Progress on Light-Ion Fusion Reactions with Three-Nucleon Forces

22nd European Conference on Few Body Problems in Physics

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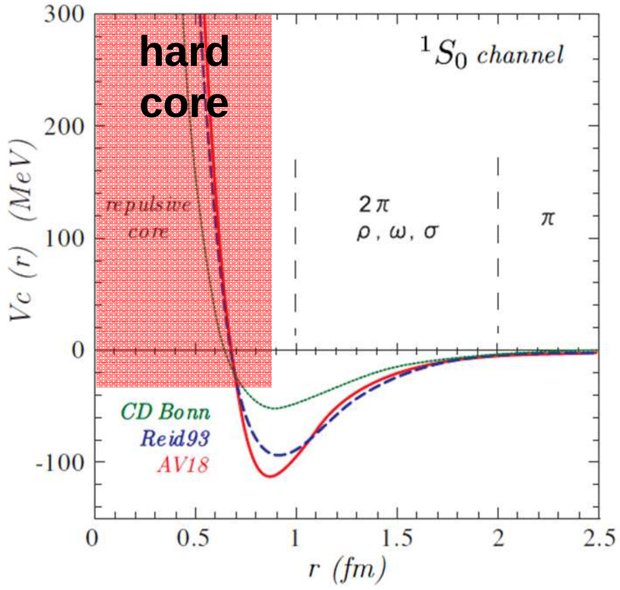
Some words about the ingredients of an *ab initio* calculation

A high precision nuclear Hamiltonian

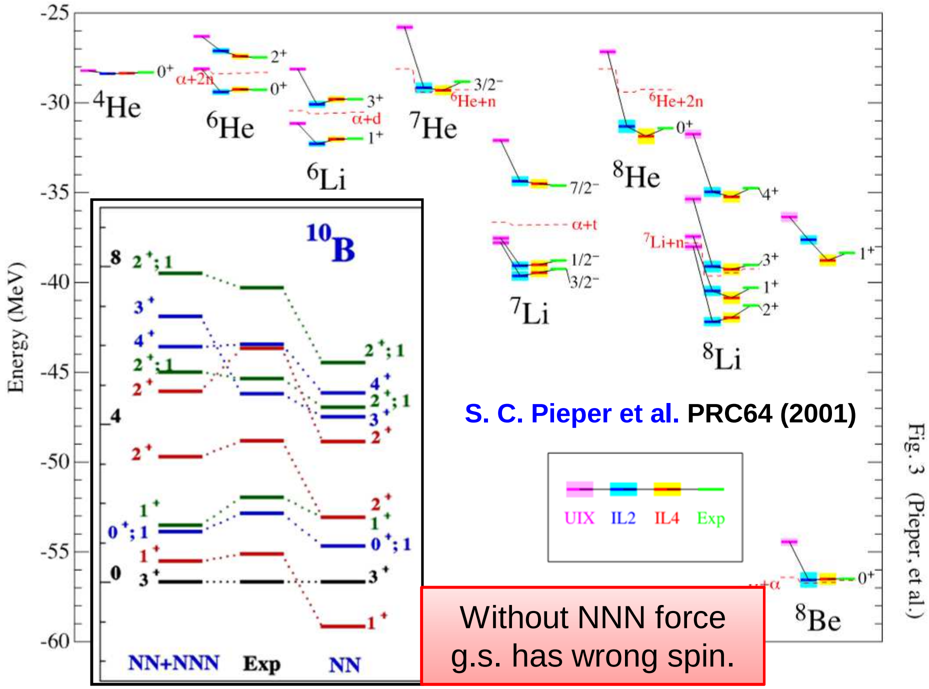
We have a NN interaction…



N. Ishii et al. PRL99 (2007)

- The nuclear interaction has a strong repulsive core.
- This makes nuclear structure calculations to converge slowly.

…and also NNN interaction



S. C. Pieper et al. PRC64 (2001)

Without NNN force
g.s. has wrong spin.

P. Navrátil et al. PRL 99 (2007)

- NNN interaction effects are important.
- This is ~100 times numerically costlier.

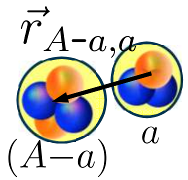
Ab initio NCSM/RGM: formalism for binary clusters

S. Quaglioni and P. Navrátil, Phys. Rev. Lett. 101, 092501 (2008); Phys. Rev. C 79, 044606 (2009)

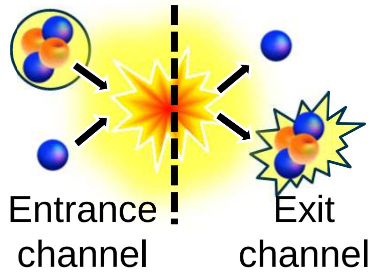
- Starts from:

$$\Psi^{(A)}_{RGM} = \sum_{\nu} \int d\vec{r}\, g_\nu(\vec{r})\, \hat{A}_\nu \left| \Phi^{(A-a,a)}_{\nu\vec{r}} \right\rangle \Longleftrightarrow \psi^{(A-a)}_{\alpha_1} \psi^{(a)}_{\alpha_2} \delta(\vec{r} - \vec{r}_{A-a,a})$$

Relative wave function (unknown) — Antisymmetrizer — Channel basis — Cluster expansion technique



Ex: n-^{4}He scattering



- Schrödinger equation on channel basis:

$$H\Psi^{(A)}_{RGM} = E\Psi^{(A)}_{RGM} \Rightarrow \sum_{\nu} \int d\vec{r} \left[H_{\nu'\nu}(\vec{r}',\vec{r}) - E\, N_{\nu'\nu}(\vec{r}',\vec{r}) \right] g_\nu(\vec{r}) = 0$$

$$\left\langle r' \left| \hat{A}_{\nu'} H \hat{A}_\nu \right| r \right\rangle \qquad \left\langle r' \left| \hat{A}_{\nu'} \hat{A}_\nu \right| r \right\rangle \quad \propto \text{ NCSM densities}$$

Hamiltonian kernel — Norm (overlap) kernel

RGM accounts for: 1) interaction (Hamiltonian kernel), 2) Pauli principle (Norm kernel) between clusters and NCSM accounts for: internal structure of clusters

Going around the hard core problem

E. Jurgenson, Navrátil, R. J. Furnstahl Phys. Rev. Lett. 103 (2009)

In configuration interaction methods we need to soften interaction to address the hard core

We use the Similarity-Renormalization-Group (SRG) method

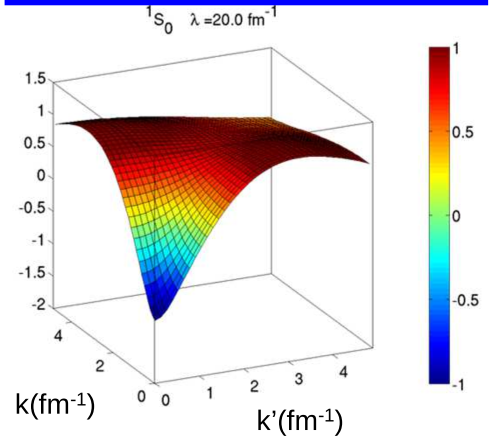
$$\mathrm{H}_\lambda = \boxed{U_\lambda} \mathrm{H} \boxed{U_\lambda^+}$$

Unitary transformations

$$\begin{cases} \dfrac{d\mathrm{H}_\lambda}{d\lambda} = \left[\eta(\lambda), \mathrm{H}_\lambda\right] \\ \eta(\lambda) = \dfrac{dU_\lambda}{d\lambda} U_\lambda^+ \end{cases}$$

Flow parameter

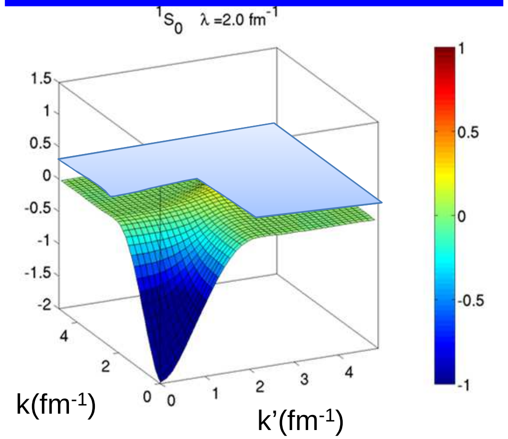
Bare potential



Evolution with flow parameter λ

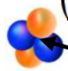
- Preserves the physics
- Decouples high and low momentum
- Induces many-body forces

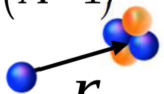
Evolved potential



Including the NNN force into the NCSM/RGM approach

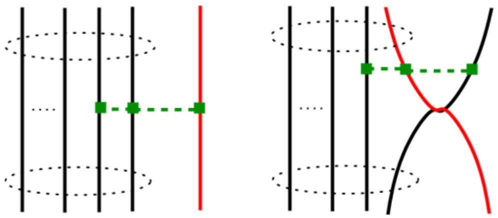
nucleon-nucleus formalism





$$\left\langle \Phi^{J^\pi T}_{\nu' r'} \middle| \hat{A}_{\nu'} V^{NNN} \hat{A}_{\nu} \middle| \Phi^{J^\pi T}_{\nu r} \right\rangle = \left\langle (A-1) \;\; r' \;\; (a'=1) \middle| V^{NNN}\left(1-\sum_{i=1}^{A-1}\hat{P}_{iA}\right) \middle| (a=1) \;\; r \;\; (A-1) \right\rangle$$

$$\mathcal{V}^{NNN}_{\nu'\nu}(r,r') = \sum R_{n'l'}(r')R_{nl}(r)\left[\frac{(A-1)(A-2)}{2}\langle\Phi^{J^\pi T}_{\nu' n'}|V_{A-2A-1A}(1-2P_{A-1A})|\Phi^{J^\pi T}_{\nu n}\rangle\right.$$

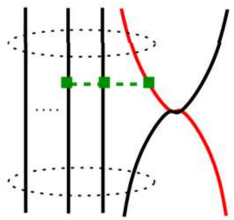


(a) (b)

Direct potential:

$$\propto {}_{SD}\left\langle \psi^{(A-1)}_{\alpha_1'} \middle| a_i^+ a_j^+ a_l a_k \middle| \psi^{(A-1)}_{\alpha_1} \right\rangle_{SD}$$

$$\left. - \frac{(A-1)(A-2)(A-3)}{2}\langle\Phi^{J^\pi T}_{\nu' n'}|P_{A-1A}V_{A-3A-2A-1}|\Phi^{J^\pi T}_{\nu n}\rangle\right] .$$



(c)

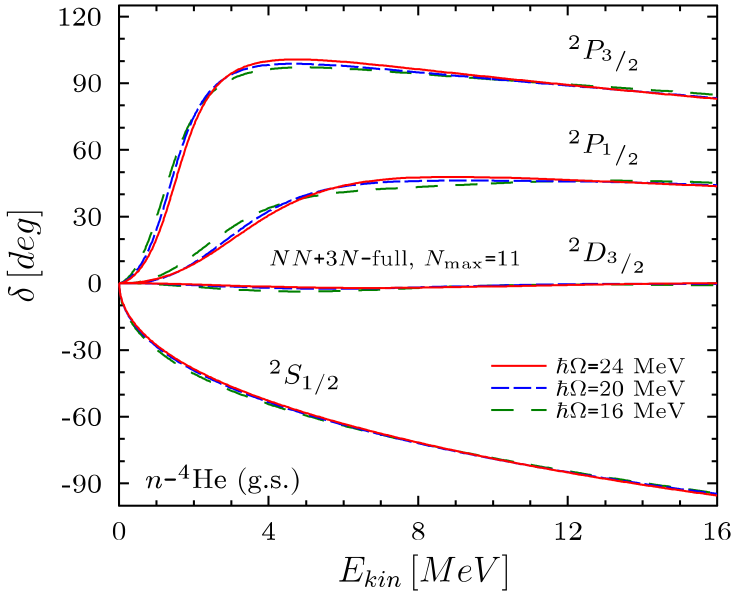
Exchange potential:

$$\propto {}_{SD}\left\langle \psi^{(A-1)}_{\alpha_1'} \middle| a_h^+ a_i^+ a_j^+ a_m a_l a_k \middle| \psi^{(A-1)}_{\alpha_1} \right\rangle_{SD}$$

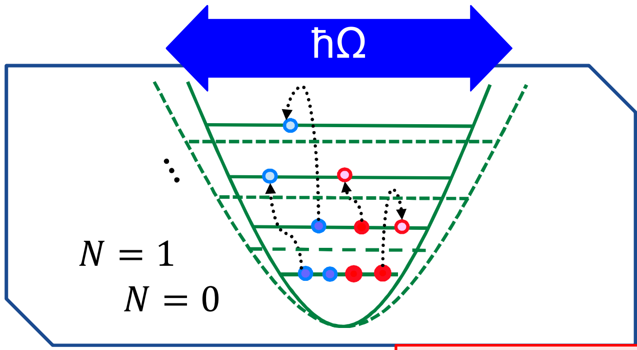
n-^{4}He scattering: study of the model space convergence

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700

Convergence with respect to the Harmonic Oscillator frequency



Convergence of the phase-shifts as a function of the HO frequency.



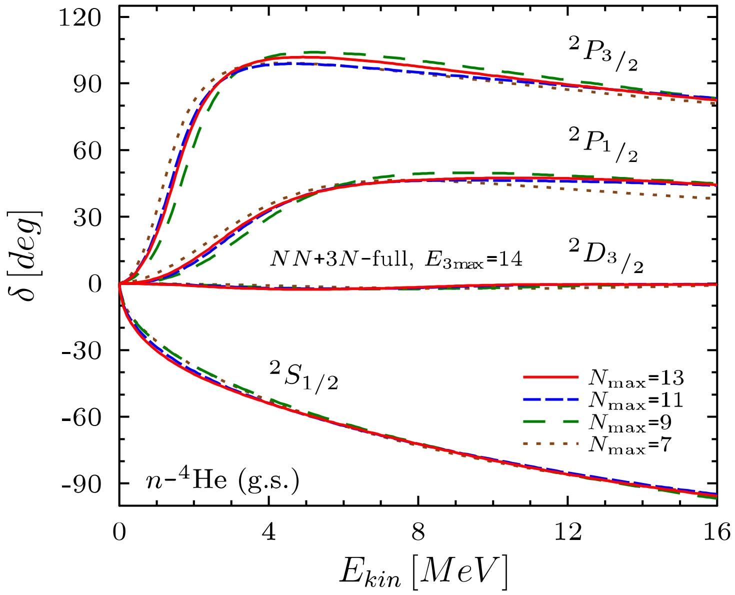
Change the energy resolution of the model-space.

- Calculations are done using the chiral EFT interaction at N^{3}LO for the NN and N^{2}LO for the NNN.
- SRG flow parameter is λ=2.0 fm^{-1}
- Phase-shifts are to a good extent independent from ħΩ.

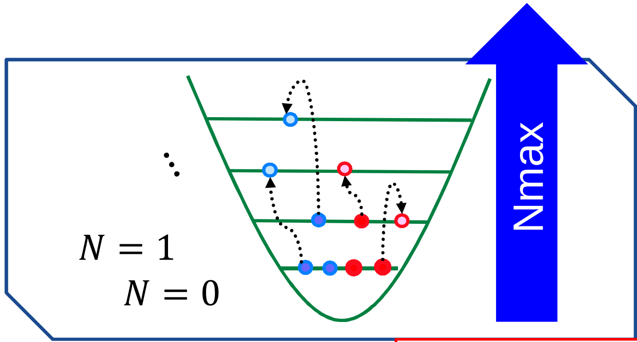
n-^{4}He scattering: study of the HO model-space convergence

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700

Convergence with respect to the number of HO major shells



Convergence of the phase-shifts as a function of the HO basis size.



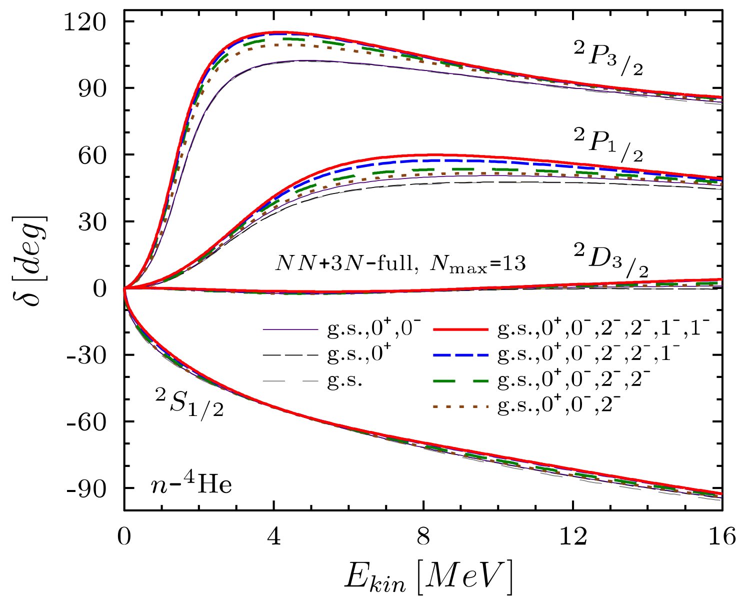
Expanding the size in the HO space.

- **Convergence pattern is good.**
- **In the following, we adopt ħΩ=20 MeV.**

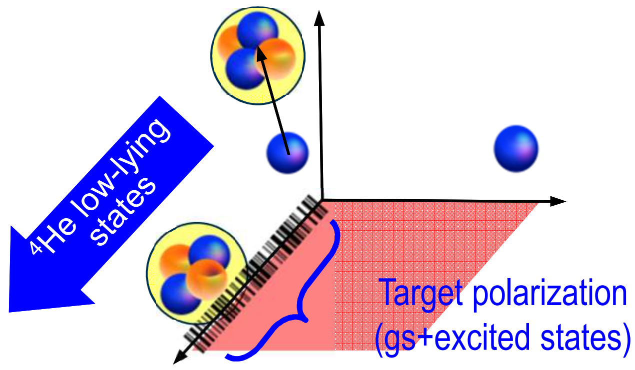
n-^{4}He scattering: study of the RGM convergence in the NNN case

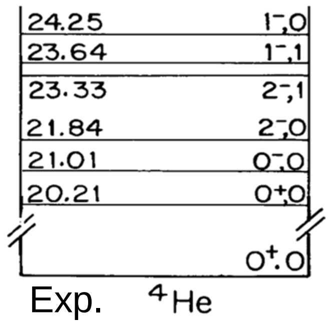
G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700

Convergence with respect to the target polarization



Convergence of the phase-shifts as a function of the ^{4}He excited states.



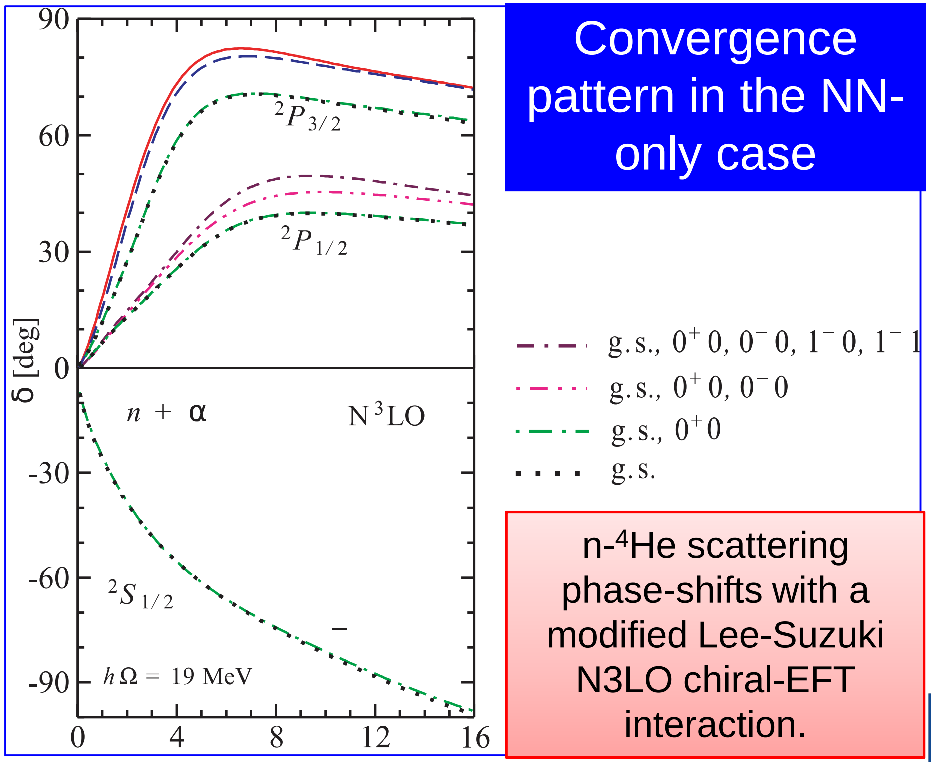


Expanding the size in the RGM space.

- **The convergence of the phase-shifts is reached when the first six excited states of ^{4}He are included.**

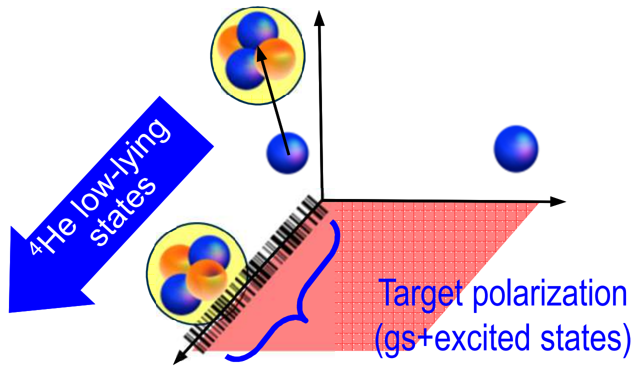
n-^{4}He scattering: study of the RGM convergence in the NN case

P. Navrátil and S. Quaglioni, PRC83 044609 (2011)



Convergence pattern in the NN-only case

n-^{4}He scattering phase-shifts with a modified Lee-Suzuki N3LO chiral-EFT interaction.

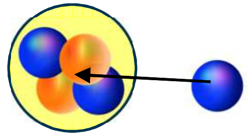


Expanding the size in the RGM space.

- The phase-shifts are sensitive to the inclusion of the first six excited states of ^{4}He.
- The convergence pattern of the NNN-full is similar.

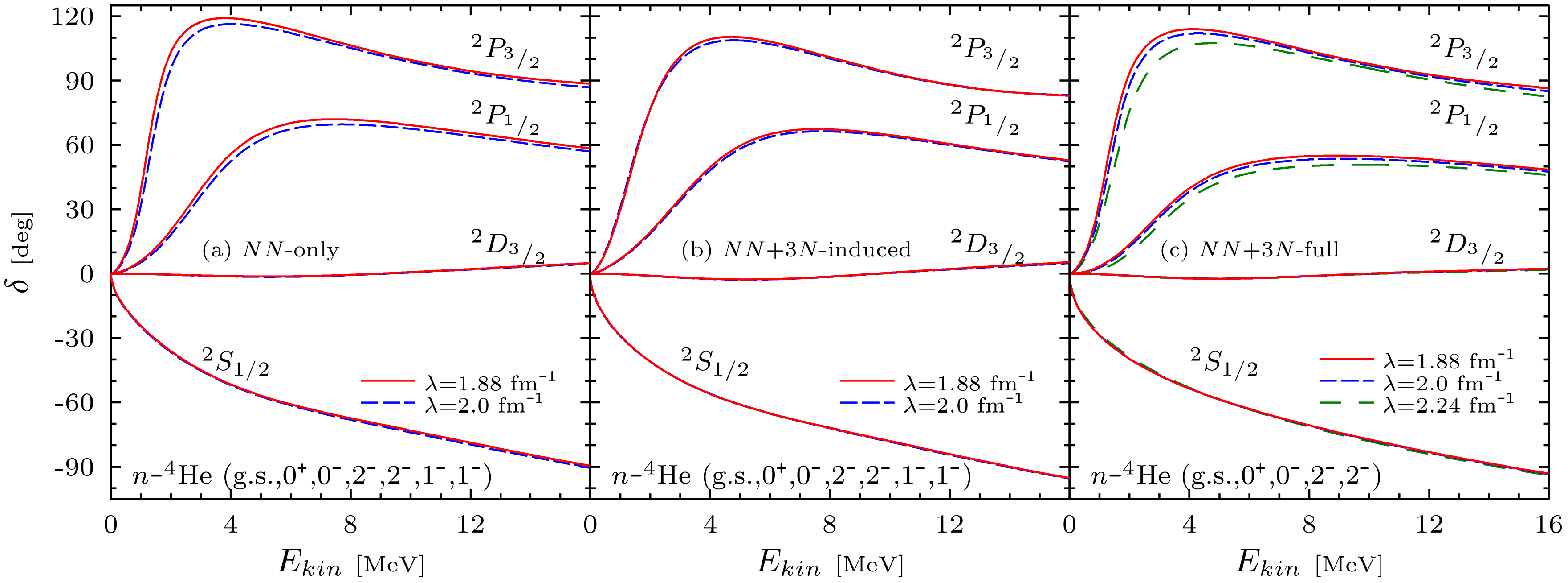
Study of the SRG flow parameter dependence

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700



n-^{4}He scattering

Unitarity behavior of the calculated phase-shifts

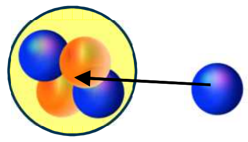


Comparison at Nmax=13 between NN-only, NN+NNN-ind and NN+NNN between λ=2.0 and 1.88 fm^{-1}

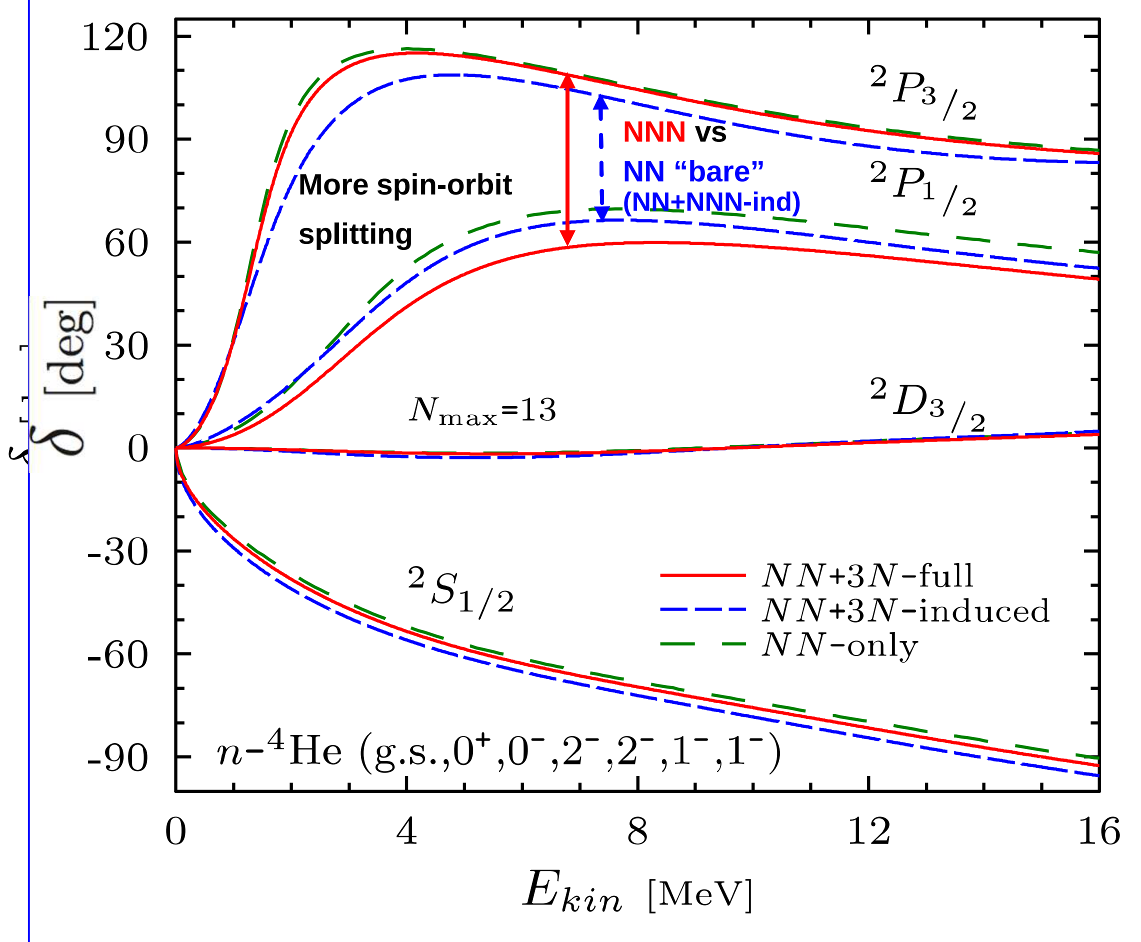
- **The SRG evolution is to a good extend unitary.**

n-^{4}He scattering: NN versus NNN interactions

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700



Three scenarii of nuclear Hamiltonians

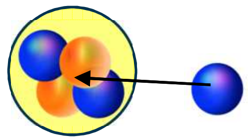


- The NNN interactions affect mostly the P waves.
- The largest spin-orbit splitting between *P* waves is obtained with NN+NNN.
- The agreement of the $P_{3/2}$ phase-shifts between NN-only and NN+NNN forces is accidental.

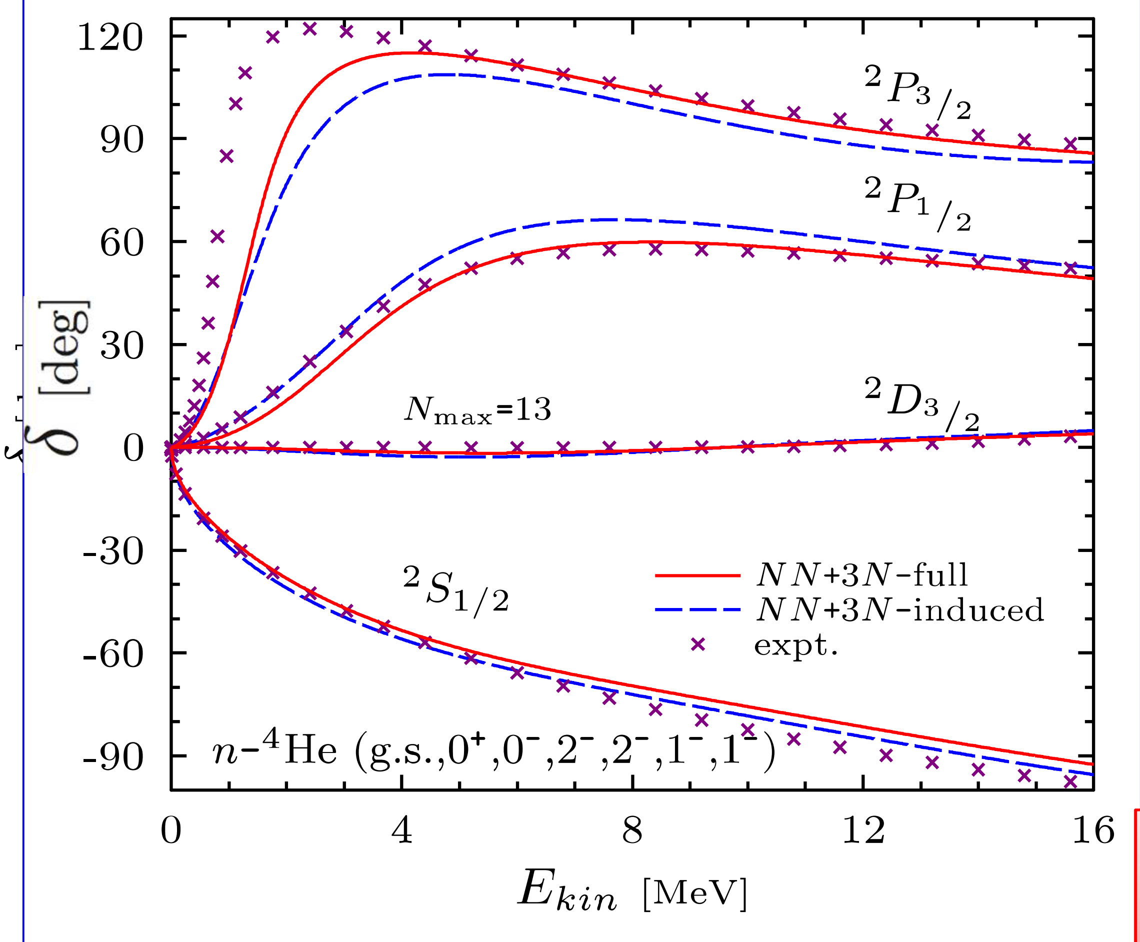
Comparison between NN+NNN -ind and NN+NNN at Nmax=13 with six ^{4}He states.

n-^{4}He scattering: NCSM/RGM versus experiment

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700



Initial NN and NN+NNN compared to experiment

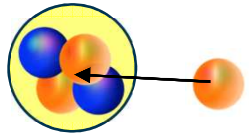


- **The present NNN force is incomplete (N^{2}LO only versus N^{3}LO for NN).**
- **The missing d+^{3}H channel could account for the shift in the P$_{3/2}$ resonance?**

Comparison between NN+NNN –ind, NN+NNN and experiment at Nmax=13 and λ=2.0 fm^{-1}.

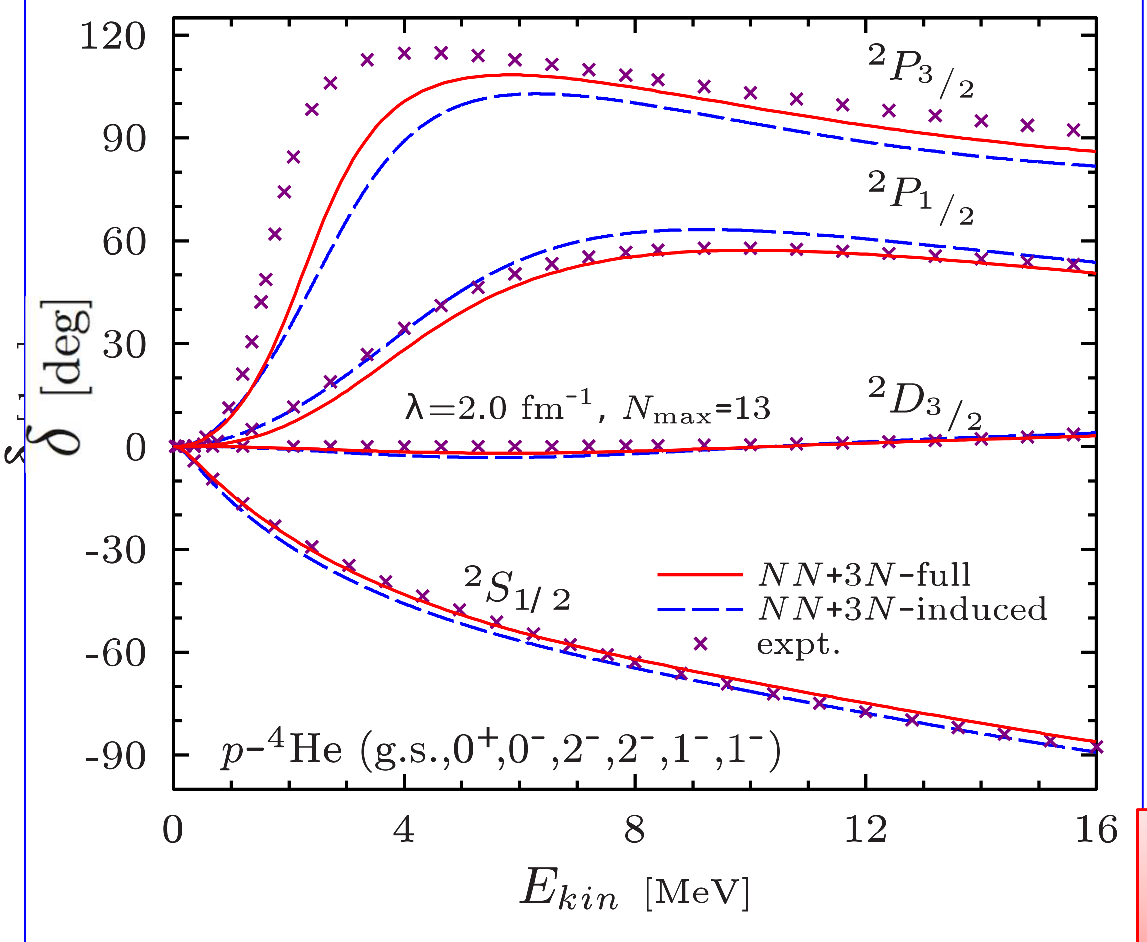
p-^{4}He scattering: NCSM/RGM versus experiment

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700



p-^{4}He scattering

Initial NN and NN+NNN compared to experiment

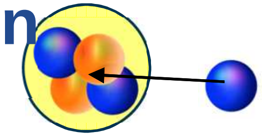


- **The present NNN force is incomplete (N^{2}LO only versus N^{3}LO for NN).**
- **The missing d+^{3}He channel could account for the shift in the $P_{3/2}$ resonance?**

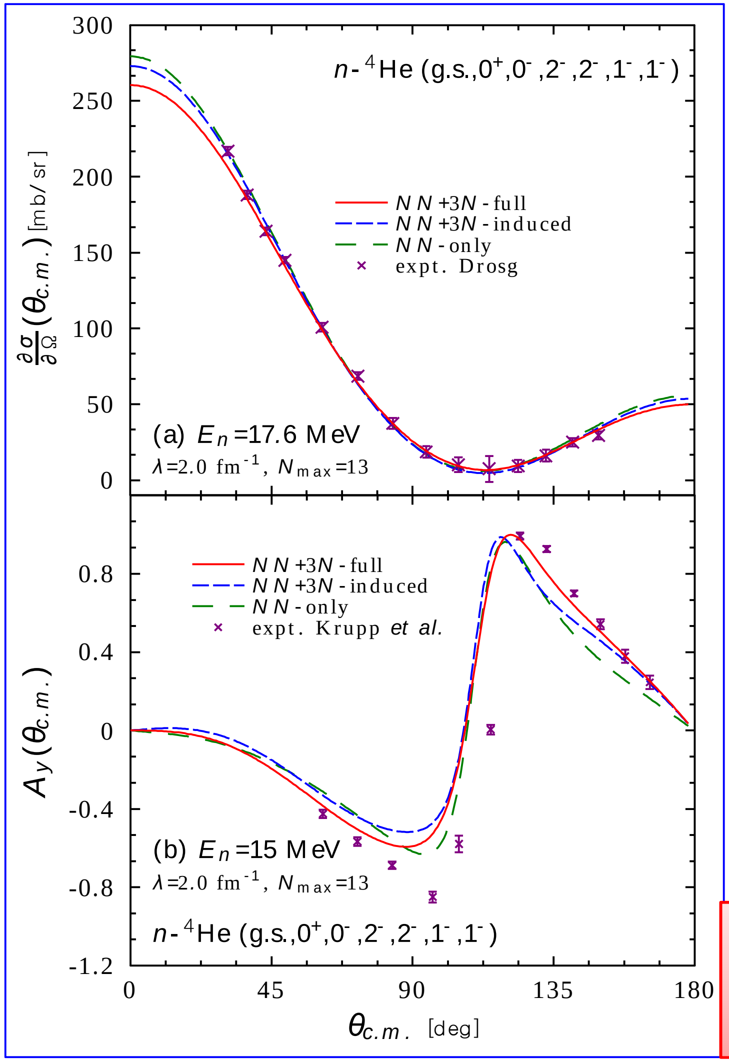
Comparison between NN+NNN –ind, NN+NNN and experiment at Nmax=13 and λ=2.0 fm^{-1}.

Analyzing power and differential cross section

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700



n-^{4}He scattering

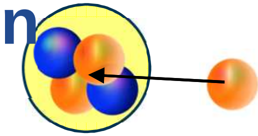


- Good overall reproduction of the experimental data yet it cannot fully explain the A_y puzzle.
- So far we have to restrict the number of partial waves in the calculation due to the model-space truncation related to NNN interactions.

Comparison between NN-only, NN+NNN –ind, NN+NNN and experiment of the differential cross section at E_n=17.6 MeV and analyzing power at E_n=15 MeV.

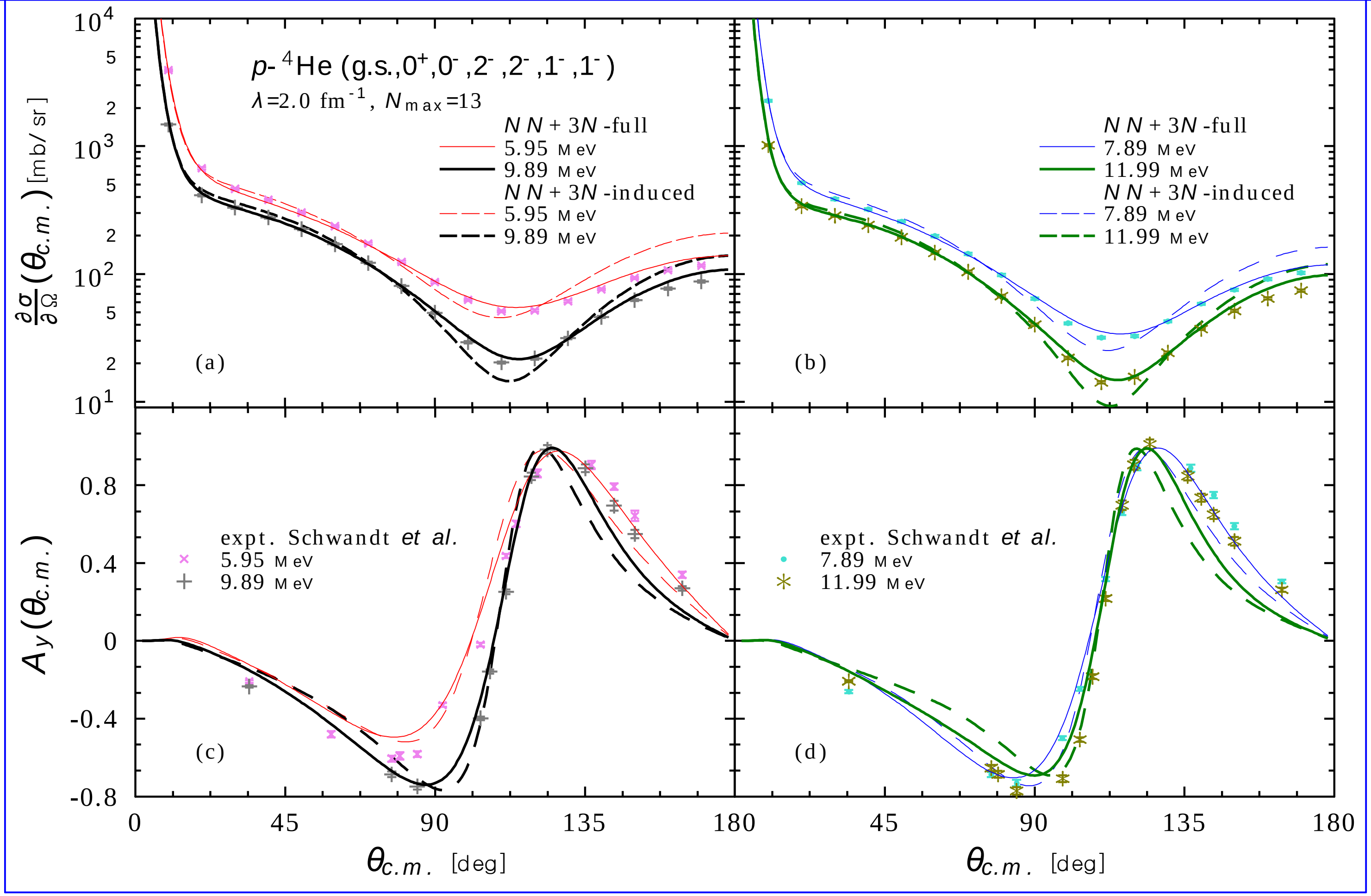
Analyzing power and differential cross section

G. Hupin, J. Langhammer *et al.* ArXiv:1308.2700



p-^{4}He scattering

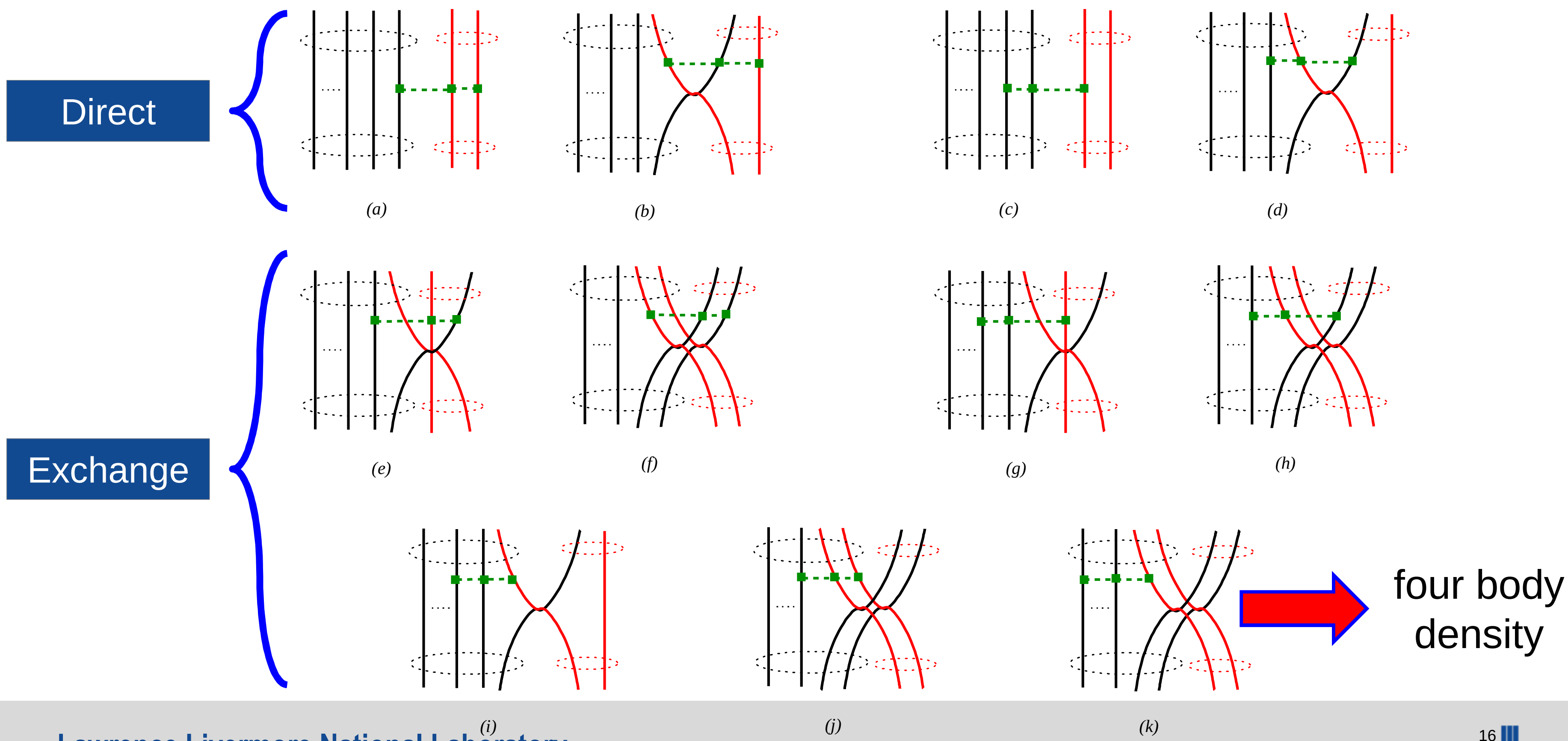
p-^{4}He reaction observables compared to experiment



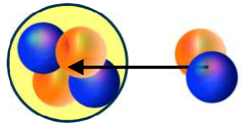
Including the NNN force into the NCSM/RGM approach

deuteron-nucleus formalism

$$\left\langle \Phi^{J^\pi T}_{\nu' r'} \middle| \hat{A}_{\nu'} V^{NNN} \hat{A}_\nu \middle| \Phi^{J^\pi T}_{\nu r} \right\rangle = \left\langle \begin{matrix}(A-2)\\ r' \\ (a'=2)\end{matrix} \right| V^{NNN}\left(1-\sum_{i=1}^{A-2}\sum_{k=A-1}^{A}\hat{P}_{i,k} + \sum_{i<j=1}^{A-2}\hat{P}_{i,A-1}\hat{P}_{j,A}\right)\left| \begin{matrix}(A-2)\\ r \\ (a=2)\end{matrix}\right\rangle$$

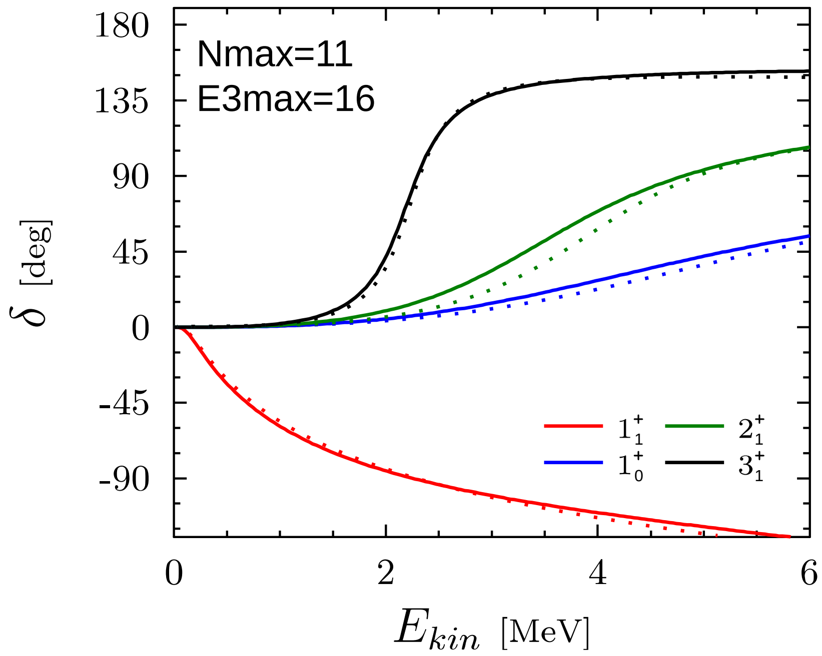


^{4}He(d,d)^{4}He: seven pseudo-states



d-^{4}He scattering

^{4}He(d,d)^{4}He phase shifts: preliminary results with NNN-full

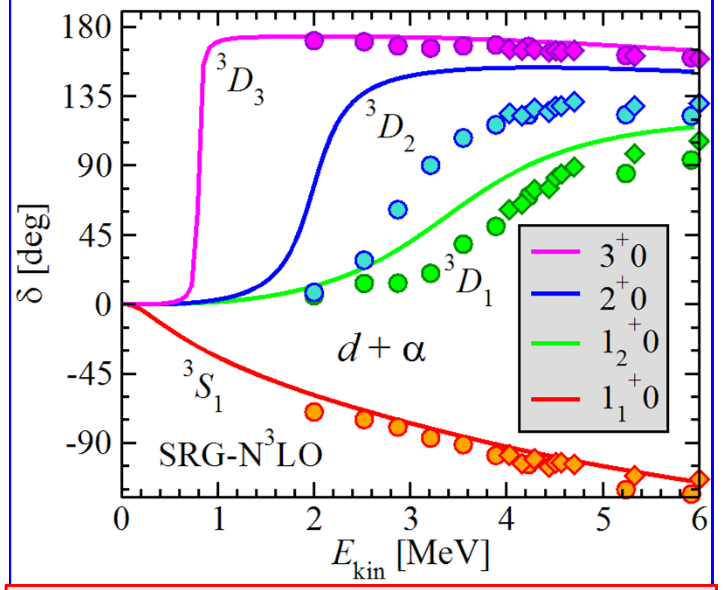


NN+NNN potential with λ=2.0 fm^{-1}, ħΩ=20 MeV.

7
5
3
1

Pseudo-states in each channel

Comparison Expt. R-matrix



N_{max}= 12 d(g.s.,$^3S_1$-$^3D_1$, $^3D_2$, $^3D_3$-$^3G_3$) + ^{4}He(g.s.) SRG-N^{3}LO NN potential with λ=1.5 fm^{-1}, ħΩ=14 MeV.

Recent progress: coupling of NCSM and NCSM/RGM "NCSMC"

S. Baroni, P. Navrátil and S. Quaglioni PRL110 (2013)

- Methods develop in this presentation to solve the many body problem

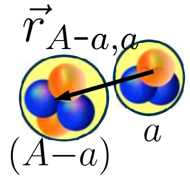
$$\Psi^{(A)}_{NCSM} = \sum_{\lambda} c_{\lambda} \left| A \lambda J^{\pi} T \right\rangle$$

Mixing coefficients(unknown)

A-body harmonic oscillator states

$$\left| A \lambda J^{\pi} T \right\rangle_{SD} \varphi_{00}\left(\vec{R}^{A}_{c.m.}\right)$$

Second quantization



$$\Psi^{(A)}_{RGM} = \sum_{\nu} \int d\vec{r}\, g_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| \Phi^{(A-a,a)}_{\nu\vec{r}} \right\rangle$$

$$\psi^{(A-a)}_{\alpha_1} \psi^{(a)}_{\alpha_2} \delta\left(\vec{r} - \vec{r}_{A-a,a}\right)$$

Relative wave function (unknown)

Antisymmetrizer

Channel basis state

Cluster expansion technique

- The many body <u>quantum</u> problem best describe by superposition of both

$$\Psi^{(A)}_{NCSMC} = \sum_{\lambda} c_{\lambda} \left| A \lambda J^{\pi} T \right\rangle + \sum_{\nu} \int d\vec{r}\, g_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| \Phi^{(A-a,a)}_{\nu\vec{r}} \right\rangle$$

Can address bound and low-lying resonances (short range correlations)

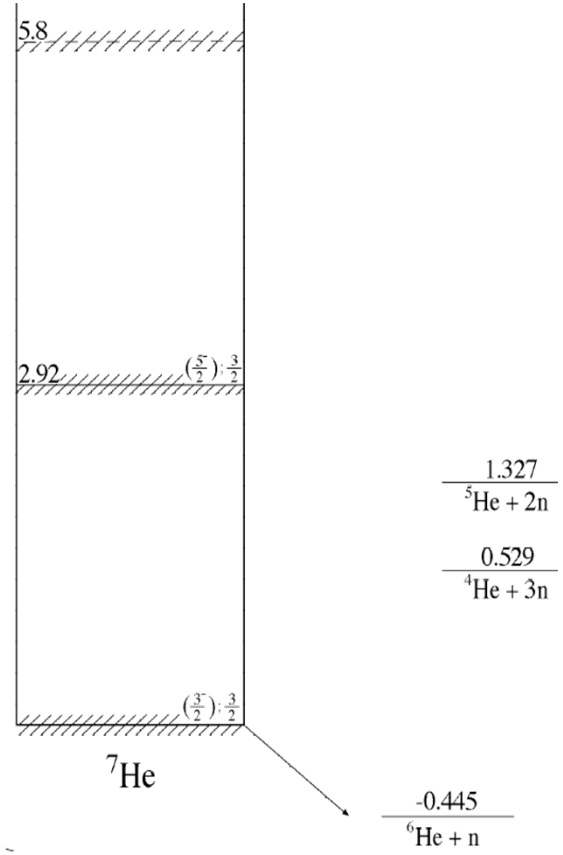
Design to account for the continuum of scattering state (long range correlations)

NCSMC

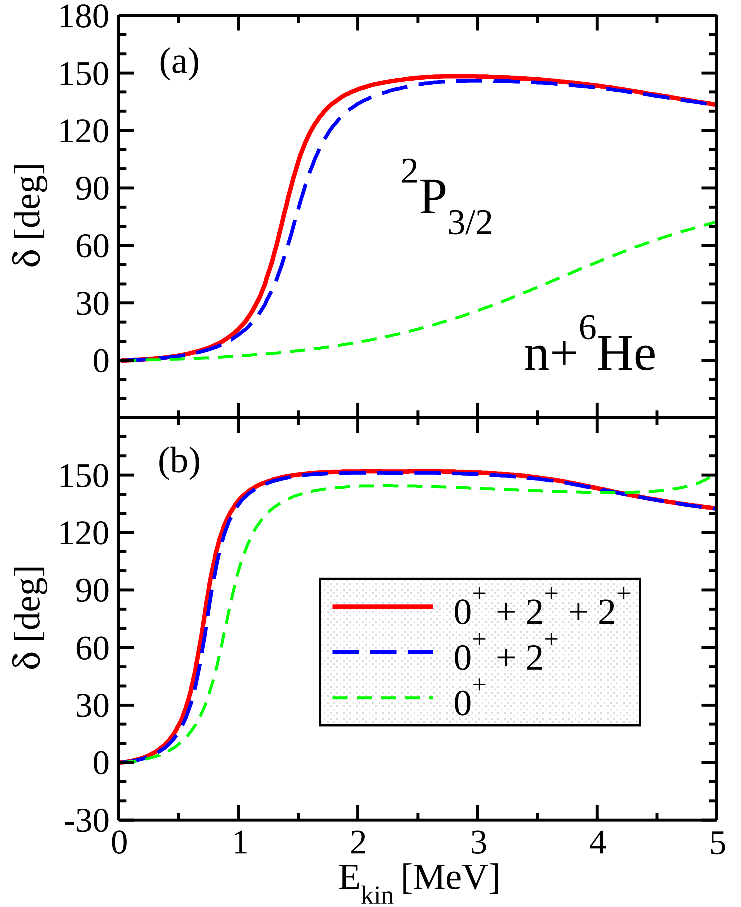
NCSMC speeds up convergence of reaction observables

S. Baroni, P. Navrátil and S. Quaglioni PRC87 (2013)

A=7 and Z=2 system from experiment



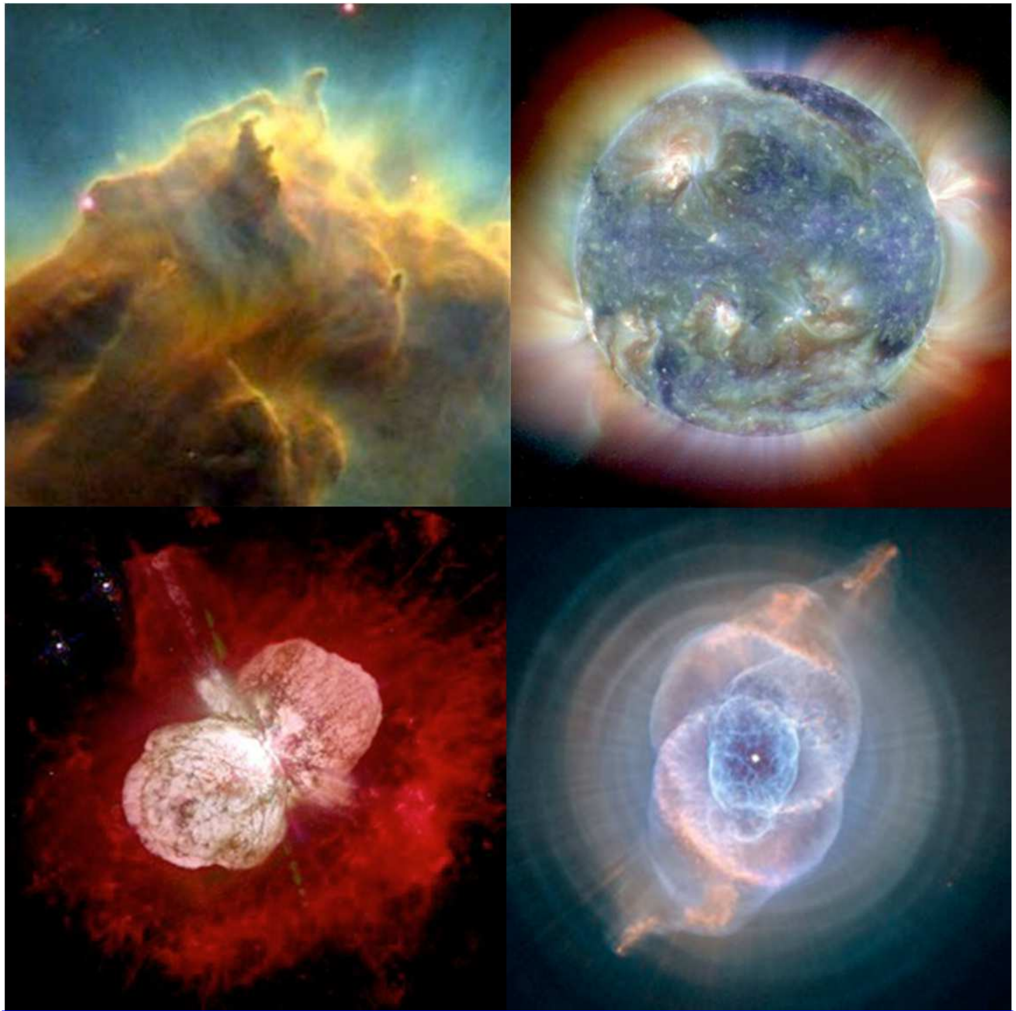
^{6}He(n,n)^{6}He phase shifts, convergence properties



NCSM/RGM
with up to three ^{6}He states.

NCSMC
with up to three ^{6}He states and four ^{7}He eigenstates.
More 7-nucleon correlations and fewer target states needed.

Conclusions and Outlook



Evolution of stars, birth, main sequence, death

- We are extending the *ab initio* NCSM/RGM approach to describe low-energy reactions with two- and three-nucleon interactions.

- We are able to describe:
  - Nucleon-nucleus collisions with NN+NNN interaction
  - Deuterium-nucleus collisions with NN+NNN interaction
  - NCSMC for single- and two-nucleon projectile

- Work in progress
  - Fusion reactions with our best complete *ab initio* approach
  - The present NNN force is "incomplete", need to go to N^3LO
  - Scattering of heavier target