Efimov spectrum in bosonic and fermionic systems with increasing number of particles

A. Kievsky

INFN, Sezione di Pisa (Italy)

The 22nd European Conference on Few-Body Problems in Physics
Krakow, Poland 9-13 September 2013

- M. Gattobigio - (INLN, Nice)
- N. Timofeyuk - (Surrey)

Efimov spectrum in bosonic and fermionic systems with increasing number of particles

A. Kievsky

INFN, Sezione di Pisa (Italy)

The 22nd European Conference on Few-Body Problems in Physics
Krakow, Poland 9-13 September 2013

- M. Gattobigio - (INLN, Nice)
- N. Timofeyuk - (Surrey)

Increasing N: Preliminary things

Efimov physics with potential models

- Tunable strength
- Finite-range versus zero-range theory
- Finite-range effects
- Equivalent to results from EFT

Discrete Scale Invariance

- Efimov effect
- Evolution with N
- Efimov physics is a well established sector of Quantum Mechanics
  Quantum Mechanics of shallow states
- Experimental activities in ultracold trapped atoms
- Experimental in nuclear physics

Increasing N: Preliminary things

Efimov physics with potential models

- Tunable strength
- Finite-range versus zero-range theory
- Finite-range effects
- Equivalent to results from EFT

Discrete Scale Invariance

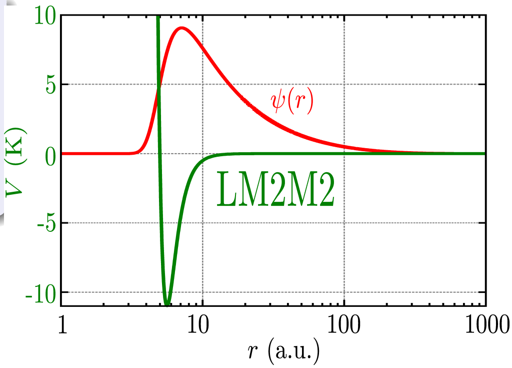
- Efimov effect
- Evolution with N
- Efimov physics is a well established sector of Quantum Mechanics
  Quantum Mechanics of shallow states
- Experimental activities in ultracold trapped atoms
- Experimental in nuclear physics

Preliminary things

Efimov physics with potential models

The He-He system as example:

$E(\mathrm{He-He}) \approx -1.3$ mK
$a = 190$ a.u.
$r_0 = 13$ a.u.
$a >> r_0$
$E(\mathrm{He-He}) \approx -\frac{\hbar^2}{m}\frac{1}{a^2}$
≈ -1.2 mK



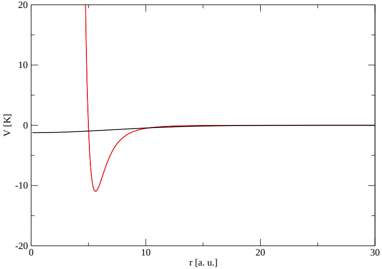
Soft Two-Body Gaussian Potential

Effective low-energy soft potential

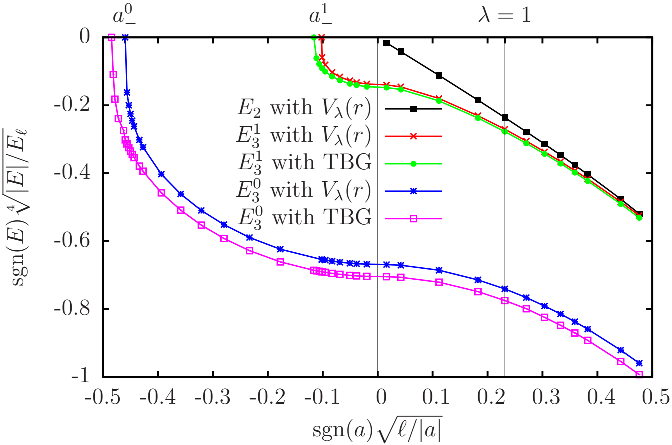
- $V(r) = V_0 \, \mathrm{e}^{-r^2/R^2}$
  - Regularized contact interaction
  - Fix V_0 to reproduce one low-energy LM2M2 datum
  - Use the cut-off R to reproduce a second datum

$V_0 = -1.2344$ K, $R = 10.0$ a.u.

	Gaussian	LM2M2
a_0 (a.u.)	189.41	189.42
r_0 (a.u.)	13.81	13.84
E_2 (mK)	-1.303	-1.303

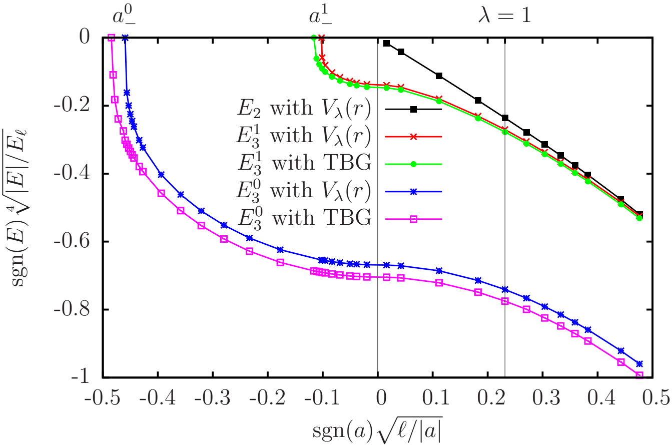


$V_\lambda(r) = \lambda V_{LM2M2}(r)$



• Soft Potentials together with the Schrödinger equation can be used to investigate Efimov physics

$V_\lambda(r) = \lambda V_{LM2M2}(r)$

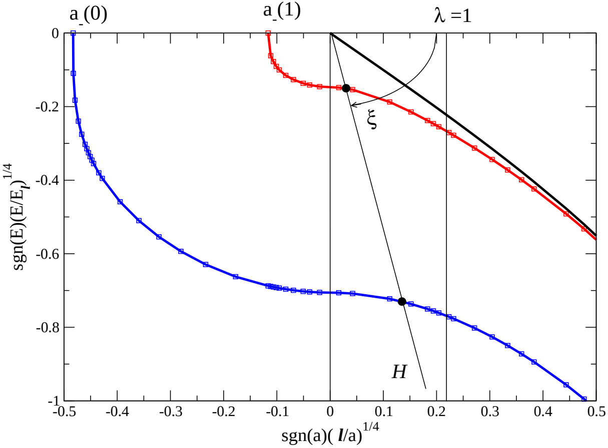


- Soft Potentials together with the Schrödinger equation can be used to investigate Efimov physics

Discrete Scale Invariance

Polar Coordinates: $1/a = H\cos\xi\,,\quad K = H\sin\xi$

$$E_3^n + \frac{\hbar^2}{ma^2} = e^{-2n\pi/s_0} e^{\Delta(\xi)/s_0} \frac{\hbar^2\kappa_*^2}{m} \rightarrow H = \kappa_*\, e^{-n\pi/s_0}\, e^{\Delta(\xi)/2s_0}$$

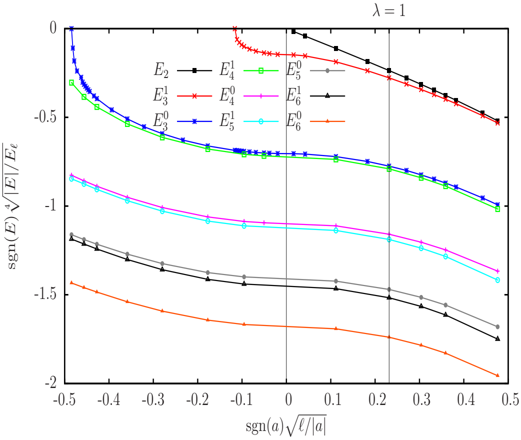


Increasing N

- We know that there is a tree of two attached states

Increasing *N*

- We know that there is a tree of two attached states



Increasing N

- We know that there is a tree of two attached states
- We know that DSI is verified:

$$E_N^n/E_2 = \tan^2 \xi$$

$$\kappa_N^n a_B + \Gamma_N^n = \frac{e^{-\Delta(\xi)/2s_0}}{\cos \xi}$$

- Going to $N > 6$ what we can expect?
- It is clear that increasing the number of particles the potential energy can grow faster than the kinetic energy destroying the Efimov picture (more excited states to appear)

Increasing N

- We know that there is a tree of two attached states

- We know that DSI is verified:

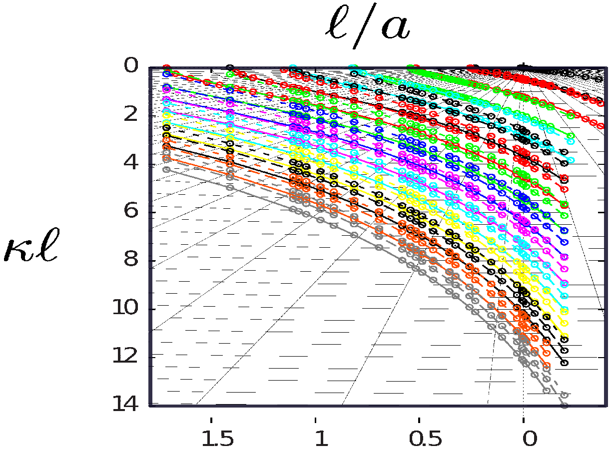
$$E_N^n / E_2 = \tan^2 \xi$$

$$\kappa_N^n a_B + \Gamma_N^n = \frac{e^{-\Delta(\xi)/2s_0}}{\cos \xi}$$

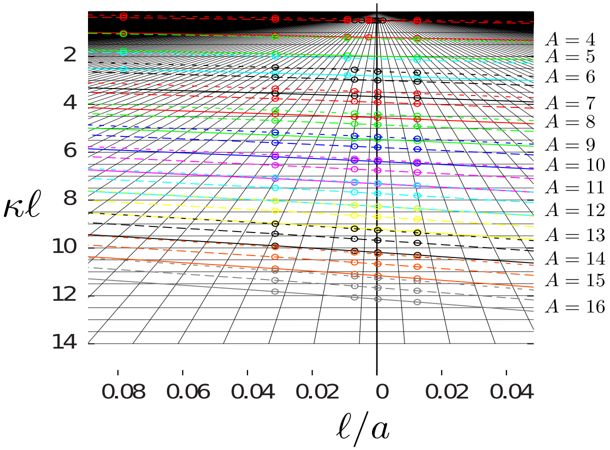
- Going to $N > 6$ what we can expect?

- It is clear that increasing the number of particles the potential energy can grow faster than the kinetic energy destroying the Efimov picture (more excited states to appear)

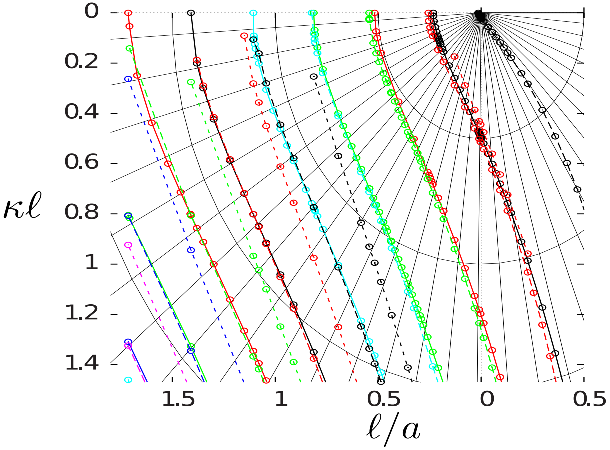
Efimov plot up to $N = 16$ using soft potentials



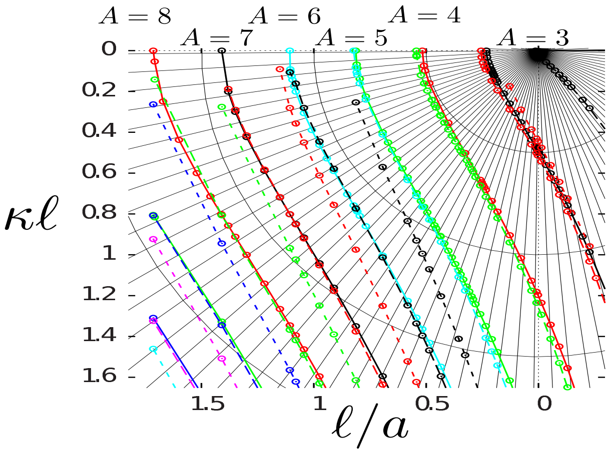
Efimov plot up to $N = 16$ close to $1/a = 0$



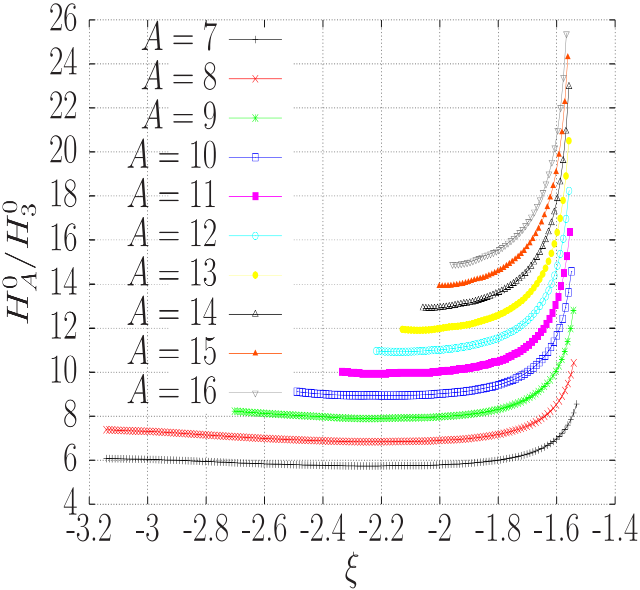
Looking the Efimov plot to the right



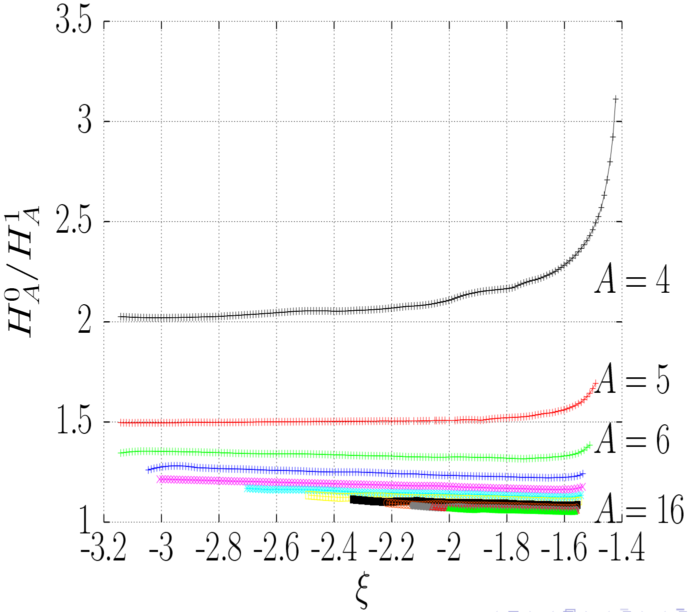
Looking the Efimov plot to the left



Looking at relevant ratios: H^0_A/H^0_3



Looking at relevant ratios: H_A^0/H_A^1



Analysis of the results

regular values of $1/a_N^-$ at threshold

The DSI imposes that:

$$E_N^0 + \frac{\hbar^2}{ma^2} = \exp\left[\Delta(\xi)/s_0\right]\frac{\hbar^2\kappa_N^2}{m}$$

and we have found:

$$\frac{1}{|a_N^-|} = \frac{1}{|a_3^-|} + \frac{N-3}{d}$$

with d a constant that can be extracted from the results. Moreover, κ_N and a_N^- are related trough the universal function at $\xi = -\pi$:

$$\frac{1}{(a_N^-)^2} = \exp\left[\Delta(-\pi)/s_0\right]\kappa_N^2$$

Therefore,

$$a_N^- = \frac{-1.56}{\kappa_N}$$

Analysis of the results

Linear dependence of κ_N

$$\kappa_N = \kappa_3 + \frac{1.56}{d}(N-3)$$

or

$$\kappa_N/\kappa_3 = 1 + \frac{1.56}{\kappa_3 d}(N-3)$$

But $\kappa_4/\kappa_3 = 2.147$ is an universal quantity (A. Deltuva, FBS 54, 569 (2013)). From our calculations we obtain ≈ 2.2 (not bad!). Therefore

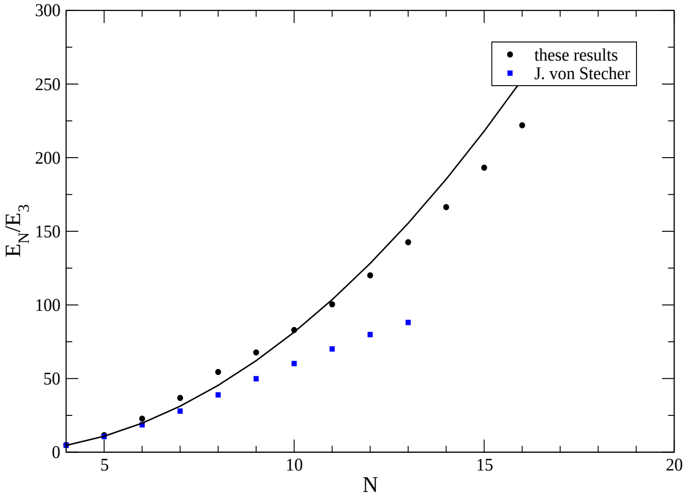
$$\kappa_N/\kappa_3 = 1 + 1.147(N-3)$$

that can be transformed to an energy ratio:

$$\frac{E_N}{E_3} = [1 + 1.147(N-3)]^2 \approx (1.15N - 2.44)^2$$

Preliminary result for the N-boson energy at unitary limit

Testing the E_N/E_3 formula at $\xi = -\pi/2$



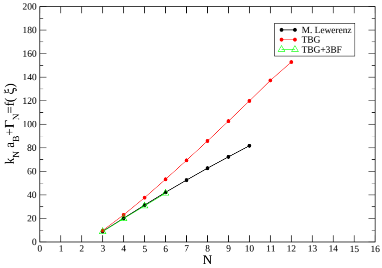
Testing the linear dependence on κ_N at constant a

$$E_N/E_2 = \tan^2\xi$$

$$\kappa_N a_B + \Gamma_N = \frac{e^{-\Delta(\xi)/2s_0}}{\cos\xi}$$

M. Lewerenz, J. Chem. Phys. 106, 4596 (1997) for the TTY potential:

	TTY	TBG
a_0 (a.u.)	189.0	189.4
r_0 (a.u.)	13.94	13.84
E_2 (mK)	-1.310	-1.303
E_3 (mK)	-126	-151
E_4 (mK)	-558	-751
E_5 (mK)	-1302	-1945
.	.	.
.	.	.



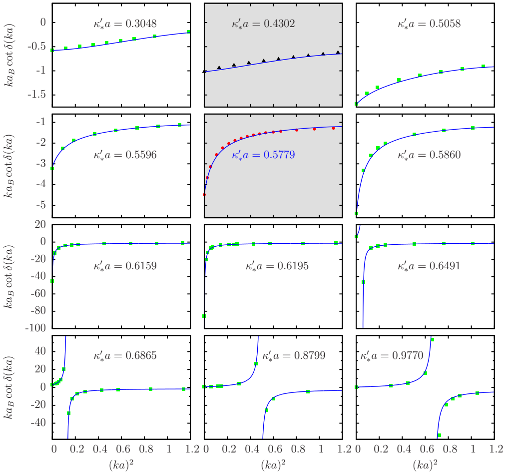
Conclusions - bosons

- The Efimov spectrum for N bosons has been analyzed
- We can call these studies: Physics of shallow states
- It shows universality through the function $\Delta(\xi)$
- It shows a DSI
- The left part of spectrum shows less finite-range effects
- The values of a at threshold, a_N^-, at which the N-body cluster goes into the N-body continuum are regular.
- Using this regularity the linear relation $\kappa_N/\kappa_3 = 1 + 1.147(N-3)$ has been obtained
- The relation $E_N/E_3 \approx (1.15N - 2.44)^2$ has been derived too.
- These are preliminary results. A deeper analysis of the results on the $-\pi$ axis is underway.
- The old data from Lewerenz has been analyzed using the DSI showing the predicted linear relation.
- We hope that this work will estimulate more experimental activity

Universal Effective Range Function (Efimov)

$$ka_B \cot\delta = c_1(ka) + c_2(ka)\cot[s_0 \ln(\kappa_* a + \Gamma) + \phi(ka)]$$

Universal functions from E. Braaten, H.-W Hammer, Phys. Rep. 428, 259 (2006)



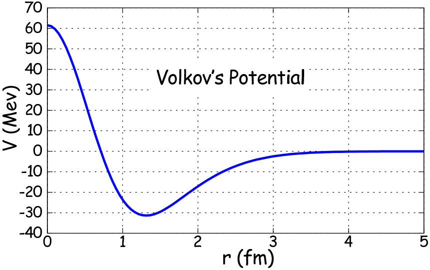
A.K. and M. Gattobigio, PRA87, 052719 (2013)

The Fermion case

- Application to Nuclear Physics
- Volkov Potential

$$V(r) = E_1\, e^{-r^2/R_1^2} + E_2\, e^{-r^2/R_2^2}$$

- $E_1 = 144.86$ Mev, $R_1 = 0.82$ fm, $E_2 = -83.34$ Mev, $R_2 = 1.6$ fm



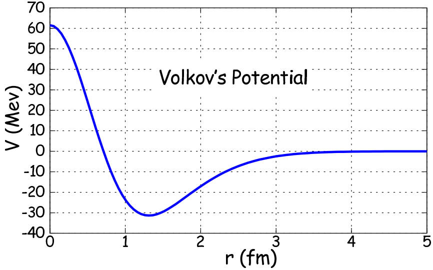
- all-waves potential – acts on all l_{ij} values
- s-wave potential – only acts on $l_{ij} = 0$

The Fermion case

- Application to Nuclear Physics
- Volkov Potential

$$V(r) = E_1\, \mathrm{e}^{-r^2/R_1^2} + E_2\, \mathrm{e}^{-r^2/R_2^2}$$

- $E_1 = 144.86$ Mev, $R_1 = 0.82$ fm, $E_2 = -83.34$ Mev, $R_2 = 1.6$ fm



- all-waves potential – acts on all l_{ij} values
- s-wave potential – only acts on $l_{ij} = 0$

All-waves Volkov – Summary

0.546 MeV [2] 0^+

$A = 2$

0.599 MeV [3] 0^+

8.465 MeV [3] 0^+

$A = 3$

8.562 MeV [4] 0^+

10.406 MeV [3 1] 1^-

30.418 MeV [4] 0^+

$A = 4$

28.72 MeV [4 1] 0^+

31.72 MeV [5] 0^+

43.03 MeV [4 1] 1^-

68.28 MeV [5] 0^+

$A = 5$

66.49 MeV [4 2] 0^+

70.28 MeV [5 1] 0^+

73.49 MeV [6] 0^+

122.78 MeV [6] 0^+

$A = 6$

S-wave Volkov – Physical Spectrum