Core-Excitation Three-Cluster Model on Borromean Nuclei

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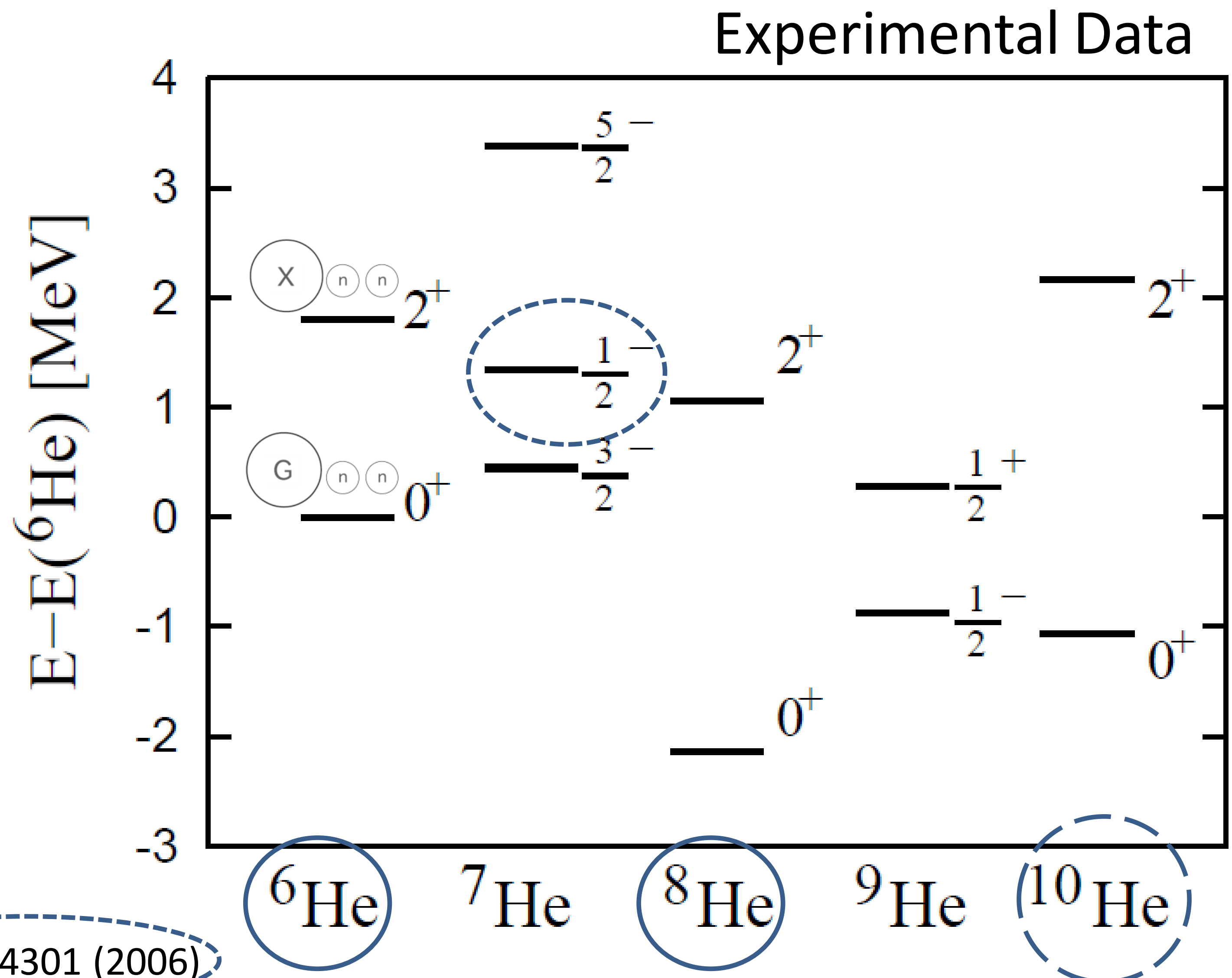
Outline

- Spectra of ^{6,7,8,9,10}He
- Core-Excitation Three-Cluster Model
- Numerical Result

- Spectra of ^{11}Li
- Numerical Result

- Conclusion

Spectra of $^{6,7,8,9,10}He$



PRC **73** 44301 (2006)

Core-Excitation Three-Cluster Model

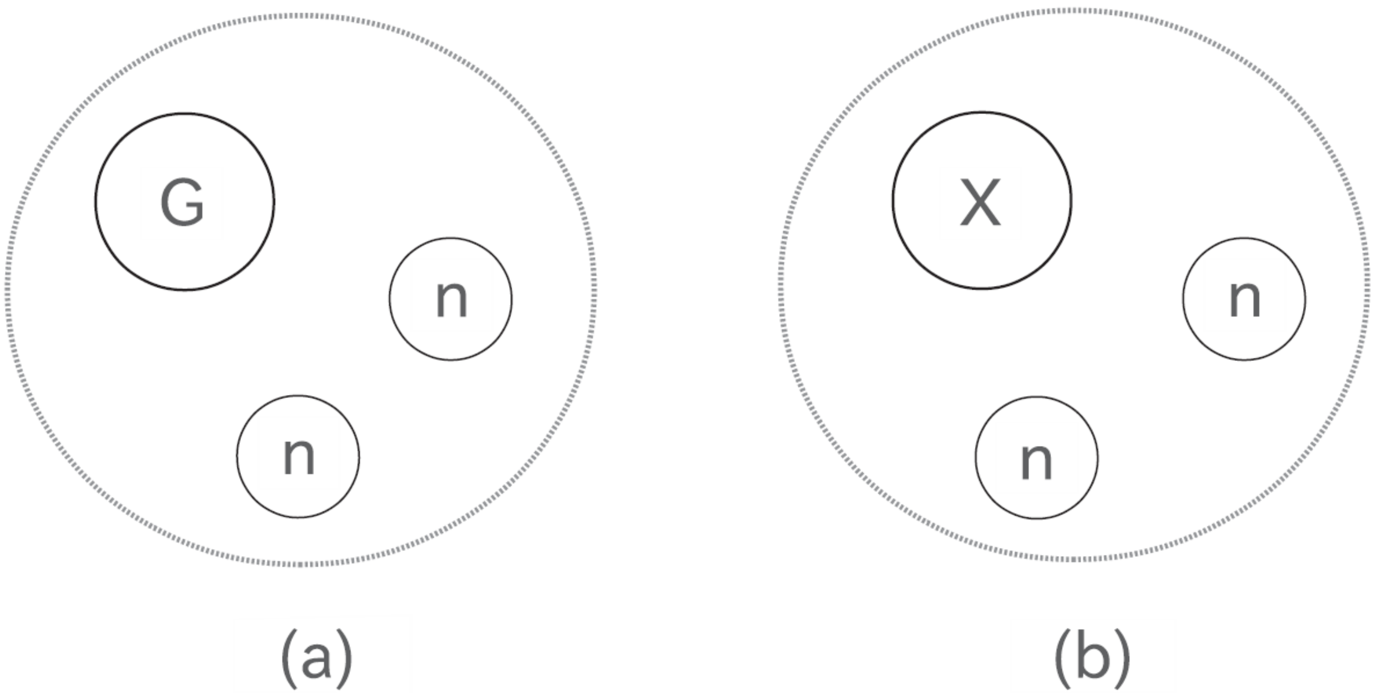


FIG. 1. Illustration of core-excitation cluster. The core cluster of the ground state and excited state are labeled G and X, respectively. Neutrons are labeled n.

The Hilbert space $\mathcal{H}$ of the model consists of two Hilbert ones

$$\mathcal{H} = \mathcal{H}(G) \oplus \mathcal{H}(X). \tag{1}$$

Using the wave function, we have

$$|\Psi\rangle = |G\rangle|\Psi_G\rangle + |X\rangle|\Psi_X\rangle, \tag{2}$$

where $|G\rangle$ and $|X\rangle$ are orthonormal bases to distinguish their spaces,

$$\langle G|G\rangle = \langle X|X\rangle = 1, \quad \langle G|X\rangle = \langle X|G\rangle = 0. \tag{3}$$

$$\hat{H}_0^{\text{2clust.}}|G\rangle \equiv \frac{p^2}{2\nu}|G\rangle, \quad \hat{H}_0^{\text{2clust.}}|X\rangle \equiv \left(\delta m + \frac{p^2}{2\nu}\right)|X\rangle, \tag{4}$$

δm～1．8MeV

A. Two-body interaction

In our model the potential of a two-cluster system has a rank 1 separable Yamaguchi form using a simple form factor $g(p)$. For instance, the neutron-neutron potential of the ${}^1S_0$ partial wave is given as

$$V_{nn}(p, p') = -\gamma_{nn}^2 g_{nn}(p) g_{nn}(p') \tag{5}$$

with

$$g_{nn}(p) = \frac{1}{p^2 + \beta_{nn}^2}, \tag{6}$$

where we choose parameters $\beta_{nn} = 1.1648\ \text{fm}^{-1}$ and $\gamma_{nn}^2 = 0.3943\ \text{fm}^{-3}$ from [10].

Let us introduce a new form factor h, which is combined with the partial waves $|l_I S_I j_I\rangle$ and the particle basis $|I\rangle$:

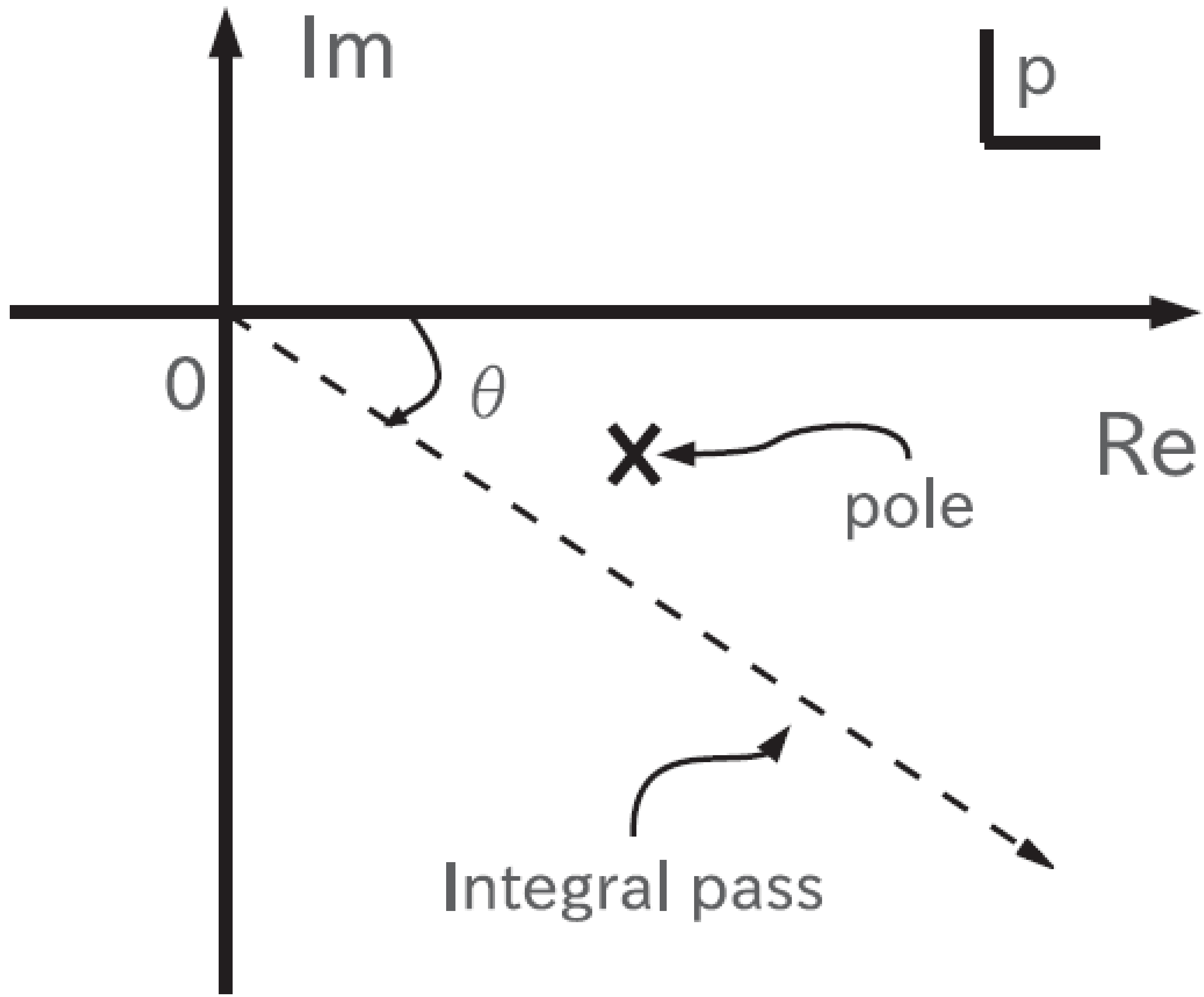
$$\langle p|h\rangle = \sum_{I=G,X}\sum_{l_I,S_I,j_I} \gamma_{In;l_I,S_I,j_I} g_{In;l_I,S_I,j_I}(p)|l_I S_I j_I\rangle|I\rangle \quad (7)$$

with

$$g_{In;l_I,S_I,j_I}(p) = \frac{p^{l_I}}{\left(p^2+\beta^2_{In;l_I,S_I,j_I}\right)^{l_I+1}}, \quad (8)$$

where l_I, S_I, and j_I are angular momentum, total spin, and total angular momentum of the two-body subsystem ($j_I = l_I + S_I$), respectively. The core-nuclei neutron potential V is given by the form factor h,

$$\hat{V} = -|h\rangle\langle h|. \quad (9)$$



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According to the separable scheme, the t matrix $t(p, p'; E_2)$,

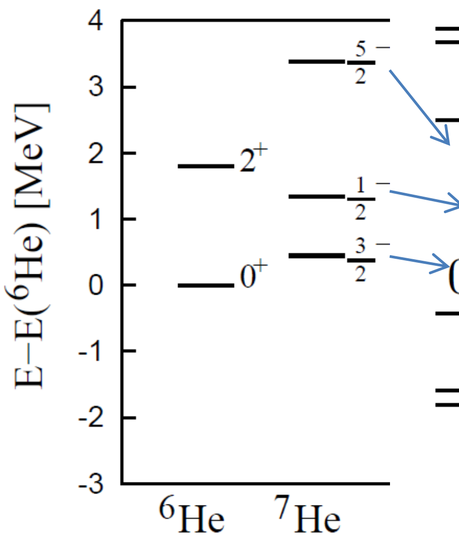
$$t_{In;l_I,S_I,j_I,I'n;l'_I,S'_I,j'_I}(p, p'; E_2) \equiv \langle I|\langle l_I S_I j_I|\langle p|h\rangle\tau(E_2)\langle h|p'\rangle|l'_I S'_I j'_I\rangle|I'\rangle, \qquad (12)$$

fulfills the Lippmann-Schwinger equation with the result

$$\tau(E_2) = -1 - \tau(E_2)\langle h|\hat{G}_0^{\text{2clust.}}(E_2)|h\rangle. \qquad (13)$$

$$1 + \gamma^2_{Gn;l_G,j_G}\langle g_{Gn;l_G,j_G}|\frac{1}{E_2^{\text{res}} - \hat{p}^2/2\nu + i\epsilon}|g_{Gn;l_G,j_G}\rangle + \gamma^2_{Xn;l_X,j_X}\langle g_{Xn;l_X,j_X}|\frac{1}{E_2^{\text{res}} - \delta m - \hat{p}^2/2\nu + i\epsilon}|g_{Xn;l_X,j_X}\rangle = 0. \qquad (14)$$

TABLE I. Parameters for ^{6}He(0^+)-n+^{6}He(2^+)-n potential. The resonance energies are measured from the ^{6}He + n threshold. The strengths γ^2 are in units of fm^{-5} for P wave, and fm^{-7} for F wave. The parameters β_G and β_X are commonly taken to be 1.5166 fm^{-1}. (β_{nn} = 1.1648 fm^{-1}.)



E_2^{res} [MeV]	partial wave	l_G	γ_G^2	l_X	γ_X^2
3.37 -i 0.995	$^2F_{5/2}+^{4,2}P_{5/2}$	3	116.80	1	7.6144
1.345 -i 0.5	$^2P_{1/2}+^4P_{1/2}$	1	5.3966	1	4.0418
0.445 -i 0.075	$^2P_{3/2}+^{4,6}P_{3/2}$	1	4.1655	1	6.1580
nn channel	$^1S_0$	0	0.3943	0	0.3943

$$1+\gamma^2_{Gn;l_G,j_G}\langle g_{Gn;l_G,j_G}|\frac{1}{E_2^{\rm res}-\hat{p}^2/2\nu+i\epsilon}|g_{Gn;l_G,j_G}\rangle$$
$$+\gamma^2_{Xn;l_X,j_X}\langle g_{Xn;l_X,j_X}|\frac{1}{E_2^{\rm res}-\delta m-\hat{p}^2/2\nu+i\epsilon}|g_{Xn;l_X,j_X}\rangle$$
$$=0. \tag{14}$$

Faddeev Equation

The total wave function $|\Psi^{J^\pi T}\rangle$ with total angular momentum J, parity π, and total isospin T consists of the Faddeev components $\psi^{J^\pi T}$ labeled by particle channels α, β, and γ:

$$|\Psi^{J^\pi T}\rangle = \left|\psi_\alpha^{J^\pi T}\right\rangle + \left|\psi_\beta^{J^\pi T}\right\rangle + \left|\psi_\gamma^{J^\pi T}\right\rangle. \tag{16}$$

The AGS equations for the Faddeev component is given by

$$\left|\psi_\alpha^{J^\pi T}\right\rangle = G_0 t_\alpha \sum_{\beta \neq \alpha} \left|\psi_\beta^{J^\pi T}\right\rangle \tag{17}$$

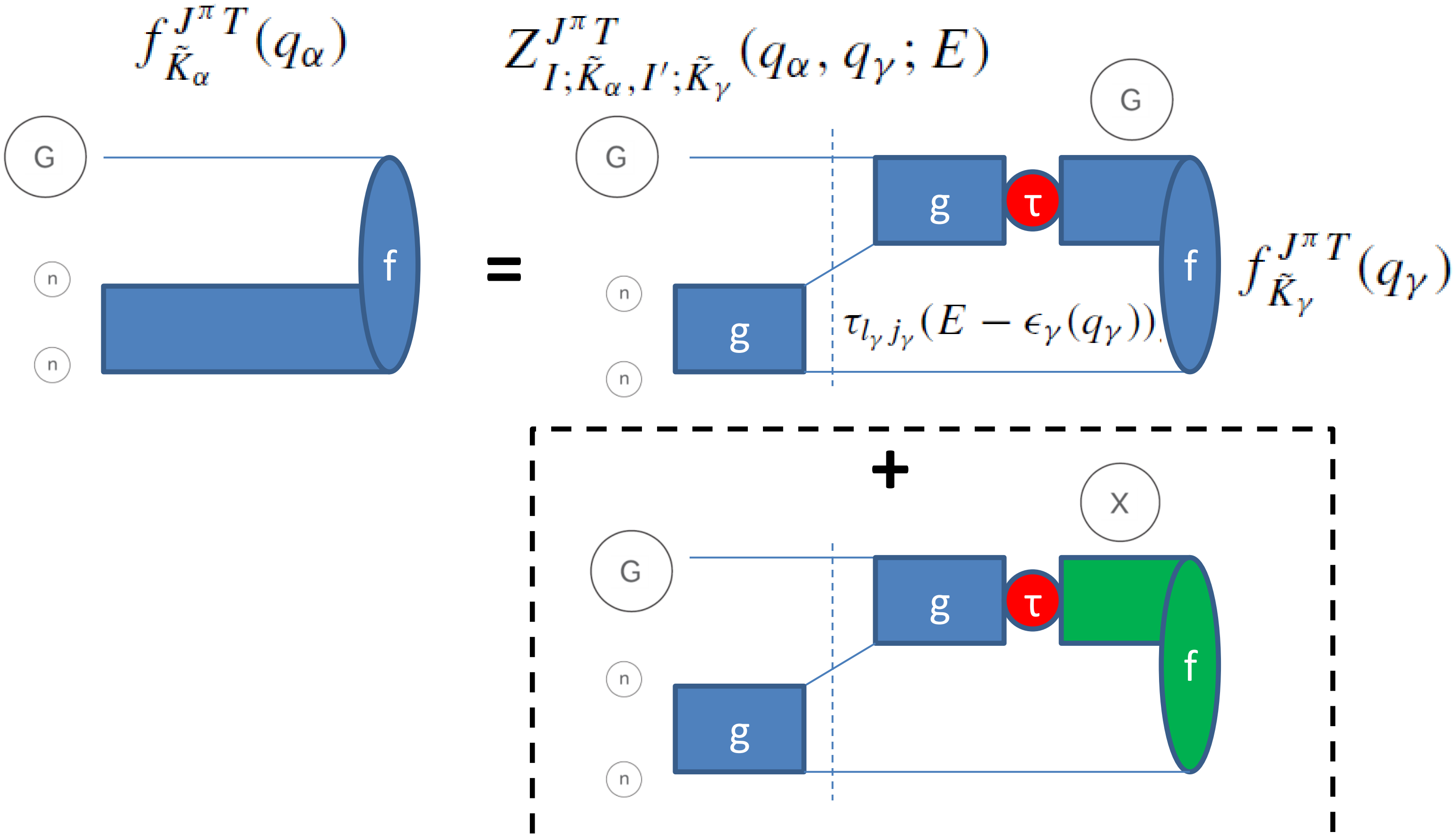
$$= G_0 |h_\alpha\rangle \tau_\alpha \langle h_\alpha| \sum_{\beta \neq \alpha} \left|\psi_\beta^{J^\pi T}\right\rangle. \tag{18}$$

AGS Equation

The reduced wave function $f^{J^\pi T}_{I;\tilde{K}_\alpha}(q_\alpha)$ is defined by

$$\sum_{I=G,X} \langle I | \langle \tilde{K}_\alpha | \langle q_\alpha \big| f^{J^\pi T}_\alpha \rangle$$
$$= \langle q_\alpha \big| f^{J^\pi T}_{\tilde{K}_\alpha} \rangle = f^{J^\pi T}_{\tilde{K}_\alpha}(q_\alpha)$$
$$\equiv \sum_{I=G,X} \gamma_{In;l_\alpha,j_\alpha} \langle g_{In;l_\alpha j_\alpha} | \sum_{\beta \neq \alpha} \big| \psi^{J^\pi T}_{I;\beta} \rangle, \qquad (19)$$

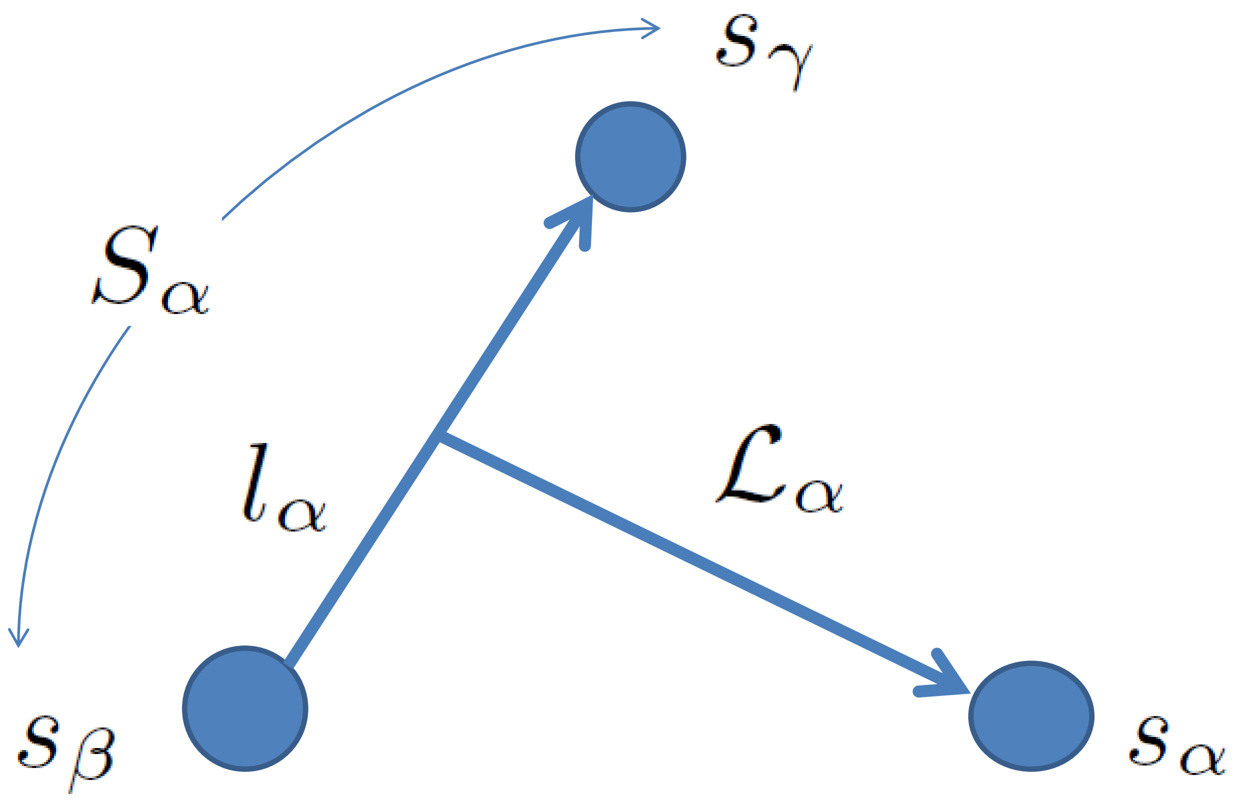
$$f^{J^\pi T}_{\tilde{K}_\alpha}(q_\alpha) = \sum_{I,I'} \sum_{\tilde{K}_\gamma} \int_0^\infty dq_\gamma q_\gamma^2 Z^{J^\pi T}_{I;\tilde{K}_\alpha, I';\tilde{K}_\gamma}(q_\alpha, q_\gamma; E)$$
$$\times \tau_{l_\gamma j_\gamma}(E - \epsilon_\gamma(q_\gamma)) f^{J^\pi T}_{\tilde{K}_\gamma}(q_\gamma), \qquad (21)$$



$f^{J^\pi T}_{\tilde{K}_\alpha}(q_\alpha)$
$Z^{J^\pi T}_{I;\tilde{K}_\alpha,I';\tilde{K}_\gamma}(q_\alpha, q_\gamma; E)$
G
n
n
f
=
G
g
τ
G
f
$f^{J^\pi T}_{\tilde{K}_\gamma}(q_\gamma)$
n
g
$\tau_{l_\gamma j_\gamma}(E - \epsilon_\gamma(q_\gamma))$
n
+
X
G
g
τ
f
n
g
n

Partial waves

$$S_\alpha = s_\beta + s_\gamma\,, \quad j_\alpha = l_\alpha + S_\alpha\,, \quad t_\alpha = \tau_\beta + \tau_\gamma\,,$$
$$\mathcal{S}_\alpha = j_\alpha + s_\alpha\,, \quad J = \mathcal{L}_\alpha + \mathcal{S}_\alpha\,, \quad T = t_\alpha + \tau_\alpha\,. \tag{20}$$



$$S_\alpha = s_\beta + s_\gamma, \quad j_\alpha = l_\alpha + S_\alpha, \quad t_\alpha = \tau_\beta + \tau_\gamma,$$
$$\mathcal{S}_\alpha = j_\alpha + s_\alpha, \quad J = \mathcal{L}_\alpha + \mathcal{S}_\alpha, \quad T = t_\alpha + \tau_\alpha. \tag{20}$$

TABLE II: Set of the quantum numbers for J^π=0^+ state of ^{8}He nucleus. The quantum numbers for the particle channel α=3 is obtained from α=1 by only cyclically label replacing $s_\alpha \longrightarrow s_\beta \longrightarrow s_\gamma \longrightarrow s_\alpha$.

K_α	$\tilde{K}_\alpha$	α	I	$\mathcal{L}_\alpha$	$\mathcal{S}_\alpha$	j_α	l_α	S_α	s_α	s_β	s_γ
1	1	1	G	1	1	3/2	1	1/2	1/2	1/2	0
2	1	1	X	1	1	3/2	1	3/2	1/2	1/2	2
3	1	1	X	1	1	3/2	1	5/2	1/2	1/2	2
4	2	1	G	1	1	1/2	1	1/2	1/2	1/2	0
5	2	1	X	1	1	1/2	1	3/2	1/2	1/2	2
6	3	1	G	3	3	5/2	3	1/2	1/2	1/2	0
7	3	1	X	3	3	5/2	1	3/2	1/2	1/2	2
8	3	1	X	3	3	5/2	1	5/2	1/2	1/2	2
9	4	2	G	0	0	0	0	0	0	1/2	1/2
10	5	2	X	2	2	0	0	0	2	1/2	1/2

Numerical Result

TABLE III. The predicted energy levels of the ^{8}He nucleus from the ^{6}He $+ n + n$ threshold. The resonance energy E equals $E^{(r)} - i\Gamma/2$. Units are MeV.

J^π	Present work		Expt.	
	$E^{(r)}$	Γ	$E^{(r)}$	Γ
0^+	-1.35		-2.14	
2^+	2.01	2.12	1.06 ± 0.5	0.6 ± 0.2

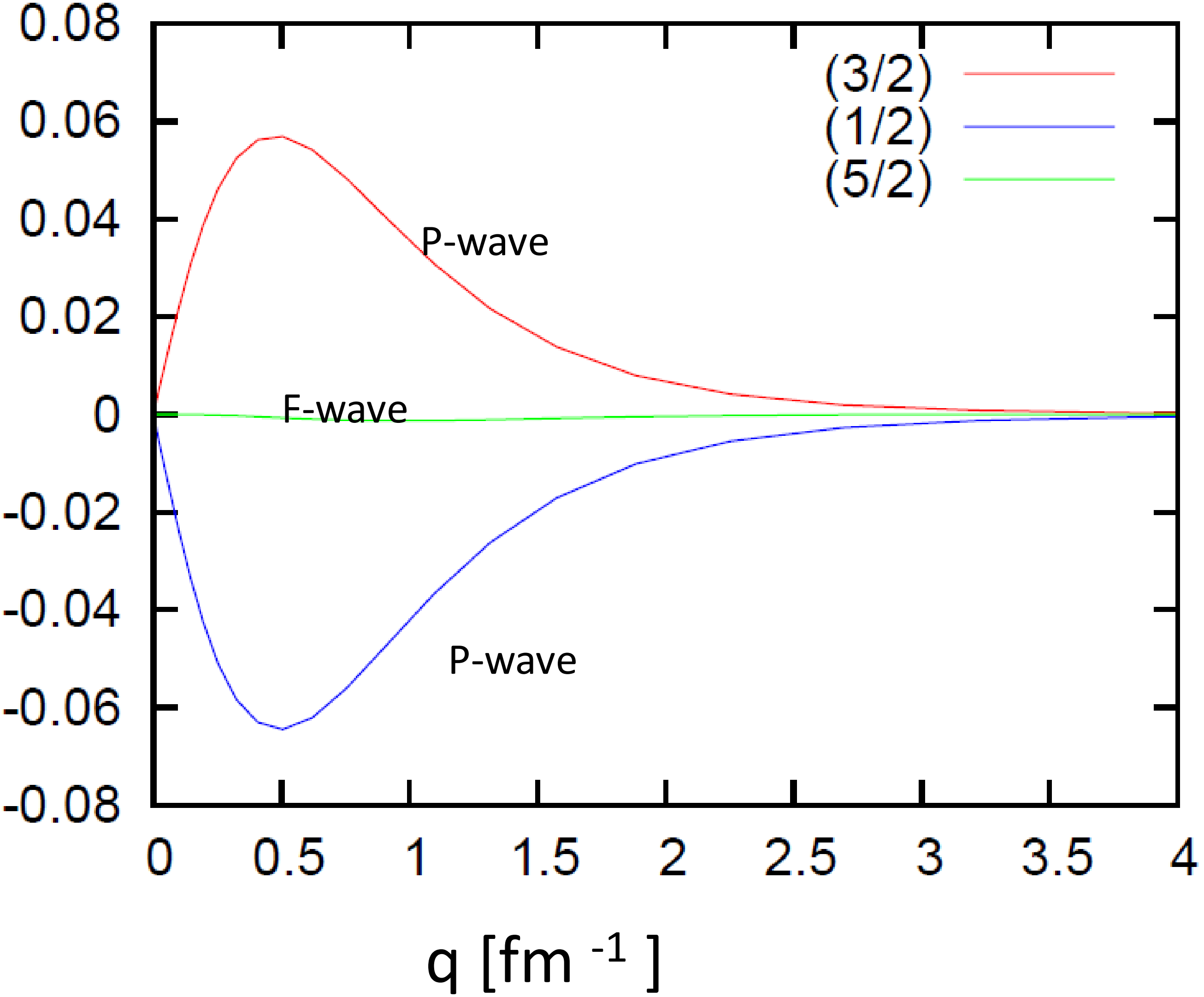
0^+ ground state of ^8He

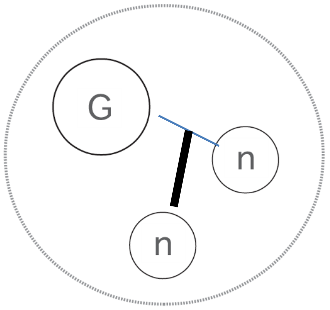
K_α	$\tilde{K}_\alpha$	α	I	$\mathcal{L}_\alpha$	$\mathcal{S}_\alpha$	j_α	probability (%)
1	1	1	G	1	1	3/2	18.1
2	1	1	X	1	1	3/2	10.1
3	1	1	X	1	1	3/2	
4	2	1	G	1	1	1/2	17.5
5	2	1	X	1	1	1/2	11.1
6	3	1	G	3	3	5/2	0.001
7	3	1	X	3	3	5/2	0.007
8	3	1	X	3	3	5/2	
9	4	2	G	0	0	0	38.3
10	5	2	X	2	2	0	4.8

P_G=73.9%

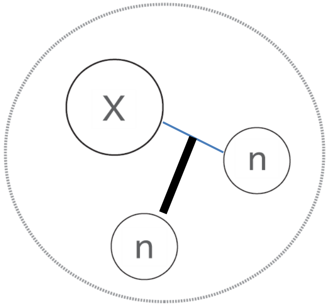
P_X=26.1%

Reduced wave function $f_{\tilde{K}_\alpha}^{J^\pi T}(q_\alpha)$



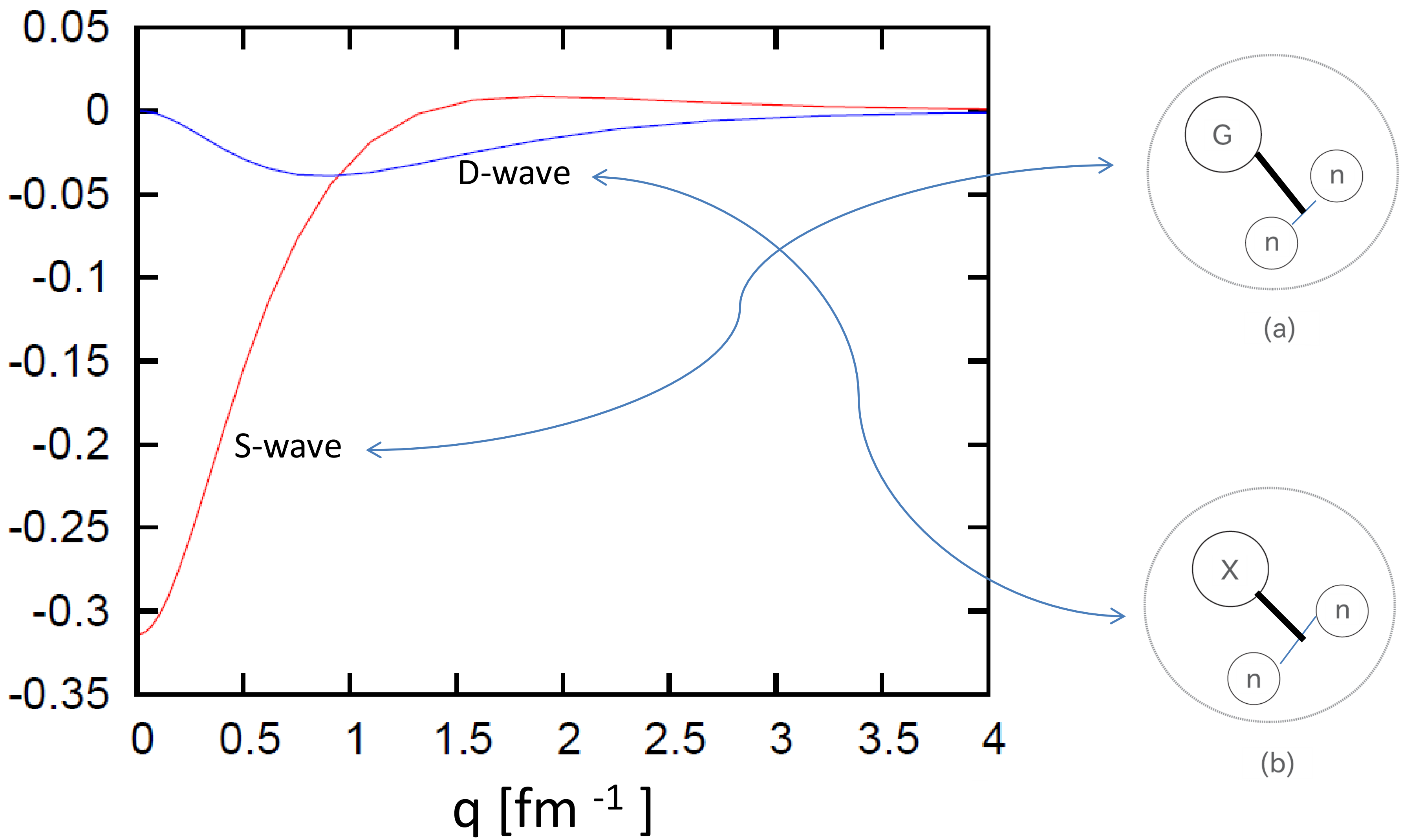


(a)

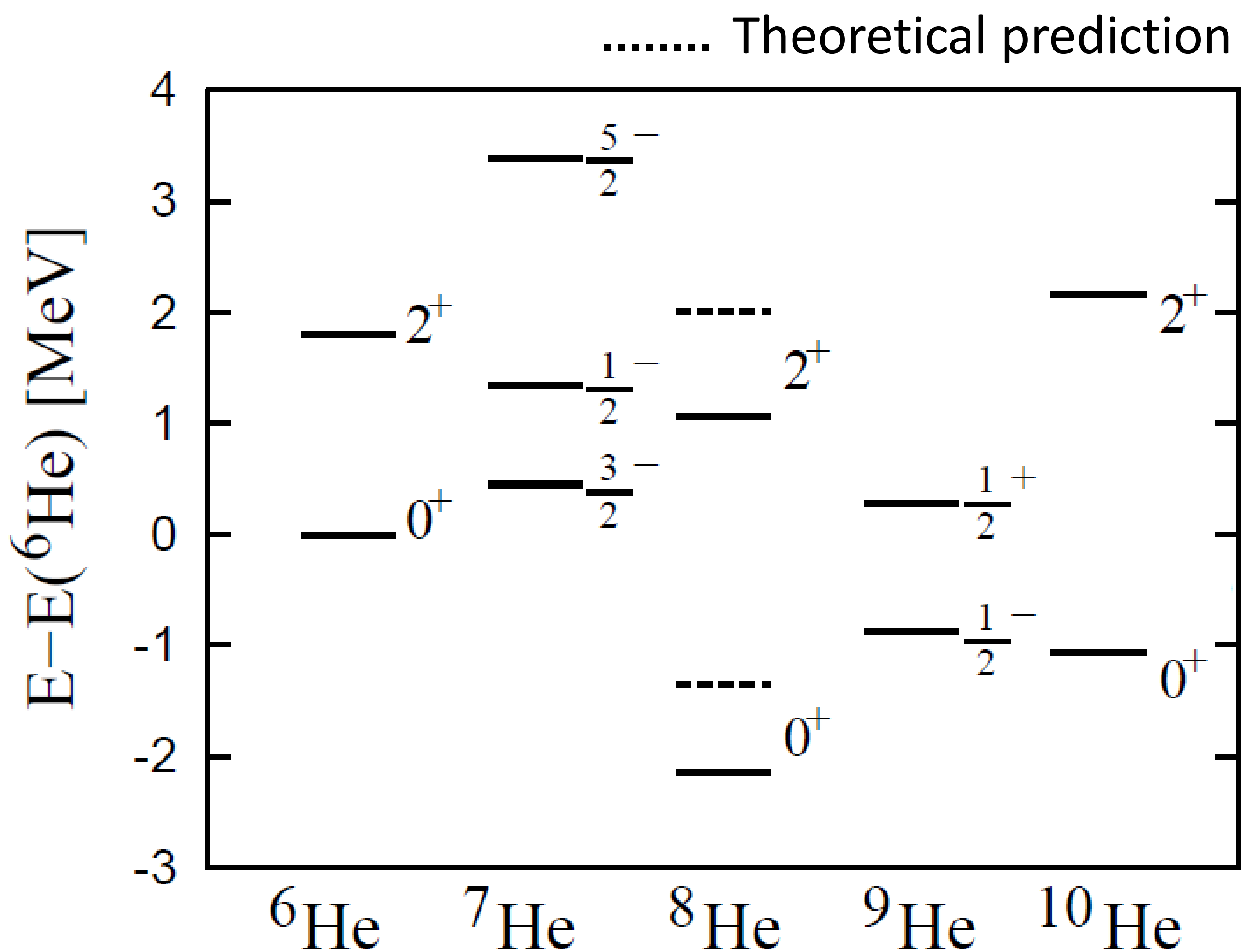


(b)

Reduced wave function $f_{\tilde{K}_\alpha}^{J^\pi T}(q_\alpha)$



Spectra of $^{6,7,8,9,10}He$



^{10}He

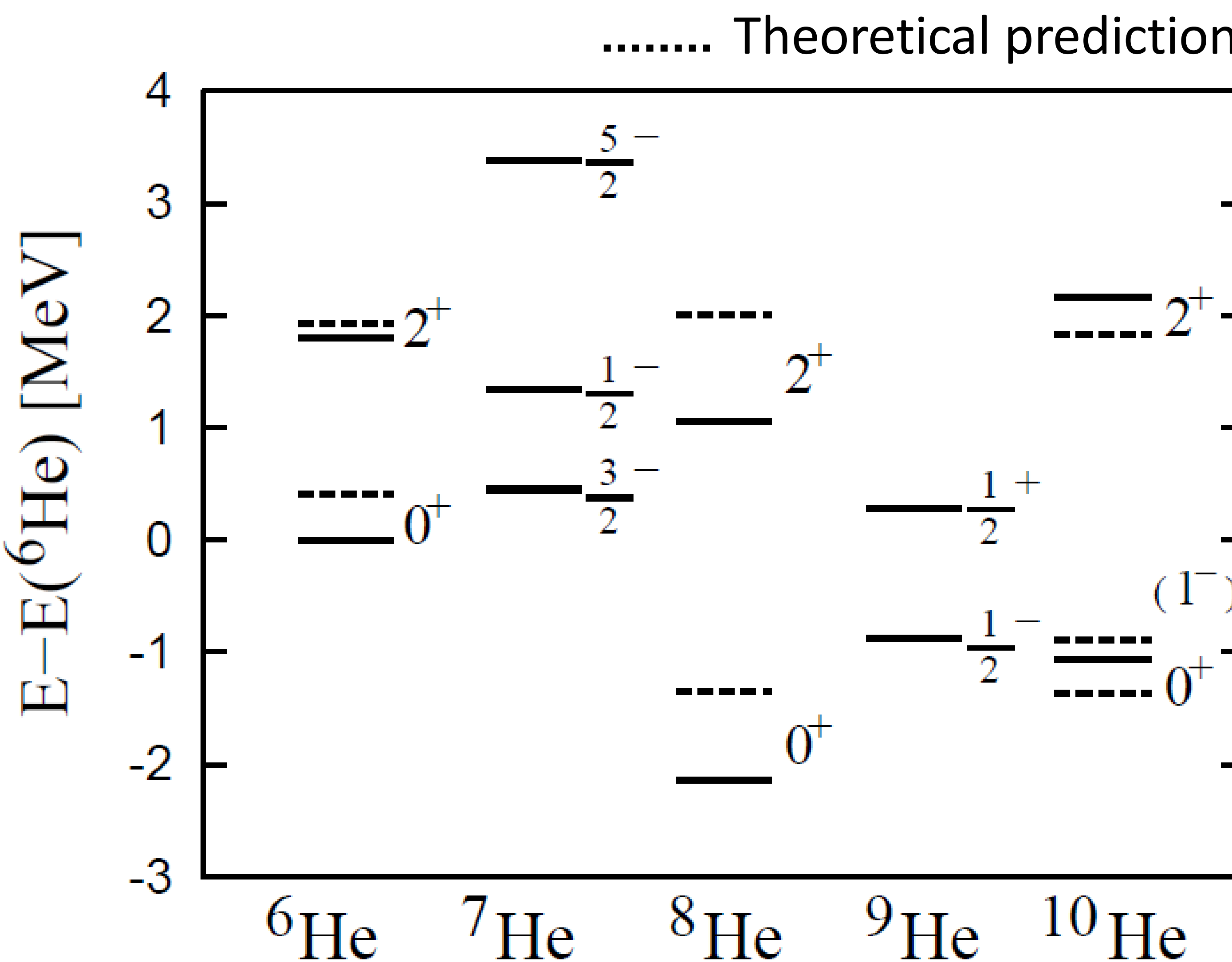
TABLE VI. The predicted energy levels of the ^{10}He nucleus from the ^{8}He $+ n + n$ threshold. The resonance energy E equals $E^{(r)} - i\Gamma/2$. Units are MeV.

J^π	Present work		Expt.	
	$E^{(r)}$	Γ	$E^{(r)}$	Γ
0^+	0.803	0.665	1.069	0.3 ± 0.2
1^-	1.25	0.21		
2^+	3.97	4.71	4.31 ± 0.20	0.6 ± 0.3

^{6}He

A. Eskandarian and I. R. Afnan, Phys. Rev. C **46**, 2344 (1992).

Spectra of $^{6,7,8,9,10}He$



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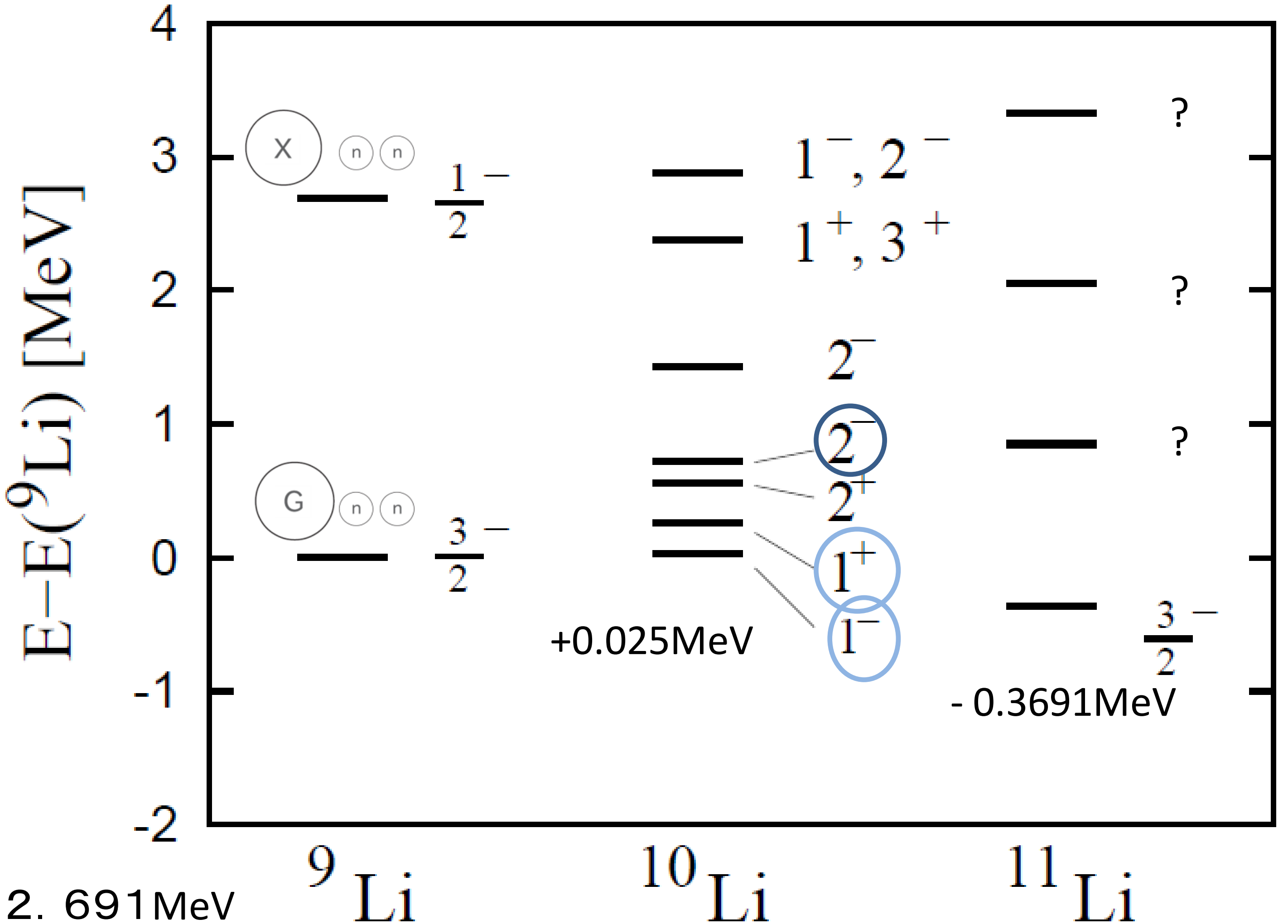
Spectra of ^{11}Li

Experimental Data



δm〜2. 691MeV

TABLE II: Set of the quantum numbers for $J^\pi=(3/2)^-$ state of ^{11}Li nucleus. The quantum numbers for the particle channel α=3 is obtained from α=1 by only cyclically label replacing $s_\alpha \rightarrow s_\beta \rightarrow s_\gamma \rightarrow s_\alpha$.

K_α	$\tilde{K}_\alpha$	α	I	$\mathcal{L}_\alpha$	$\mathcal{S}_\alpha$	j_α	l_α	S_α	s_α	s_β	s_γ
1	1	1	G	0	3/2	1	0	1	1/2	1/2	3/2
2	1	1	X	0	3/2	1	0	1	1/2	1/2	1/2
3	2	1	G	1	1/2	1	1	1	1/2	1/2	3/2
4	2	1	X	1	1/2	1	1	1	1/2	1/2	1/2
5	2	1	G	1	3/2	1	1	1	1/2	1/2	3/2
6	2	1	X	1	3/2	1	1	1	1/2	1/2	1/2
7	3	1	G	0	3/2	2	0	2	1/2	1/2	3/2
8	3	1	X	0	3/2	2	2	0	1/2	1/2	1/2
9	4	2	G	0	3/2	0	0	0	3/2	1/2	1/2
10	5	2	X	2	1/2	0	0	0	1/2	1/2	1/2

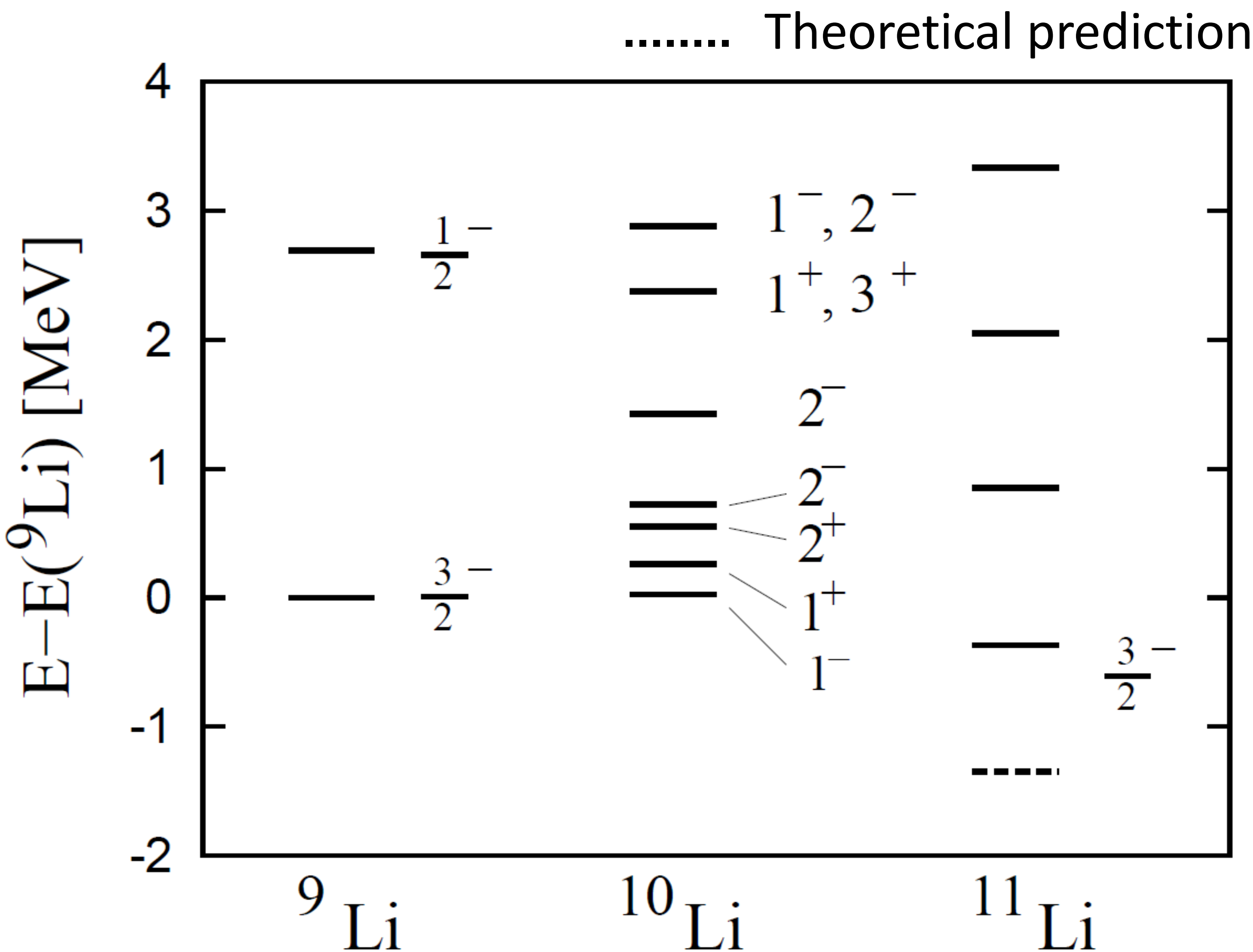
Numerical Result

TABLE III: The predicted energy levels of ^{11}Li nucleus from ^{9}Li + n + n threshold. Unit is in MeV.

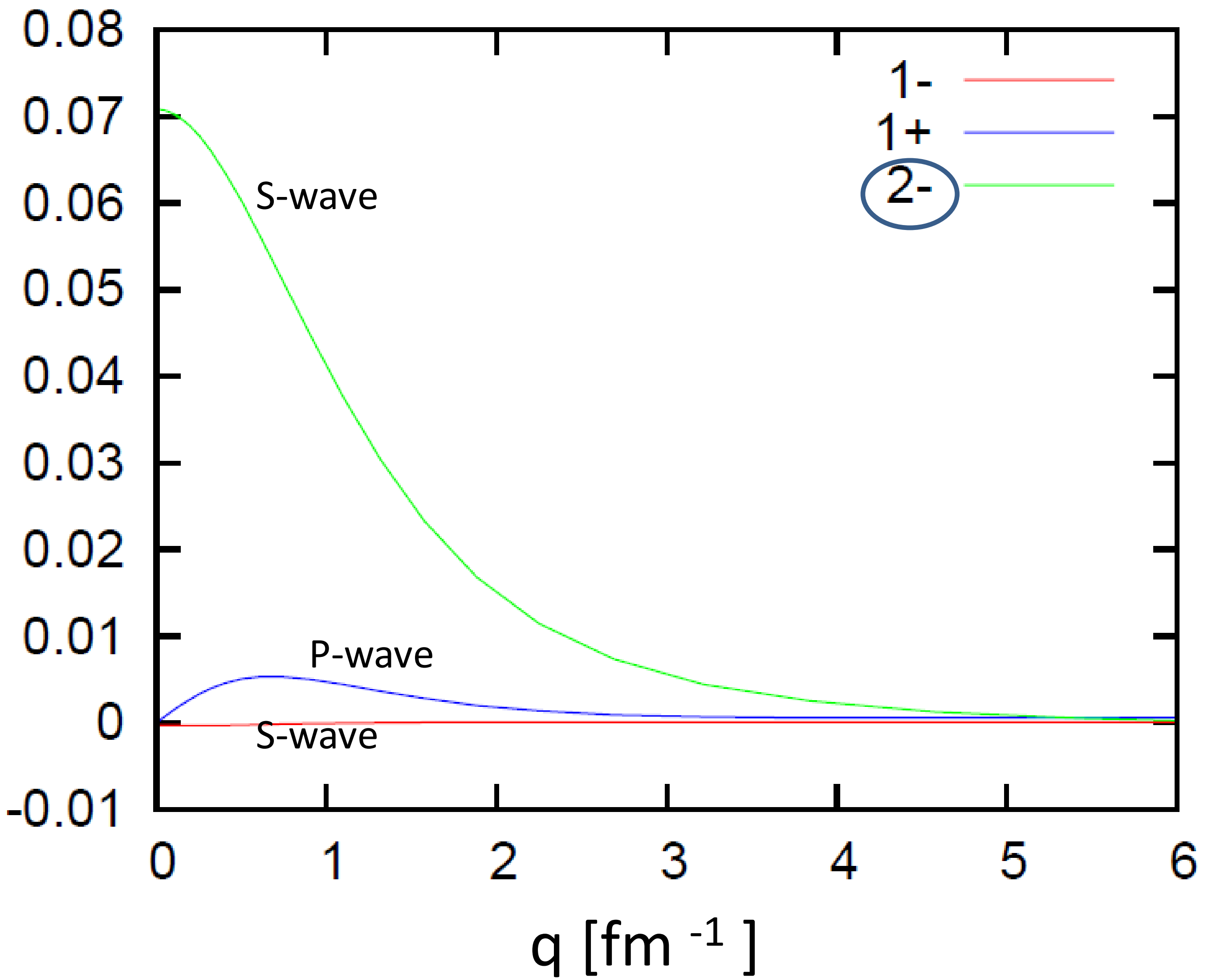
J^{π}	Present work	Expt.
$(3/2)^{-}$	-1.35	-0.3691

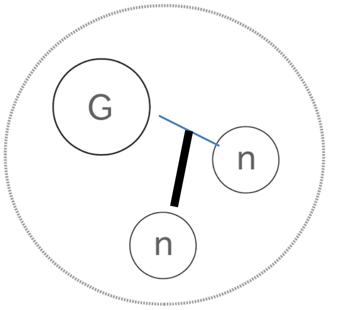
Preliminary Result

Spectra of ^{11}Li



Reduced wave function $f^{J^\pi T}_{\tilde{K}_\alpha}(q_\alpha)$





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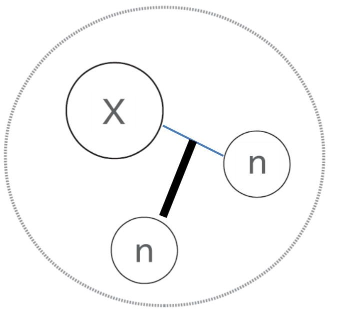
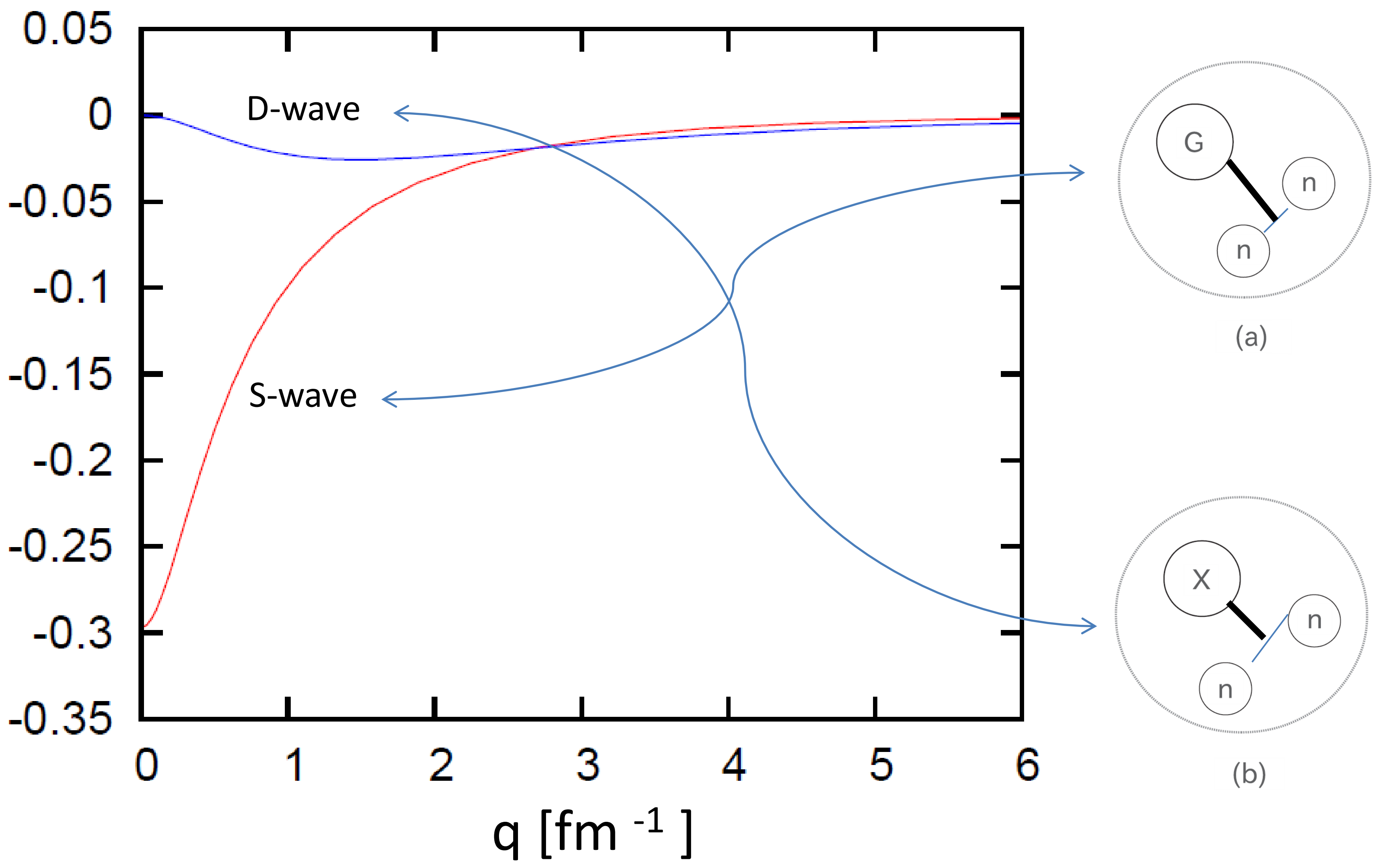
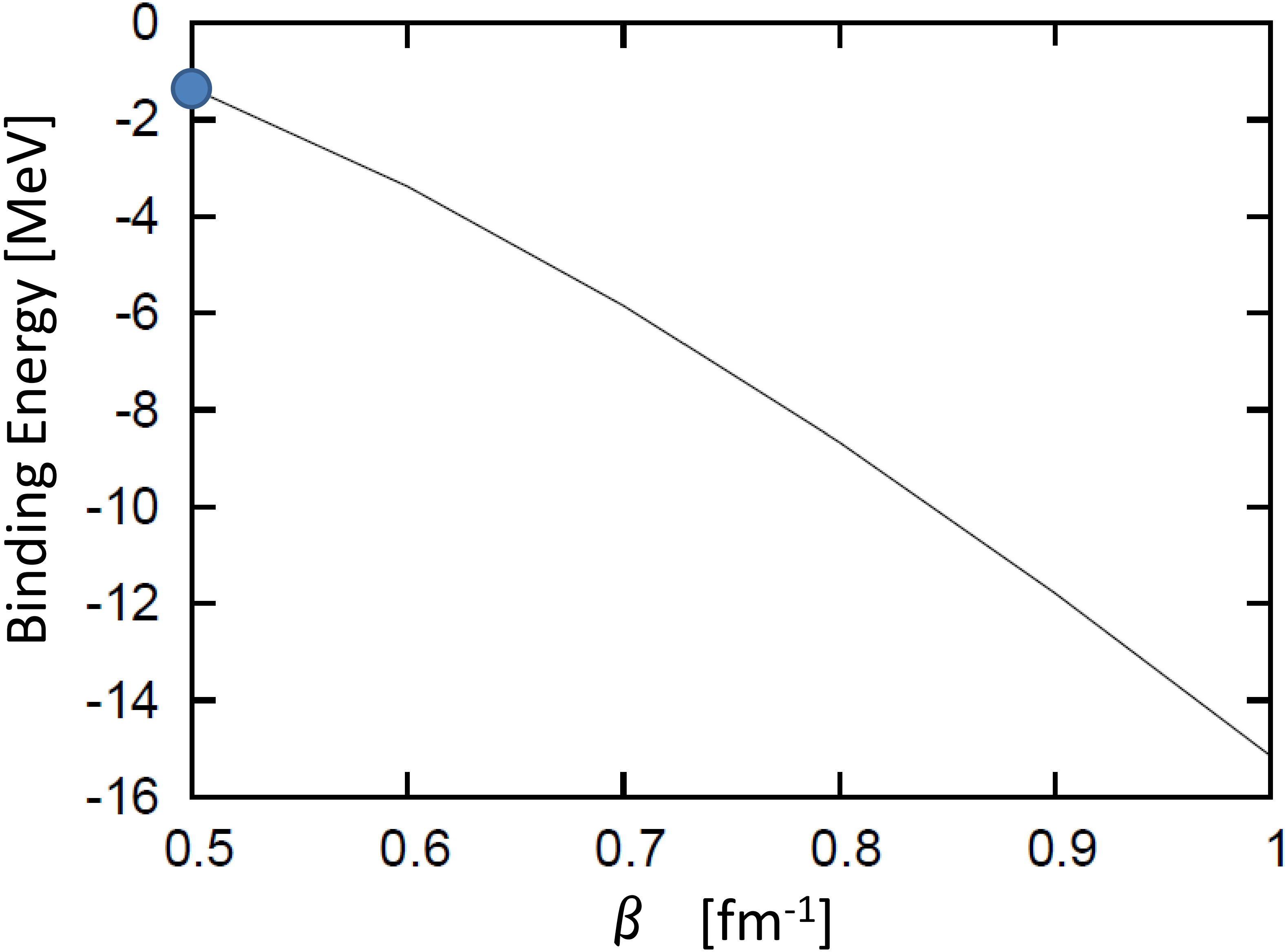


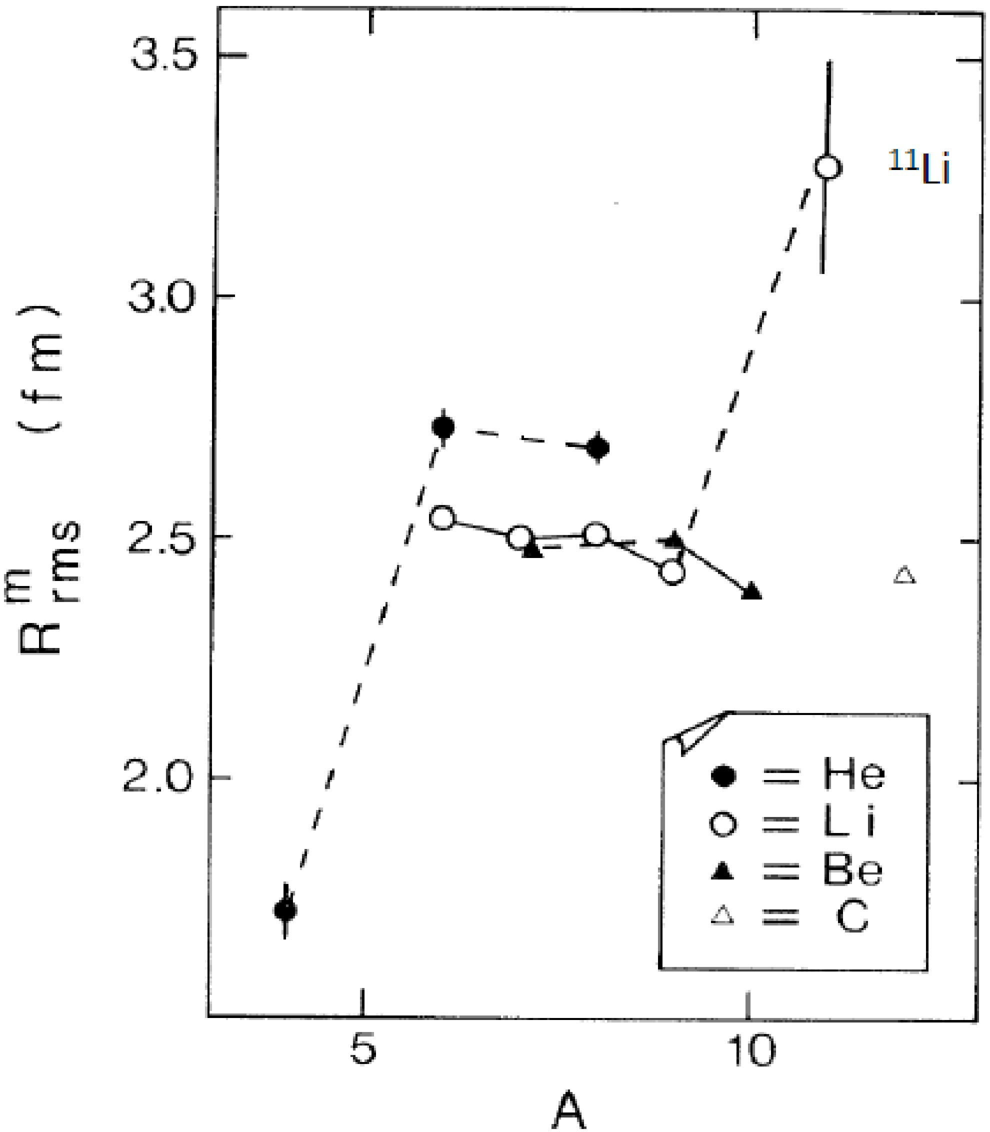
(b)

Reduced wave function $f_{\tilde{K}_\alpha}^{J^\pi T}(q_\alpha)$



Dependence of β





I. Tanihata,et al.,
Phys. Rev. Lett. 55,2676 (1985)

Conclusion

- We introduce a new model applying to the core-nucleus and two neutrons system.
- The Faddeev equations of ^{6}He-n-n system and ^{8}He-n-n system for ^{8}He and ^{10}He, are solved, respectively.
- Inputting only the information of subsystem energy levels and widths the model gives the coupling constant between the core-nucleus and neutron.
- Using the obtained potentials some theoretical predictions in the low-lying level of ^{8}He and ^{10}He were presented.
- Trying ^{11}Li case we need a small parameter $\boldsymbol{\beta}$.