. Phys. A676 (2000) 311*

1/m power counting used in FM work ⇒ $\frac{p}{m} \sim \frac{q}{\Lambda_\chi}$

- Difference in Weinberg's power counting for NN ⇒ $\frac{p}{m} \sim \left(\frac{q}{\Lambda_\chi}\right)^2$

Refit of d_i and e_i LECs is needed

$$\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$$

$$T_{\pi N}^{ba} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\,\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^+(\omega,t)\right] + i\,\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^-(\omega,t)\right]\right)$$

CMS kinematics: $\omega = q_1^0 = q_2^0, \quad E = E_1 = E_2 = \sqrt{\vec{q}^{\,2}+m^2}, \quad \vec{q}_1^{\,2} = \vec{q}_2^{\,2} = \vec{q}^{\,2}, \quad t = (q_1-q_2)^2$

Partial wave amplitudes: $f_{l\pm}^{\pm}(s) = \frac{E+m}{16\pi\sqrt{s}}\int_{-1}^{1} dz\left[g^\pm P_l(z) + \vec{q}^{\,2}h^\pm\left(P_{l\pm1}(z) - zP_l(z)\right)\right]$

In the isospin basis: $f_{l\pm}^{1/2} = f_{l\pm}^{+} + 2f_{l\pm}^{-}, \quad f_{l\pm}^{3/2} = f_{l\pm}^{+} - f_{l\pm}^{-}$

Absence of inelasticity below the two-pion production threshold

$$\delta_{l\pm}^{I}(s) = \arctan\left(|\vec{q}|\,\mathcal{R}e\,f_{l\pm}^{I}(s)\right)$$

Explicit decoupling

Don‘t positive powers of Δ possibly spoil the convergence?

Small scale expansion parameter $\Delta/\Lambda_\chi \sim \frac{1}{3}$ is not that small!

Manifest decoupling through the choice of renormalization conditions (no positive powers of Δ)

Decoupling theorem due to Appelquist & Carrazone Phys. Rev. 11 (1974) 2856

$$\mathcal{L}_{\pi N}^{\mathrm{SSE}} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \Delta \mathcal{L}_{\pi N}^{(2)} + \Delta^2 \mathcal{L}_{\pi N}^{(1)} + \mathcal{O}(\epsilon^4)$$

Choose finite part of these LECs such that

$$\lim_{\Delta \to \infty} \text{Green Function} < \infty$$

Bernard, Fearing, Hemmert, Meißner NPA635 (1998) 121