
Low-energy ^{12}C states in three- α model

Souichi ISHIKAWA (Hosei University)



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I. INTRODUCTION

- Low-lying continuum states of ^{12}C in 3- α model

- Photodisintegration of $^{12}\text{C}(0_1^+)$ & $^{12}\text{C}(2_1^+)$



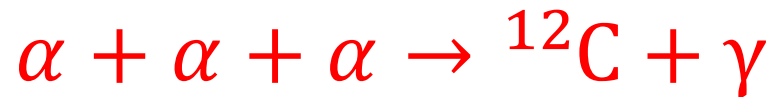
S. I., PRC **87**, 055804 (2013)



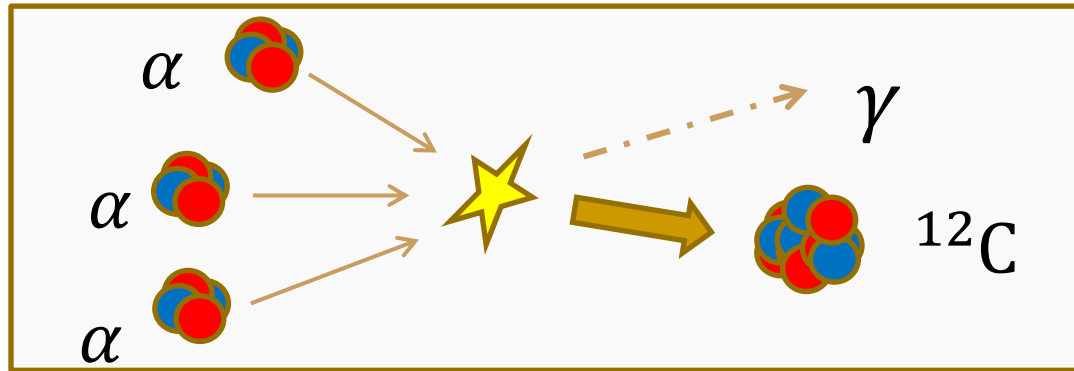
- The inverse of the triple-alpha (3α) reaction



3 α Process



- Key step to produce heavy elements in stars



- 3 α reaction rate $\langle \alpha\alpha\alpha \rangle$ is an important input for simulation of the stellar evolution
- $\langle \alpha\alpha\alpha \rangle$ is obtained from cross section of the inverse reaction

$$\langle \alpha\alpha\alpha \rangle \propto \int_0^\infty dE \frac{(E_\gamma)^2}{(kT)^3} e^{-E/kT} \underline{\sigma_\gamma(E_\gamma)} \quad (E_\gamma = E - E_c)$$

CONTENTS:

II **Formalism** to treat 3α -breakup reaction of ^{12}C

III **3α -model** : $\alpha\alpha$ and $\alpha\alpha\alpha$ (local) interactions

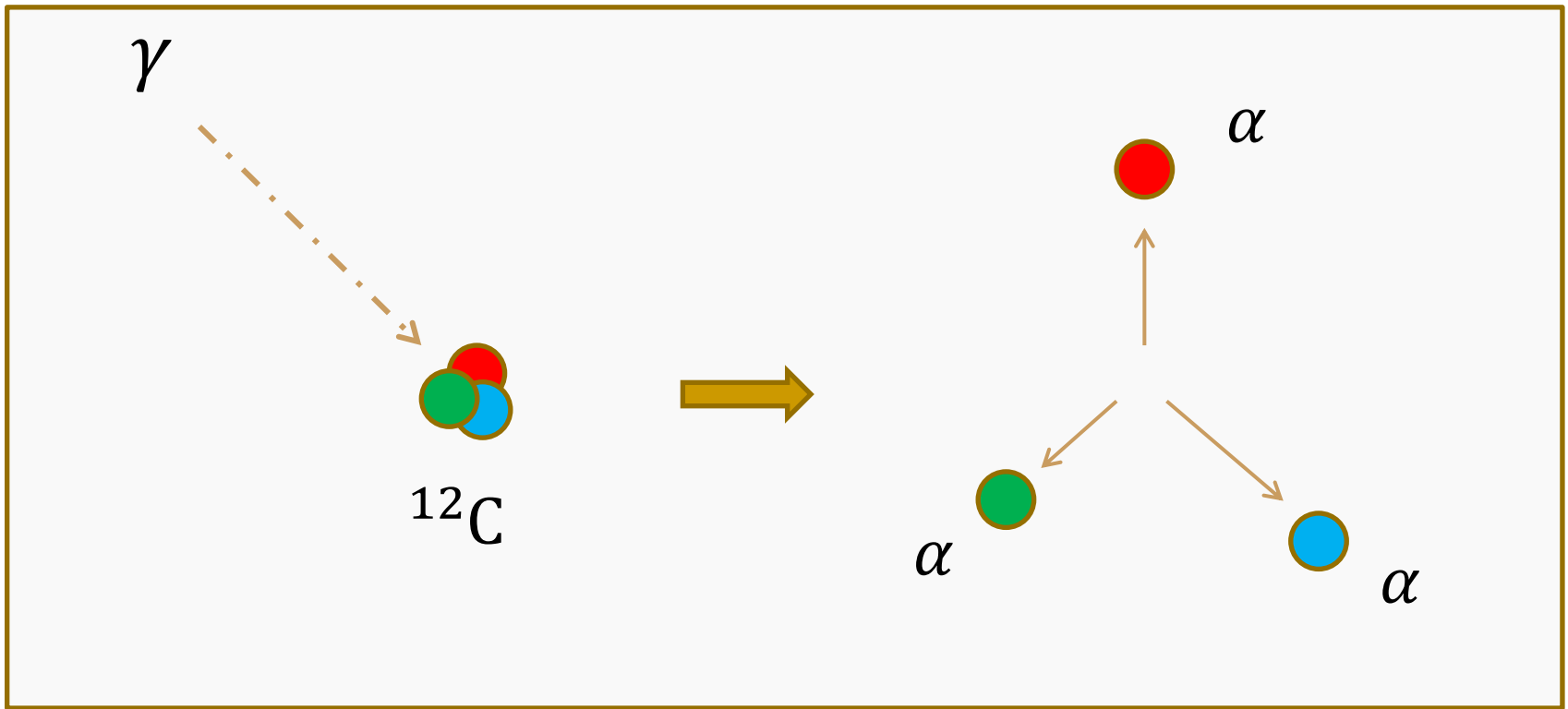
IV **Results**

- Photodisintegration cross sections



- 3α reaction rate $\langle \alpha\alpha\alpha \rangle$

V **Summary**



II. FORMALISM

Photo-induced 3α breakup of ^{12}C

1. Def. wave function for the disintegration process

$$|\Psi\rangle \equiv \frac{1}{E + i\varepsilon - H_0 - V} H_\gamma |\Psi_b\rangle \xrightarrow{R \rightarrow \infty} \frac{e^{iKR}}{R^{5/2}} f^{(B)}(\Theta, x, y)$$

R: Hyper-radius, Θ : Hyper-angle

$$\sigma_\gamma(E) \propto \iint d\hat{x}d\hat{y} \int_0^{\pi/2} d\Theta \cos^2 \Theta \sin^2 \Theta |f^{(B)}(\Theta; \hat{x}, \hat{y})|^2$$

2. Apply the Faddeev theory to calculate $|\Psi\rangle$

$$|\Psi\rangle = \frac{1}{E + i\varepsilon - H_{3\alpha}} H_\gamma |\Psi_b\rangle = |\Phi_1\rangle + |\Phi_2\rangle + |\Phi_3\rangle$$
$$|\Phi_1\rangle = \frac{1}{E + i\varepsilon - H_0 - V_1} H_{\gamma,1} |\Psi_b\rangle + \frac{1}{E + i\varepsilon - H_0 - V_1} V_1 (|\Phi_2\rangle + |\Phi_3\rangle)$$

3. 3-body problem with Coulomb(+short-range) interactions
“Sasakawa-Sawada method”

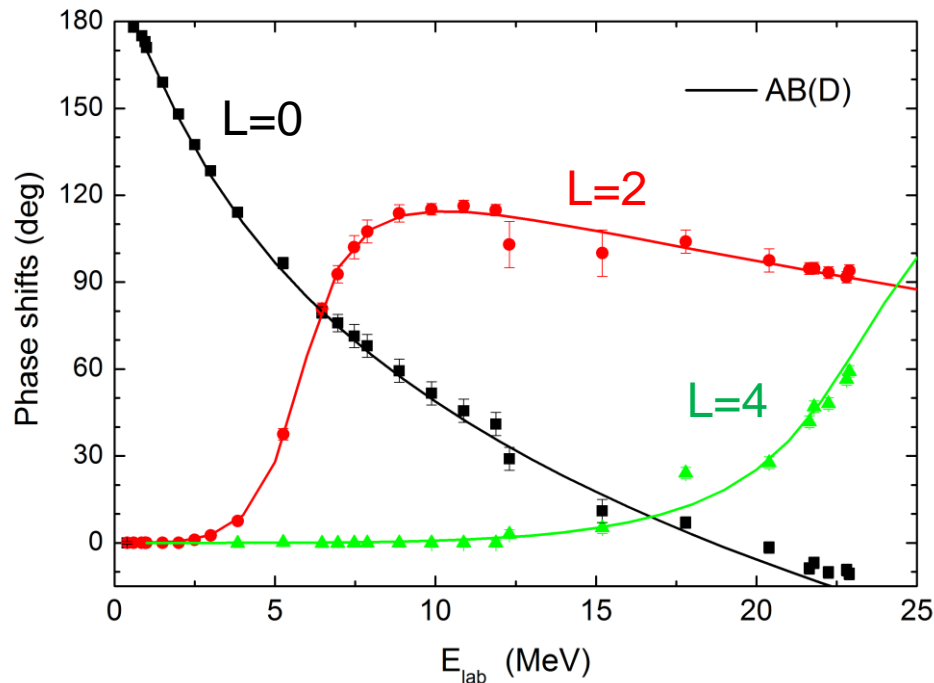
Refs. p-d system: S. I., PRC **80**, 054002 (2009).
 3α system: S. I., PRC **87**, 055804 (2013).

III. 3-ALPHA MODEL

Shallow α - α potential (no forbidden state)

$$V_{\alpha\alpha}(x) = \left(V_R^{(0)} \hat{P}_{L=0} + V_A^{(2)} \hat{P}_{L=2} \right) e^{-(x/a_R)^2} + V_A e^{-(x/a_A)^2}$$

	a_R (fm)	$V_R^{(0)}$ (MeV)	$V_R^{(2)}$ (MeV)	a_A (fm)	V_A (MeV)
AB(D)	1.40	500.0	320.0	2.11	-130.0

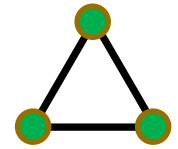


Ali & Bodmer, NP80, 99 (1966)

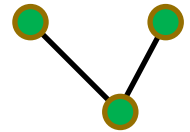
$\alpha\alpha$ phase-shift parameters

α - α - α Potentials

$$V_{\alpha\alpha\alpha} = \sum_{J=0,2} W_J \hat{P}_J e^{-\frac{(\vec{r}_1 - \vec{r}_2)^2 + (\vec{r}_2 - \vec{r}_3)^2 + (\vec{r}_3 - \vec{r}_1)^2}{a^2}} + [\text{c.p.}] \quad (1)$$



$$V_{\alpha\alpha\alpha} = \sum_{J=0,2} W_J \hat{P}_J e^{-\frac{(\vec{r}_1 - \vec{r}_2)^2 + (\vec{r}_2 - \vec{r}_3)^2}{a^2}} + [\text{c.p.}] \quad (2)$$



Model	Eq.	a (fm)	W_0 (MeV)	W_2 (MeV)
Δ	(1)	3.39	-30.95	-15.3
V_L	(2)	3.33	-12.21	-7.7
V_S	(2)	2.5	-39.64	-22.4

Strength for J=0 [J=2] is determined to reproduce $^{12}\text{C}(0_2^+)$ [$^{12}\text{C}(2_1^+)$] energy

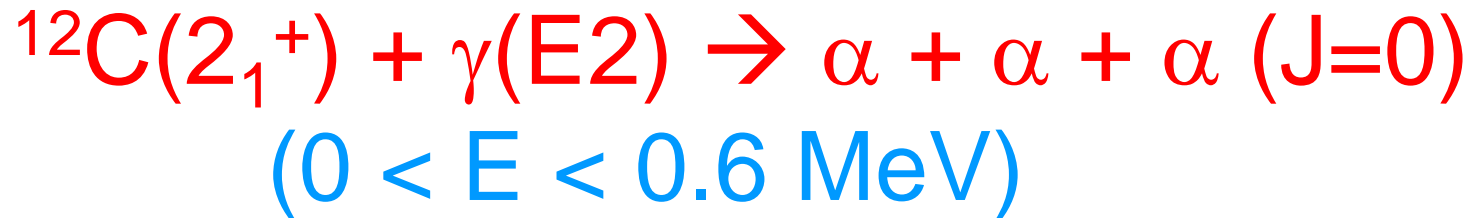
(1) D.V. Fedorov and A. S. Jensen, PLB **389**, 631 (1996)

(2) O.Portilho, D.A. Agrelo, and S.A. Coon, PRC **27**, 2923 (1983)

IV. RESULTS

Study of $0^+ - 3\alpha$ state

by



S. I., PRC **87**, 055804 (2013)

Three-body calculations of the triple- α reaction

S. Ishikawa*

Science Research Center, Hosei University, 2-17-1 Fujimi, Chiyoda, Tokyo 102-8160, Japan

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Recently, the triple- α (3α) process, by which three ${}^4\text{He}$ nuclei are fused into a ${}^{12}\text{C}$ nucleus in stars, was studied by using different methods to solve the quantum mechanical three-body problem. The results for the thermonuclear reaction rate for the process differ by several orders at low stellar temperatures of 10^7 – 10^8 K. In this paper, we will present calculations of the 3α process by using a modified Faddeev three-body formalism in which the long-range effects of Coulomb interactions are accommodated. The reaction rate of the process is calculated via an inverse process: three-alpha (3α) photodisintegration of a ${}^{12}\text{C}$ nucleus. The calculated reaction rate is about 10 times larger than that of the Nuclear Astrophysics Compilation of Reaction Rates at 10^7 K and is notably smaller than the results of recent three-body calculations. We will discuss a possible reason for the difference.

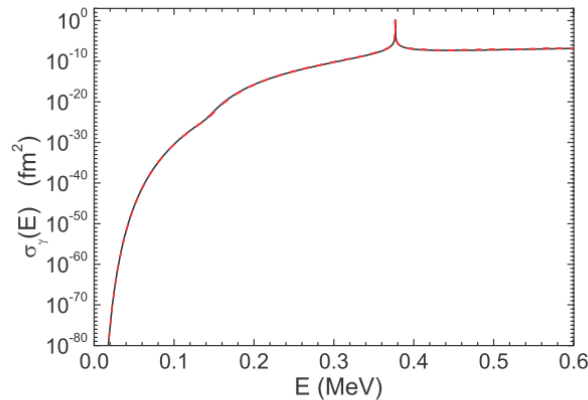


FIG. 2. (Color online) Calculated photodisintegration cross section for the process, Eq. (5), as a function of the 3α energy E . The solid line is the result for $\text{AB}(A')$ and the dashed line is for $\text{AB}(D)$.

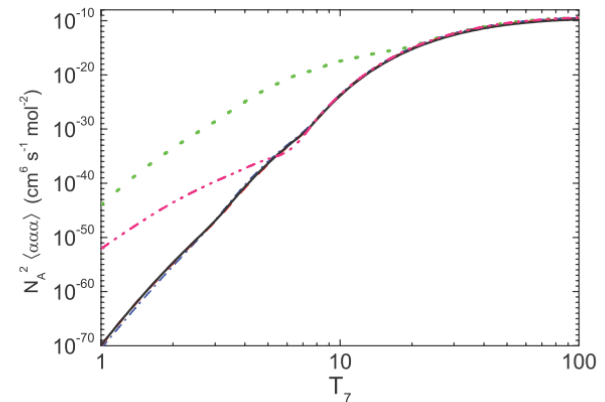
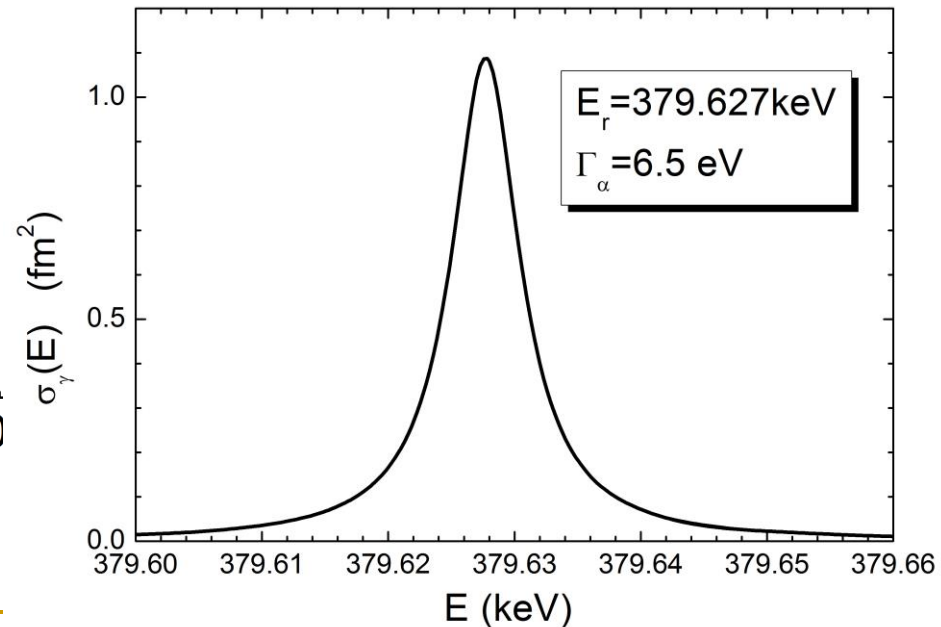
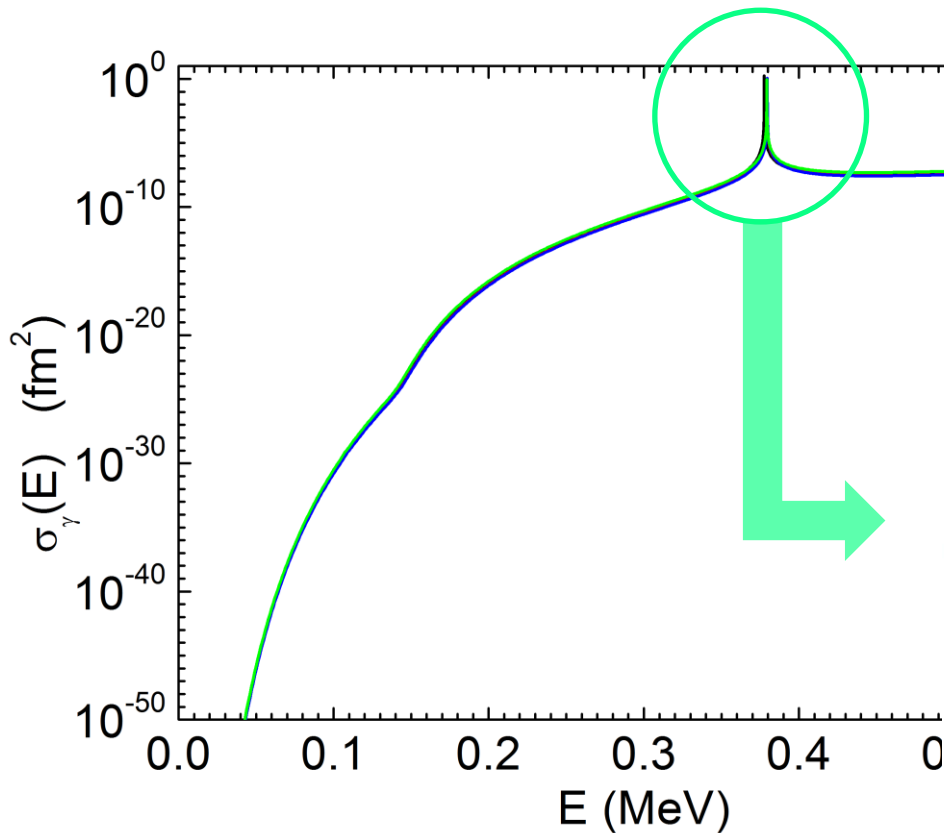
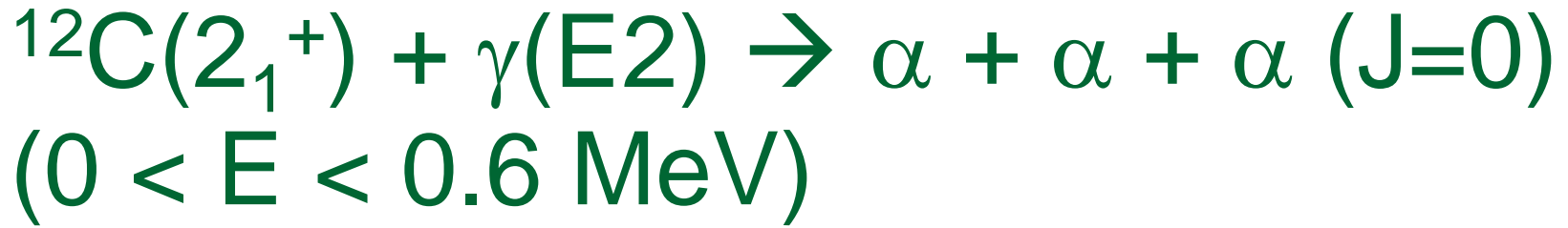


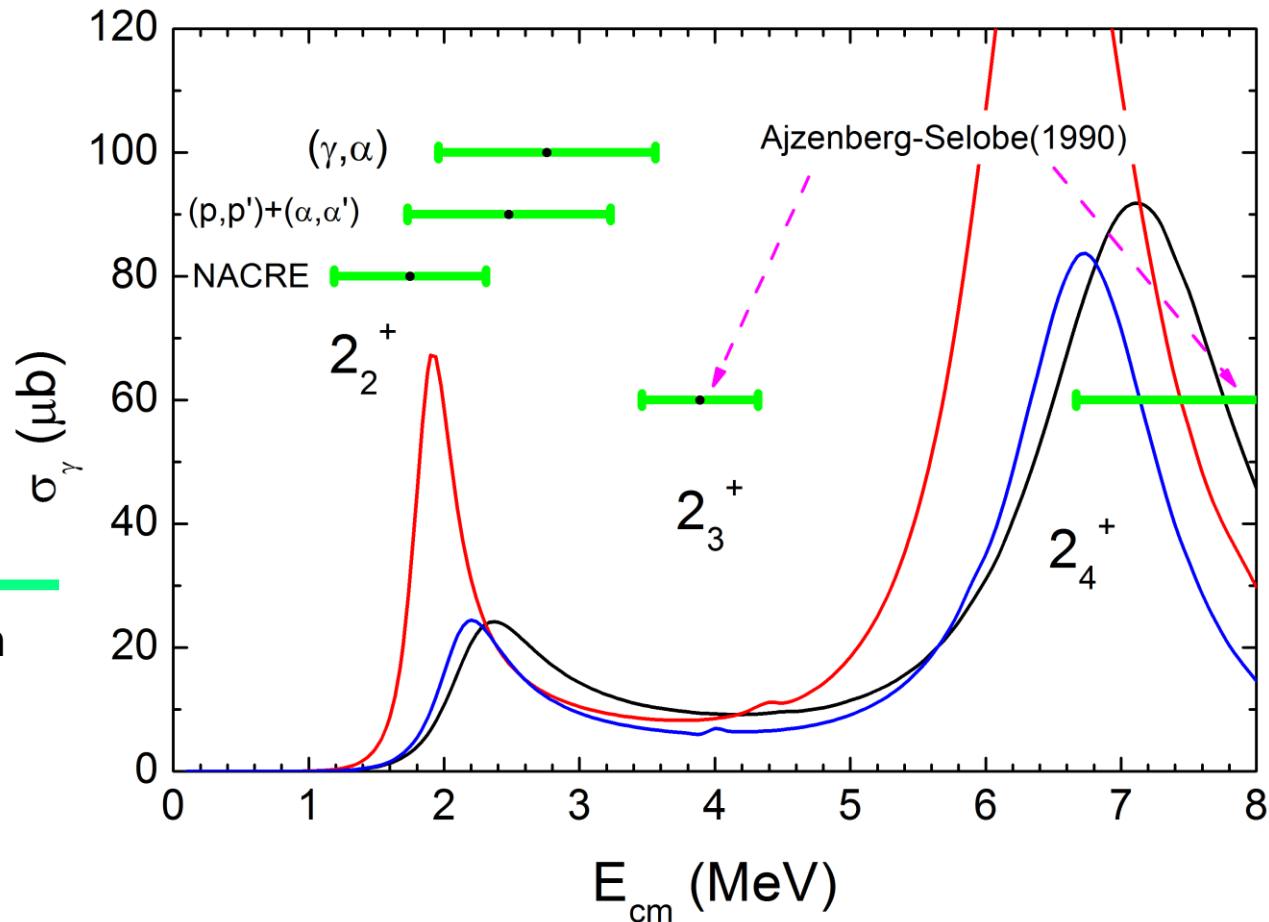
FIG. 3. (Color online) The 3α reaction rate as a function of temperature. The solid line denotes the present calculation for $\text{AB}(A')$, the dashed line is for $\text{AB}(D)$, the dot-dashed line is the NACRE rate [5], the dotted line is the OKK rate [8], and the dot-dot-dashed line is the HHR rate [17].



Study of $2^+-3\alpha$ state

by





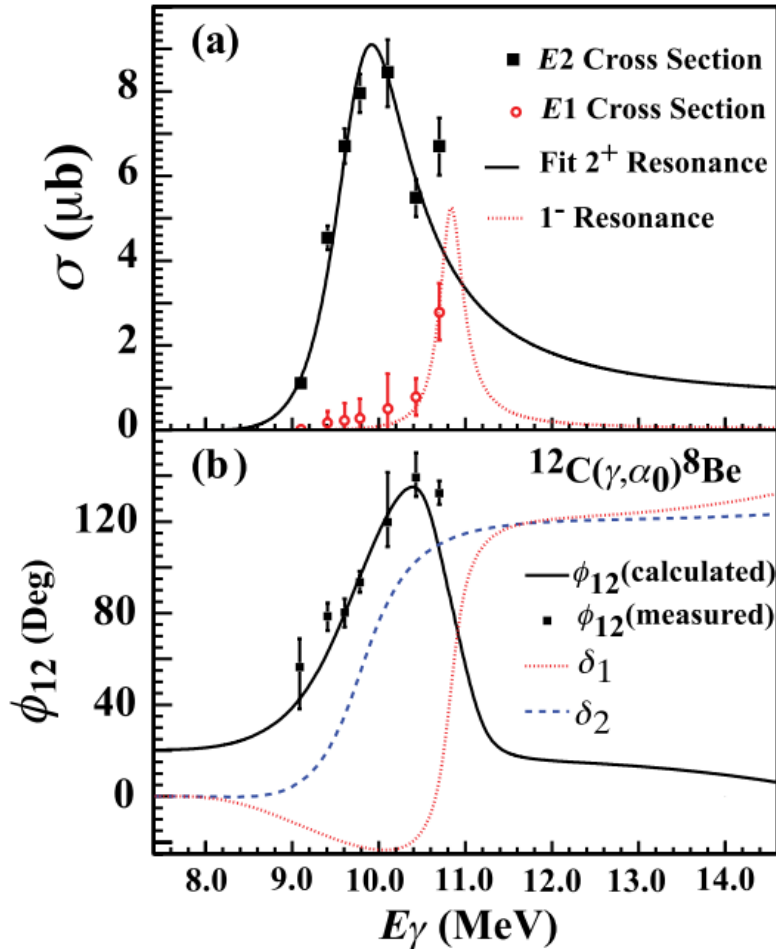
NACRE Nuclear Astrophysics Compilation of REaction Rates,
Angulo et al., [NPA656 (1999) 3]

(p,p') , (α,α') Freer et al., PRC **86**, 034320 (2012)

(γ,α) Zimmerman et al., PRL **110**, 152502 (2013)

Unambiguous Identification of the Second 2^+ State in ^{12}C and the Structure of the Hoyle State

W. R. Zimmerman,^{1,2} M. W. Ahmed,^{2,3} B. Bromberger,⁴ S. C. Stave,² A. Breskin,⁵ V. Dangendorf,⁴ Th. Delbar,⁶ M. Gai,^{1,7}
S. S. Henshaw,² J. M. Mueller,² C. Sun,² K. Tittelmeier,⁴ H. R. Weller,^{1,2} and Y. K. Wu²

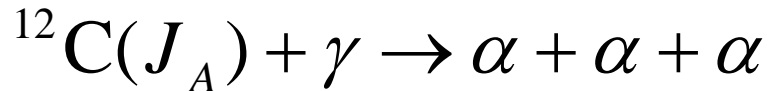


	$B(E2, 2_2^+ \rightarrow 0_1^+)$ e^2fm^4
(γ, α')	0.73(13)
$AB(D)+\Delta$	2.3
$AB(D)+V_L$	3.6
$AB(D)+V_S$	1.8

FIG. 4 (color online). (a) The measured $E1$ and $E2$ cross sections of the $^{12}\text{C}(\gamma, \alpha_0)^8\text{Be}$ reaction. (b) The measured $E1$ - $E2$ relative phase angle (ϕ_{12}) together with the phase angle — calculated from a two-resonance model.

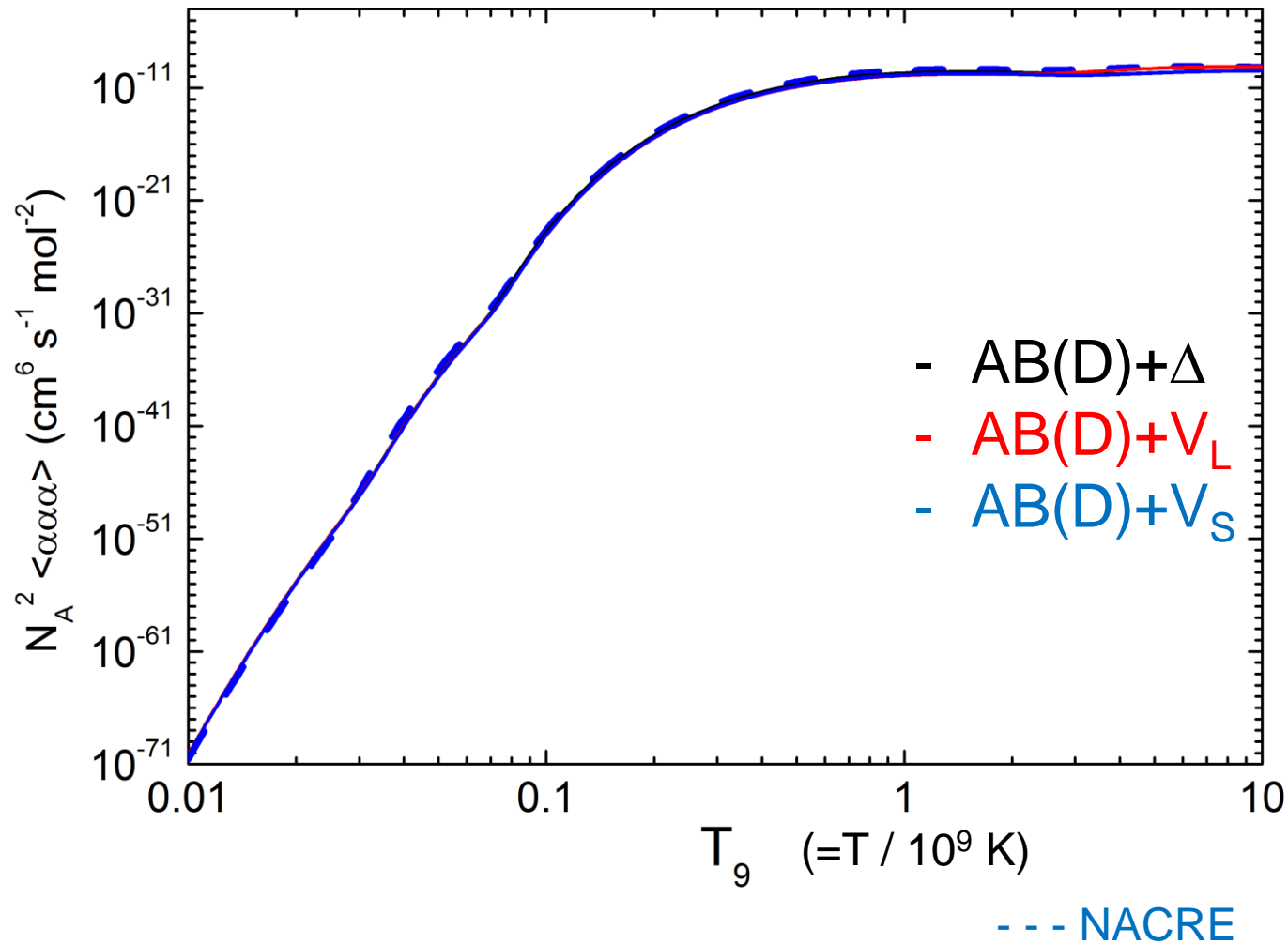
$\alpha\alpha\alpha$ reaction rate

Inverse reaction: Photo induced 3α breakup of ^{12}C



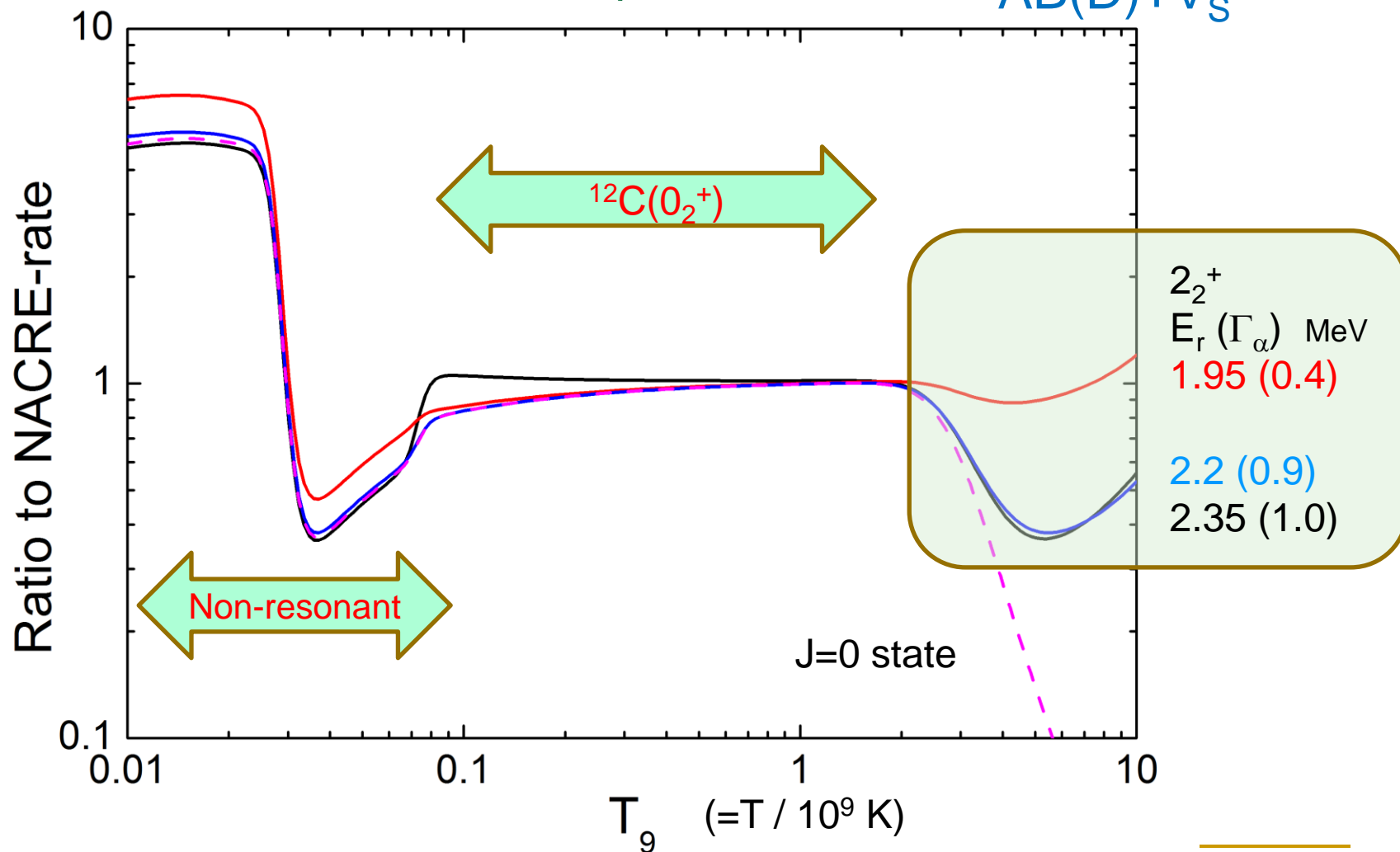
$$\langle \alpha\alpha\alpha \rangle = 48(2J_A + 1)(3)^{3/2} \pi \left(\frac{\hbar}{mc} \right)^3 c$$
$$\times \int_0^\infty \frac{dE E_\gamma^2}{(kT)^3} e^{-E/k_B T} \sigma_{^{12}\text{C}(J_A)+\gamma \rightarrow 3\alpha}(E_\gamma) \quad \left(E_\gamma = E - E_{^{12}\text{C}(J_A)} \right)$$

3α reaction rate $\langle\alpha\alpha\alpha\rangle$



$\alpha\alpha\alpha$ reaction rate (Ratio to NACRE rate)

- AB(D)+ Δ
- AB(D)+ V_L
- AB(D)+ V_S

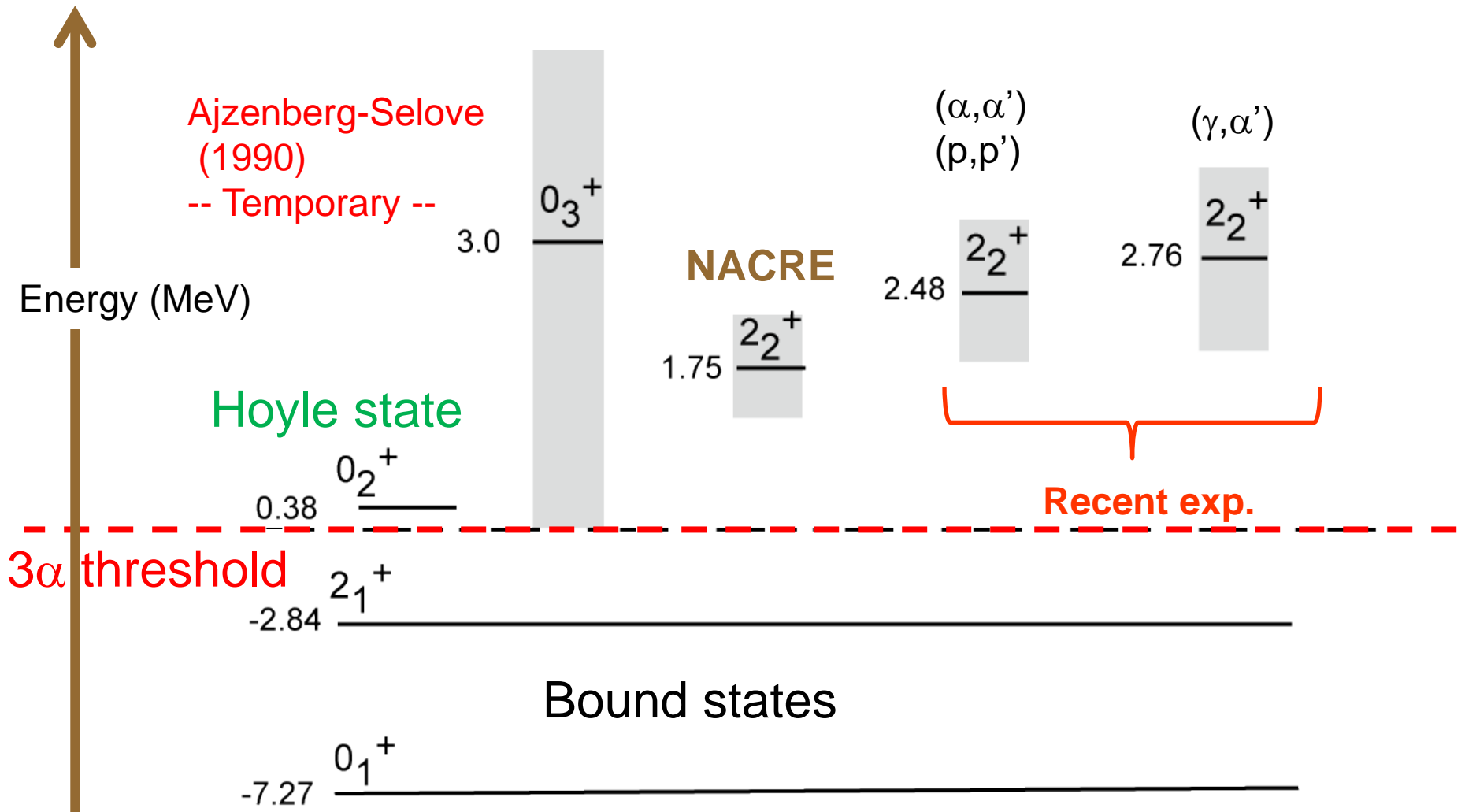


NACRE: 2_2^+ $E_r = 1.75$ MeV, $\Gamma_\alpha = 0.4$ MeV

V. SUMMARY

- 3α model calculations of ^{12}C
- Ali-Bodmer $\alpha\alpha$ potential + 3α potential
- 3α potential parameters:
 $^{12}\text{C}(0_2^+)$ [Hoyle resonance], $^{12}\text{C}(2_1^+)$
- 3α Breakup reaction 0^+ , 2^+ resonances
 $^{12}\text{C}(2_1^+) + \gamma(\text{E2}) \rightarrow \alpha + \alpha + \alpha$ (J=0)
 $^{12}\text{C}(0_1^+) + \gamma(\text{E2}) \rightarrow \alpha + \alpha + \alpha$ (J=2)
- Larger E2-Strength of $^{12}\text{C}(2_2^+) \rightarrow ^{12}\text{C}(0_1^+)$ than the experimental value
[Note: Model dependence]
- Effects on 3α reaction rate ($2 \cdot 10^9 \text{ K} < T < 10^{10} \text{ K}$)

Uncertainties of $^{12}\text{C}(2_2^+)$ resonances



$^{12}\text{C}(2_2^+)$ resonance parameters

	E_r (MeV)	Γ_α (MeV)	$B(E2, 2_2^+ \rightarrow 0_1^+)$ $e^2\text{fm}^4$
AB(D)+ Δ	2.35	1.0	2.3
AB(D)+ V_L	1.95	0.4	3.6
AB(D)+ V_S	2.2	0.9	1.8
NACRE	1.75	0.65	
Exp.			
(α, α') (p, p')	2.48(15)	0.750(150)	1.6(2), 1.83(9)
(γ, α')	2.76(11)	0.800(130)	0.73(13)

(α, α') (p, p') Freer et al., PRC **86**, 034320 (2012)

(γ, α') Zimmerman et al., PRL **110**, 152502 (2013)

Calculated ^{12}C resonance parameters

		AB(D)	AB(D)	AB(D)	Exp.
	3BP	Δ	V_L	V_S	
0_2^+	E_r (MeV)	0.379177	0.379338	0.379627	0.3794
	Γ_α (eV)	6.2	7.5	6.5	8.3(1.0)
	Γ_γ (meV)	2.2	1.4	1.5	3.7(5)
0_3^+	E_r (Γ) (MeV)	1.1 (1.2)	1.0 (1.1)	1.1 (1.6)	1.77 (1.45)
	E_r (Γ) (MeV)	3.7 (?)	4.02 (0.14)	3.3 (0.6)	3.29 (1.42)
2_2^+	E_r (Γ) (MeV)	2.35 (1.0)	1.95 (0.4)	2.2 (0.9)	2.48(0.750) 2.76(0.800) 1.75(0.65)
	E_r (Γ) (MeV)		4.4 (?)	4 (?)	3.89 (0.43)
	E_r (Γ) (MeV)	7.1 (1.8)	6.45 (1.3)	6.73 (1.2)	8.17 (1.5)