Efimov Physics from Cold Atoms to Nuclei

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Deutsche Forschungsgemeinschaft DFG

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Outline

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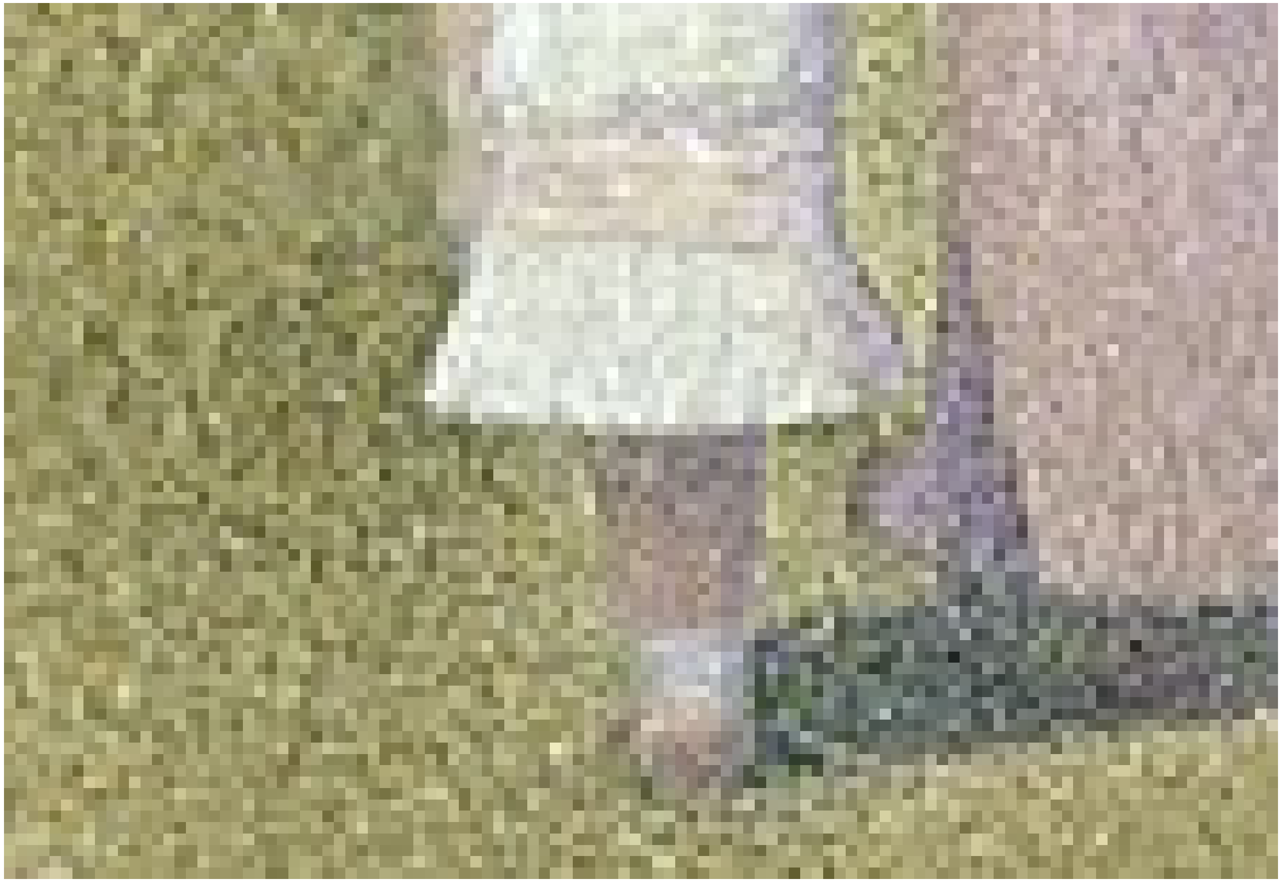
- Introduction
- Efimov physics and the unitary limit
  - Ultracold atoms
  - Halo nuclei
  - Shallow bound states in a box
- Summary and Outlook

$\Longrightarrow$ Parallel session A4: Efimov physics, Tu 17:00 h

Universality and Pointilism

- Painting at the limit of resolution of the human eye



Universality and Pointilism

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G. Seurat, A Sunday on La Grande Jatte

Physics Near the Unitary Limit

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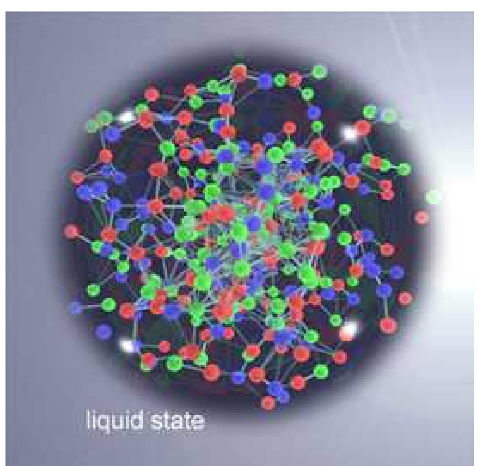
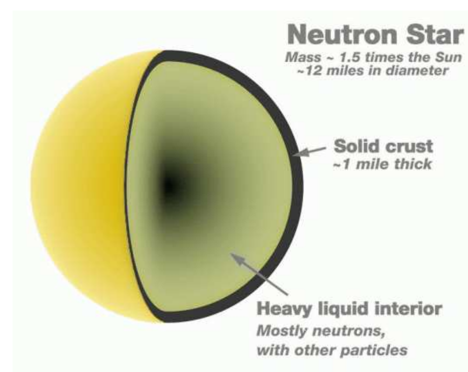
- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \implies i/k$$

  - Scattering amplitude scale invariant, saturates unitarity bound
- Interesting many-body physics: BEC/BCS crossover, universal viscosity bound $\Rightarrow$ perfect liquid, ...





Physics Near the Unitary Limit

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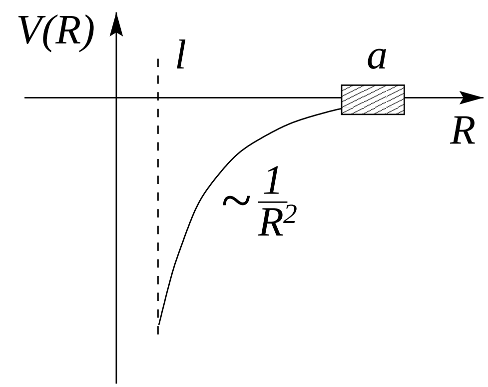
  - Scattering amplitude scale invariant, saturates unitarity bound
- Interesting many-body physics: BEC/BCS crossover, universal viscosity bound $\Rightarrow$ perfect liquid, ...
- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal dimer with energy $E_d = -1/(ma^2)$ ($a > 0$)

    size $\langle r^2 \rangle^{1/2} = a/2 \Rightarrow$ halo state

Broken Scale Invariance

- Three-boson system near the unitary limit (Efimov, 1970)
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

$\Longrightarrow$ scale invariance is anomalous

$\Longrightarrow$ observables depend on boundary condition and a

- Universality concept must be extended $\Rightarrow$ 3-body parameter

Universal EFT

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- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + \ldots$$

- 2-body amplitude:

- 2-body coupling g_2 near fixed point $(1/a = 0)$

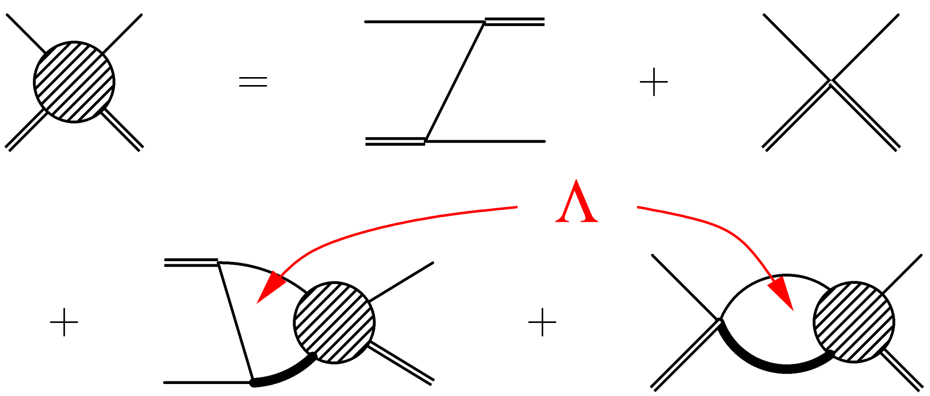
$\Rightarrow$ scale and conformal invariance $\Longleftrightarrow$ unitary limit

(Mehen, Stewart, Wise, 2000; Nishida, Son, 2007; ...)

- 3-body amplitude:

$g_3(\Lambda)$ $\Rightarrow$ limit cycle

$\Rightarrow$ discrete scale inv.



Limit Cycle

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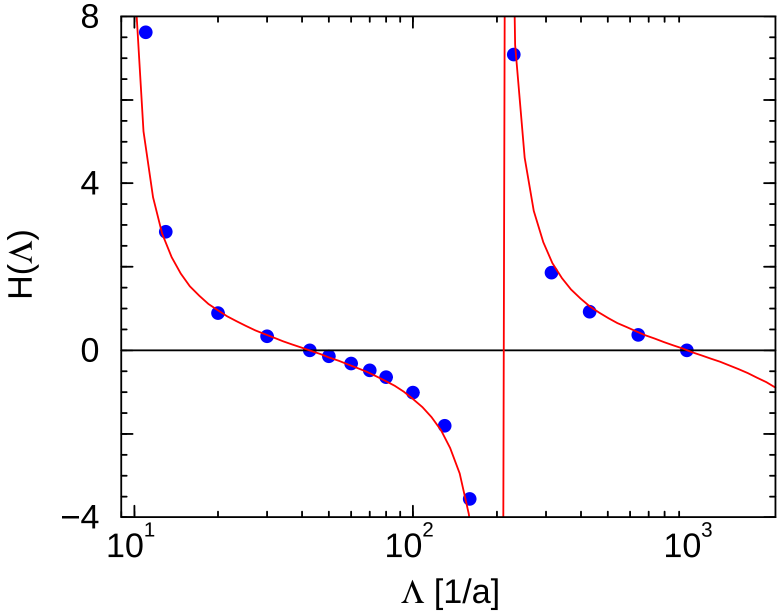
- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3 \Lambda^2/(9g_2^2)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup



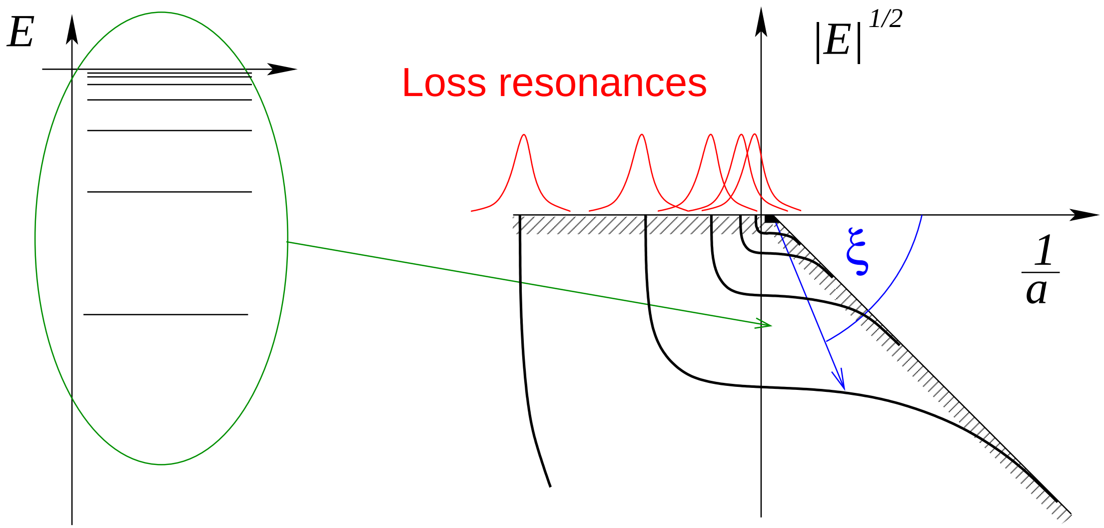
$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Three-body parameter: $\Lambda_*, \ldots$
- Limit cycle $\Longleftrightarrow$ Discrete scale invariance $\Longleftrightarrow$ Efimov physics

Limit Cycle: Efimov Effect

- Universal spectrum of three-body states
  (Efimov, 1970)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a \to 0} \quad \left(e^{\pi/s_0}\right)^2 = 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances

Efimov States in Ultracold Atoms

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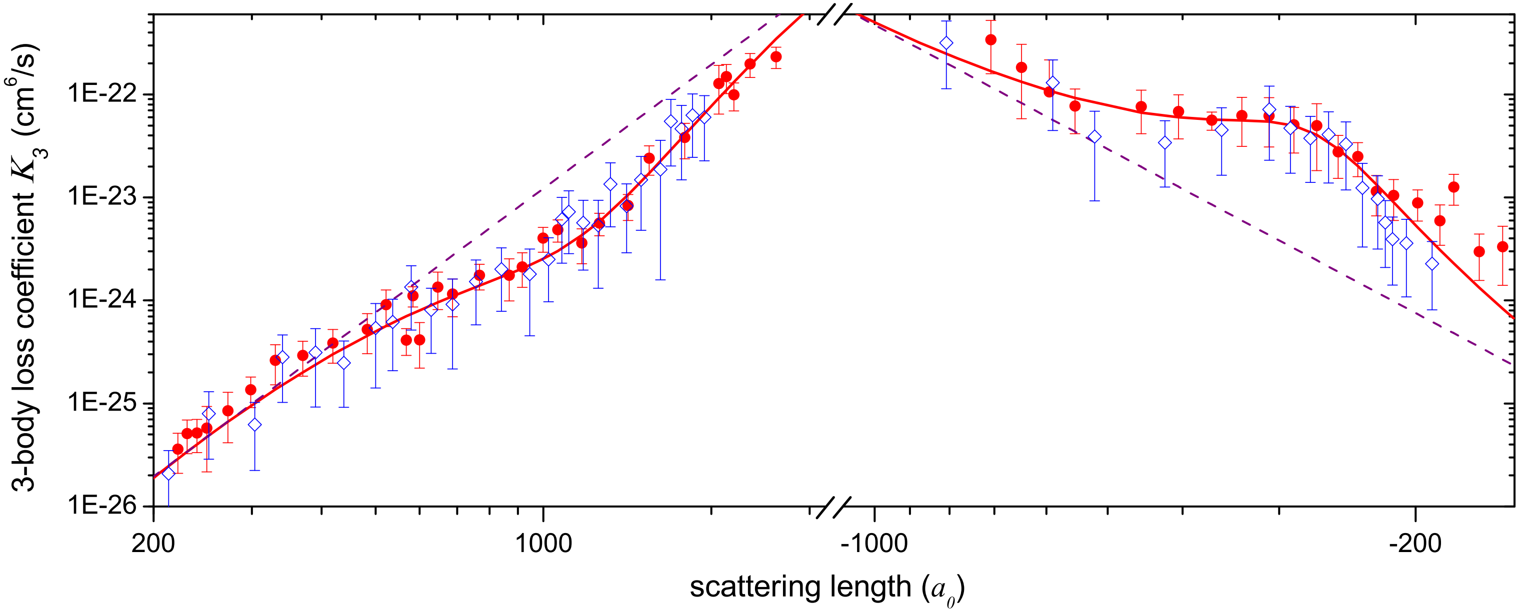
- First experimental evidence in ^{133}Cs (Kraemer et al. (Innsbruck), 2006)
  now also ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb
- Example: Efimov spectrum in ^{7}Li ($|m_F = 0\rangle$,$|m_F = 1\rangle$)
  (Gross et al. (Bar-Ilan Univ.), Phys. Rev. Lett. **105** (2010) 103203)



- Data described by universal theory (Braaten, HWH, 2000, ..., 2006)

Universal Tetramers

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- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
  $\Longrightarrow$ Universality extends to four-body system
- Universal tetramers $(1/a = 0)$

$$B_4^{(0)} = 4.610(1)\ B_3 \qquad \Gamma_4^{(0)}/2 = 0.01483(1)\ B_3$$

$$B_4^{(1)} = 1.00227(1)\ B_3 \qquad \Gamma_4^{(1)}/2 = 2.38(1) \times 10^{-4}\ B_3$$

(Platter, HWH, 2007; von Stecher et al., 2009; Deltuva 2010-2013)

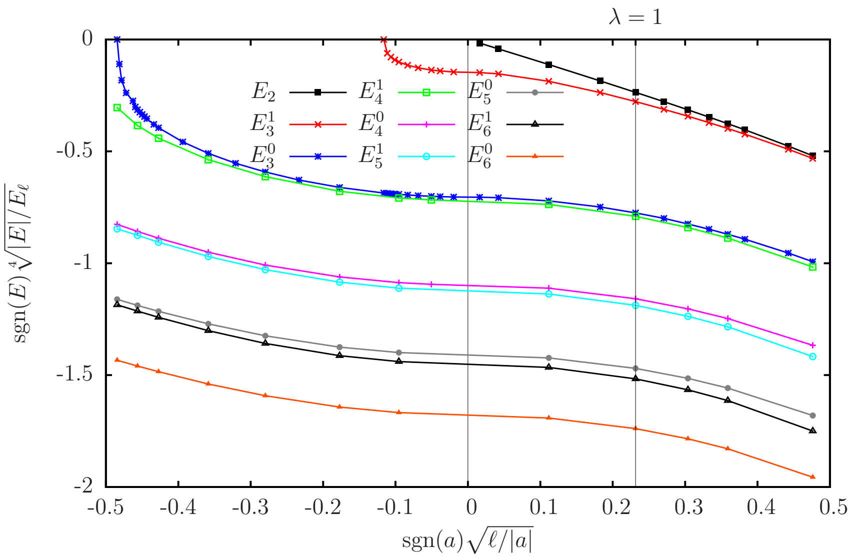
- Two tetramers attached to each trimer
- Some calculations find a very shallow third state: physical?
  (Hadizadeh et al., 2011-12; Avila, Birse, 2013)
- Universal states up to $N = 6$ calculated
  (von Stecher, 2010, 2011; Gattobigio, Kievsky, Viviani, 2011, 2012)
- Observation up to $N = 5$ in Cs losses (Grimm et al. (Innsbruck), 2009, 2012)

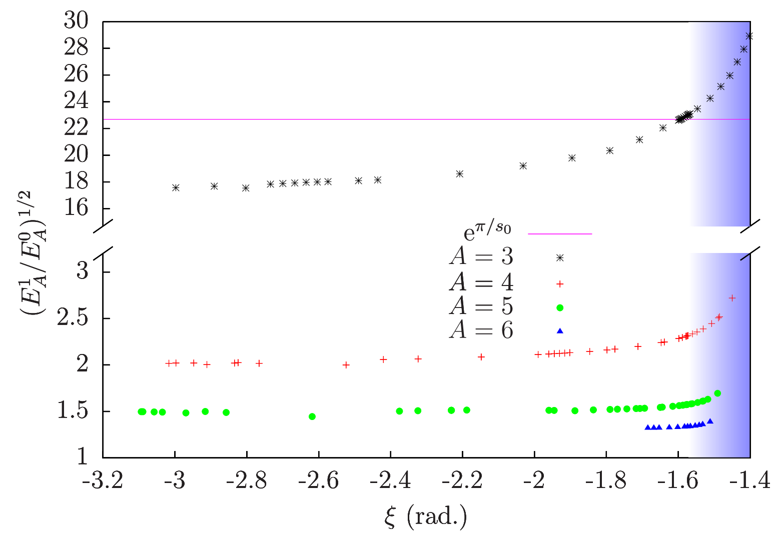
Beyond Tetramers

- Universal states up to $N = 6$ calculated

  (von Stecher, 2010, 2011; Gattobigio, Kievsky, Viviani, 2011, 2012)

- Two $(N+1)$-body states attached to each N-body state
- Hyperspherical harmonics calculation with soft Gaussian interaction



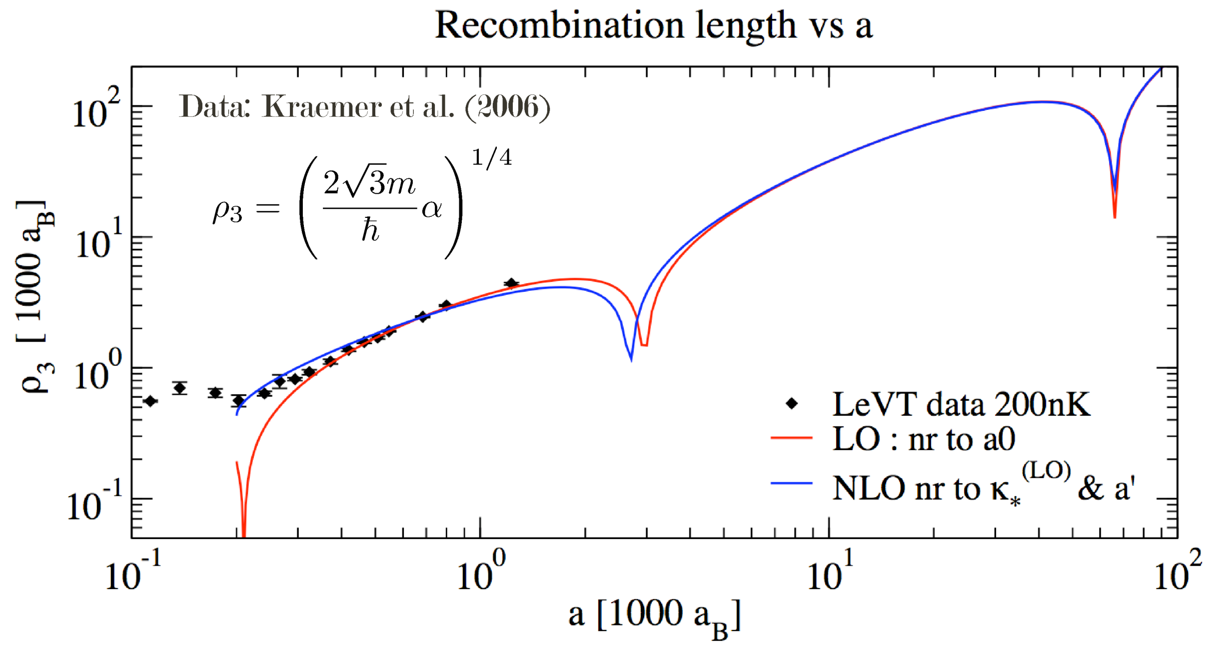


(Gattobigio, Kievsky, Viviani, Phys. Rev. A **86** (2012) 042513)

- Test of discrete scale invariance for $N \geq 3$

Beyond Universality

- Finite range corrections shift recombination features



(Ji, Phillips, Platter, private communication)

- Perturbative analysis in r_e: $\quad H_1 = H_1(a)$

$$t_1 \sim r\Lambda + \frac{r}{a}\ln\Lambda \qquad \Longrightarrow \qquad H_1 = H_{1,0} + \frac{r}{a}H_{1,1}$$

(Ji, Phillips, Platter, 2009, 2012)

Beyond Universality

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- Application to experiment: ^{7}Li (Khaykovich group, 2010, 2012)

  Minima and maxima at: $a_0 = 1160\,a_B$; $a_- = -264\,a_B$

  - NLO prediction using B-dependent r: $a_* = (210 \pm 44)\,a_B$

    (Ji, Phillips, Platter, Ann. Phys. **327** (2012) 1803)
  - Experiment: $a_* = (196 \pm 4)\,a_B$

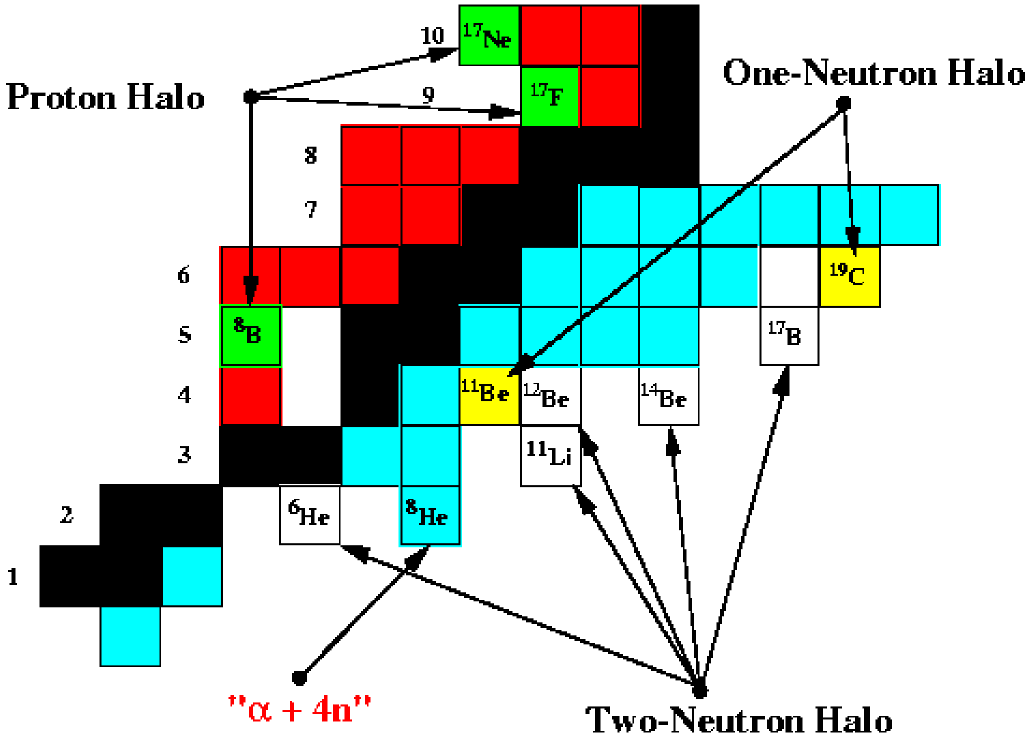
    (Machtey et al., Phys. Rev. Lett. **108** (2012) 210406)
- Momentum-dependent three-body parameter enters at N2LO

  (Bedaque, Rupak, Grießhammer, HWH, 2003; Ji, Phillips, 2013)
- Effective range from four-body recombination peak positions

  (Hadizadeh, Yamashita, Tomio, Delfino, Frederico, Phys. Rev. A **87** (2013) 013620)
- Finite range and finite temperature effects in three-body recombination (Sørensen, Fedorov, Jensen, Zinner, J. Phys. B **46** (2013) 075301, arXiv:1307.2854)

Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$
  $\longrightarrow$ close to "nucleon drip line" $\longrightarrow$ scale separation $\longrightarrow$ EFT



http://www.nupecc.org

- EFT for halo nuclei

  (Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

Scales and Antisymmetrization

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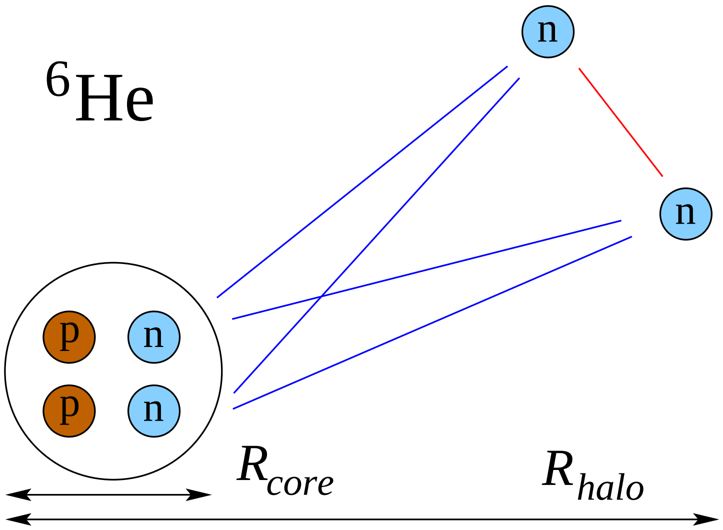
- Scales: $R_{halo} \gg R_{core} \sim \ell$
- Antisymmetrization with respect to neutrons in core?
- Core neutrons not active dof in halo EFT



- Physics: exchange of core nucleon and halo nucleon only contributes to observables if there is spatial overlap between wave functions of core and halo nucleon

$$\Longrightarrow \quad \text{small for } R_{core} \ll R_{halo}$$

- Effects subsumed in low-energy constants, included perturbatively in expansion in R_{core}/R_{halo}

Electromagnetic Structure of ^{11}Be

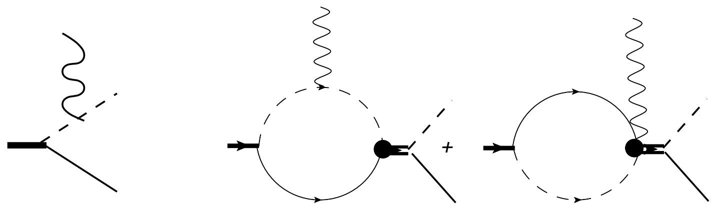
- Properties of ^{11}Be
  - Ground state: $J^P = 1/2^+$, neutron separation energy: 504 keV
  - Excited state: $J^P = 1/2^-$, neutron separation energy: 184 keV
- Properties of ^{10}Be
  - Ground state: $J^P = 0^+$
  - First excitation: 3.4 MeV above g.s.
- Separation of scales: $E_{lo}/E_{hi} \approx \frac{0.5}{3.5} = \frac{1}{7} \quad \Rightarrow \quad R_{core}/R_{halo} \approx 0.4$

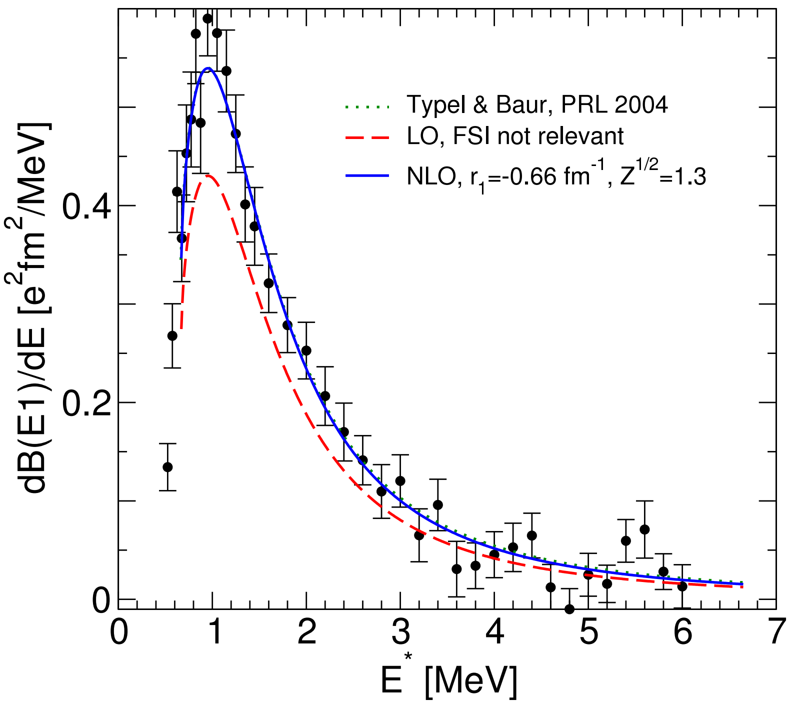
$\Rightarrow$ one neutron halo picture for ^{11}Be appropriate

- Effective range theory (Typel, Baur, 2004, 2005, 2008)
- EFT $\Longrightarrow$ straightforward coupling to external currents
- Study EM properties in halo EFT picture (HWH, Phillips, NPA **865** (2011) 17)

Coulomb Dissociation

- Transition to the continuum:





- Reasonable convergence
- At LO: impulse approximation
- At NLO: FSI from excited state

$$r_1 = -0.66 \text{ fm}^{-1} \quad [B(E1)]$$

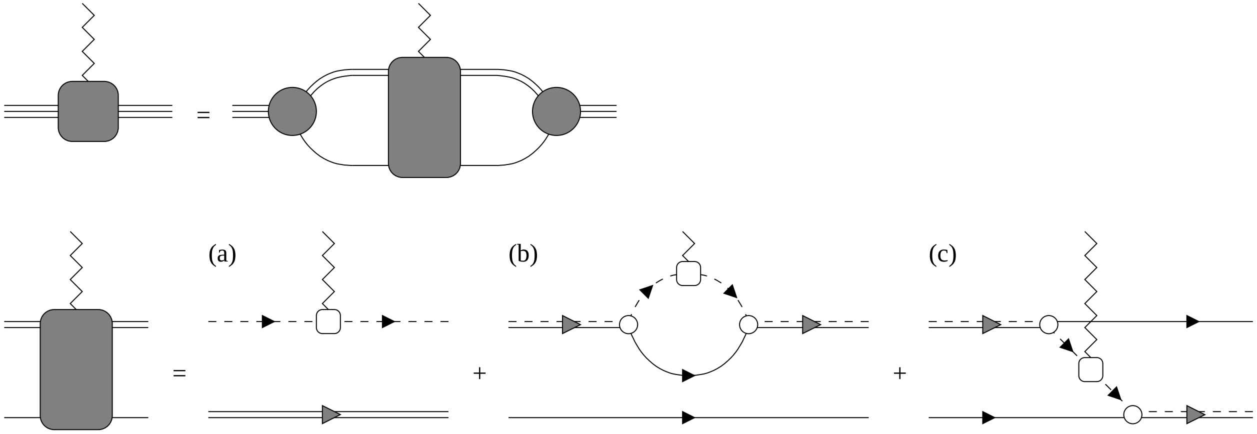
$$\sqrt{Z_\sigma} = 1.3 \quad \Rightarrow \quad r_0 = 2.7 \text{ fm}$$

- Detector resolution folded in

Data: Palit et al., PRC **68** (2003) 034318

More Halo EFT

- Radiative neutron capture on ^{7}Li
  (Rupak, Higa, Phys. Rev. Lett. **106** (2011) 222501)
- Charge form factor of $2n$ halo nuclei
  (Hagen, HWH, Platter, arXiv:1304:6516, to appear in EPJA)



- Proton halo nuclei: S-factor for ^{16}O$(p,\gamma)^{17}$F*
  (Ryberg, Forssén, HWH, Platter, arXiv:1308.5975)
- Coulomb dissociation of ^{19}C
  (Acharya, Phillips, Nucl. Phys. A **913** (2013) 103)
- ....

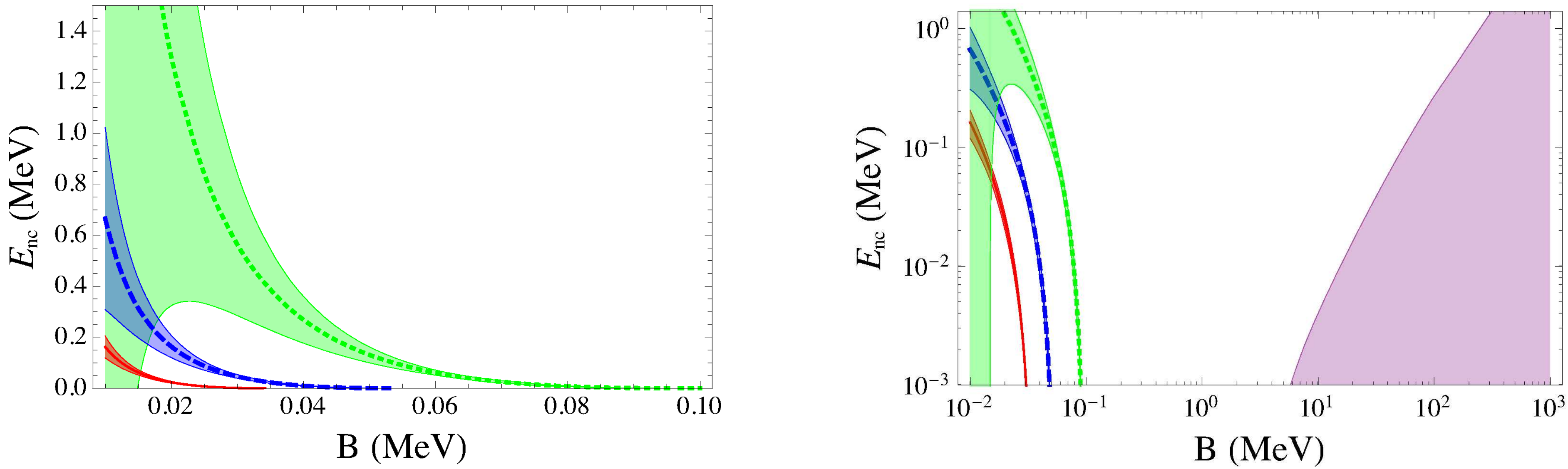
Efimov Physics in ^{22}C

- Matter radius from ^{22}C + p & Glauber: $\langle r_0^2 \rangle^{1/2} = 5.4(9)$ fm

  (Tanaka et al., Phys. Rev. Lett. **104** (2010) 062701)

- Halo EFT analysis of impact on other observables in ^{22}C

  (Acharya, Ji, Phillips, Phys. Lett. B **723** (2013) 196)



Plots for $\langle r_0^2 \rangle^{1/2} = 4.5,\ 5.4,\ 6.3$ fm

- Excited Efimov states in ^{22}C appear to be ruled out

Efimov Physics in ^{62}Ca

(G. Hagen, P. Hagen, HWH, Platter, arXiv:1306.3661, to appear in PRL)

- The Many and the Few: emergence of effective halo degrees of freedom
- Coupled cluster calculations of ^{60}Ca and ^{61}Ca using chiral N2LO two-body force and schematic three-body force:

  ^{61}Ca is a weakly bound S-wave state (or virtual state)

- Quantitative estimate: $S_n = B_{nc} = 5...8$ keV
- Scattering Parameters:

  $a_{cn} = 54(1)$ fm, $r_{cn} = 9.0(2)$ fm $\quad\Longrightarrow\quad r_{cn}/a_{cn} \approx 1/6$

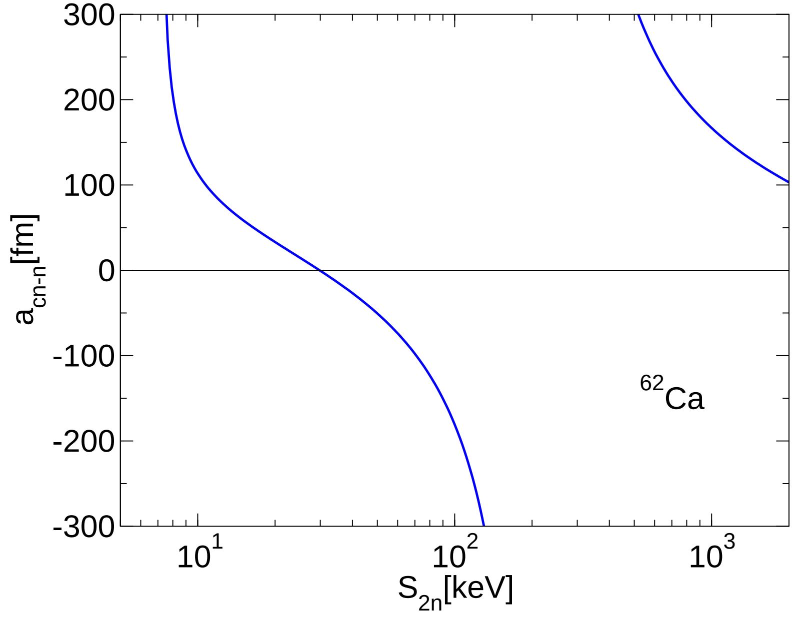
- Investigate consequences for ^{62}Ca using halo EFT
- Prospects for excited Efimov states in ^{62}Ca:

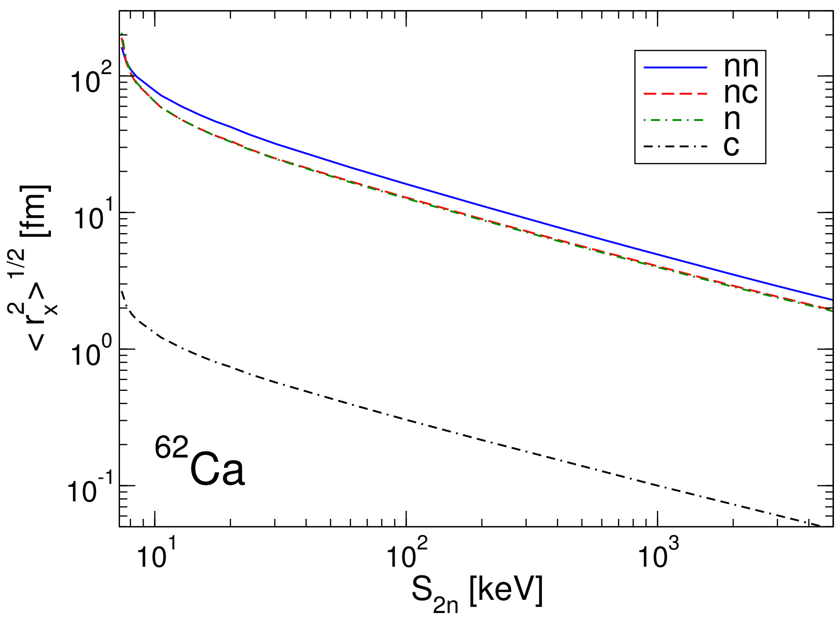
  $S_{\rm deep} = 1/(\mu_{cn} r_{cn}^2) \approx 500$ keV, scaling factor $\lambda_0 \approx 16$

  $\Longrightarrow$ possible if $S_{2n} \gtrsim 230$ keV

Efimov Physics in ^{62}Ca

- Universal correlations between S_{2n}, ^{61}Ca-n scattering length, ^{62}Ca matter, and charge radii





(G. Hagen, P. Hagen, HWH, Platter, arXiv:1306.3661, to appear in PRL)

- Excited Efimov state appears around $S_{2n} \approx 230$ keV
- Matter radii of order tens of Fermi possible

Volume Dependence of Cluster States

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- Finite volume dependence required for ab initio lattice calculations (cubic box $\sim L^3$) (cf. Epelbaum et al., PRL **106** (2011) 192501)
  - L-dependence of 2-body halo states behaves as 2-body system
- Mass shift from overlap of copies from periodic boundary cond.

$$\Delta E_B = \sum_{|\mathbf{n}|=1} \int d^3\mathbf{r}\ \psi_B^*(\mathbf{r}) V(\mathbf{r}) \psi_B(\mathbf{r} + \mathbf{n}L) + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

- S-wave bound states (Lüscher, 1986)

$$\Delta E_B = E_B(\infty) - E_B(L) = -3|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

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- Generalization to higher ℓ, e.g. P-waves (König, Lee, HWH, PRL **107** (2011) 112001)

$$\Delta E_B^{(1,0)} = \Delta E_B^{(1,\pm 1)} = +3|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

Volume Dependence of Cluster States

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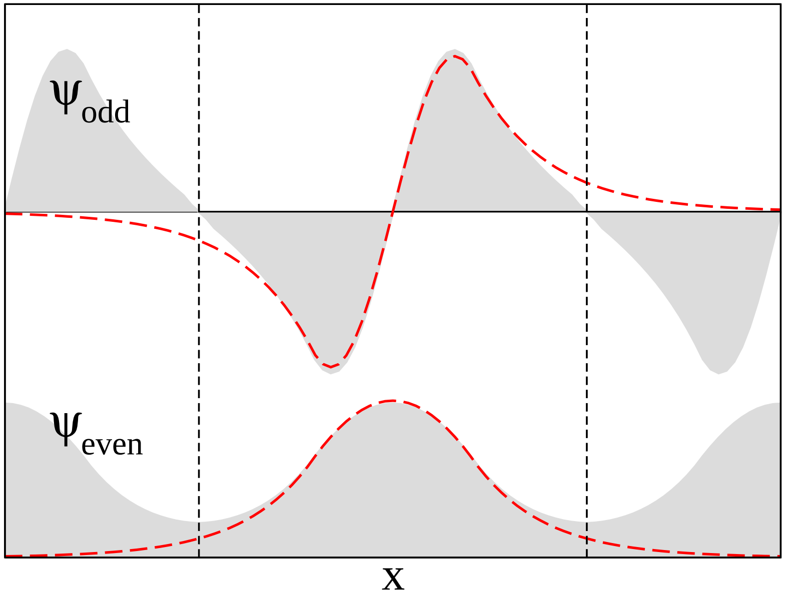
- Different sign for even and odd partial waves

$\Rightarrow$ can be understood from curvature of wave function at the boundary



- m-dependence for D- and higher waves, but

$$\sum_{m=-\ell}^{\ell} \Delta E_B^{(\ell,m)} = (-1)^{\ell+1}(2\ell+1)\cdot 3|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}\left(e^{-\sqrt{2}\kappa L}\right)$$

Topological Phase Corrections

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- Topological phase for scattering of bound states
- Factorize center-of-mass motion

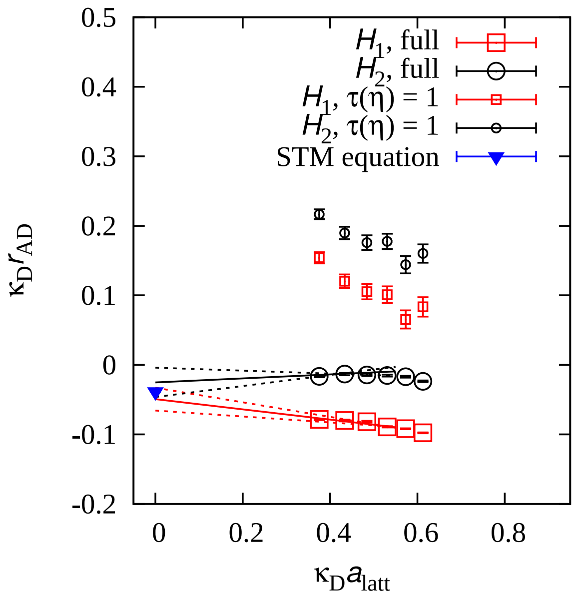
$$\psi_L\left(\vec{r}_1, \vec{r}_2\right) = e^{i2\pi\alpha\vec{k}\cdot\vec{r}_1/L} e^{i2\pi(1-\alpha)\vec{k}\cdot\vec{r}_2/L} \phi_L(\vec{r}_1 - \vec{r}_2)\,, \quad \alpha = m_1/(m_1+m_2)$$

- Periodicity of ψ_L induces topological phase for winding of ϕ_L around cube

$$\phi_L(\vec{r} + \vec{n}L) = e^{-i2\pi\alpha\vec{k}\cdot\vec{n}} \phi_L(\vec{r})$$

- Energy correction:

$$\frac{\Delta E_{\vec{k}}(L)}{\Delta E_{\vec{0}}(L)} = \frac{1}{3} \sum_{l=1,2,3} \cos\left(2\pi\alpha k_l\right)$$



Bour, König, Lee, HWH, Meißner,

Phys. Rev. D **84** (2012) 091503

Topological Phase Corrections

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- Fermion-dimer scattering (spin-1/2 fermions)
  - Lattice calculation:

$$\kappa a_{fd} = 1.174(9), \quad \kappa r_{fd} = -0.029(13)$$

  - Continuum EFT: (cf. Simenog et al. (1984))

$$\kappa a_{fd} = 1.17907(1), \quad \kappa r_{fd} = -0.0383(3)$$

Bour, HWH, Lee, Meißner, Phys. Rev. C **86** (2012) 034003

- Small, negative effective range
- Result also applies to quartet neutron-deuteron scattering
- Future extension: 4-body scattering calculations, ...

Summary and Outlook

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- Efimov physics $\Leftrightarrow$ universal aspects of DSI
  - Effective field theory for large scattering length
  - ....
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Few-body nuclei: halo nuclei, ...
  - Hadronic molecules: $X(3872)$, ...

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- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Few-body nuclei: halo nuclei, ...
  - Hadronic molecules: $X(3872)$, ...
- Future directions: exploring the unitary limit/DSI at different scales
  - Finite volume effects in lattice simulations
  - Hadronic molecules: b-quark sector, three-body molecules?, ...
  - Halo nuclei: drip line systems, external currents, reactions, ...
  - Cold atoms: dipolar interactions, confined geometries, heteronuclear systems, 2d-systems, P-waves, ...

Additional Slides

Variable Scattering Length

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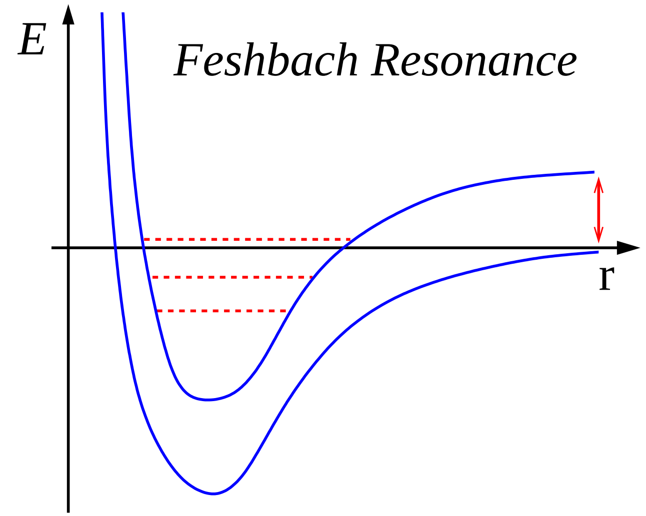
- Feshbach Resonance:

  energy of molecular state in closed channel close to energy of scattering state



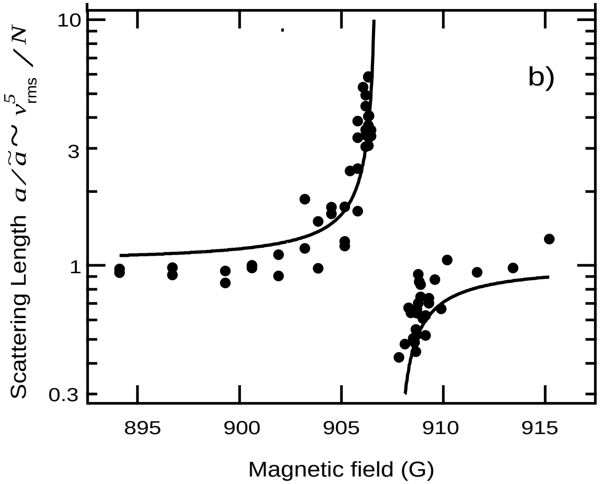
- Tune scattering length via external magnetic field

  (Tiesinga, Verhaar, Stoof, 1993)

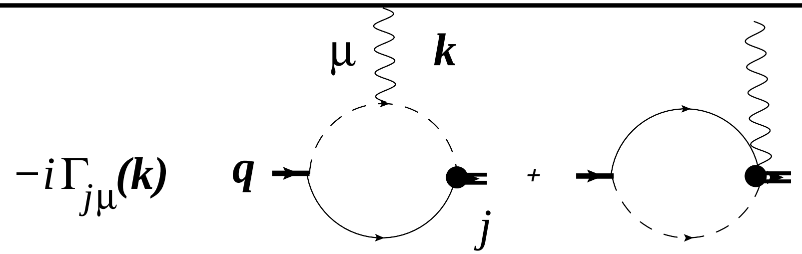
- Observation in a Na BEC

  (Inouye et al. (MIT), 1998)

$$\frac{a(B)}{a_0} = 1 + \frac{\Delta}{B_0 - B}$$



S-to-P Transition



- Irreducible transition vertex

$$\Gamma_{ji} = \delta_{ji}\Gamma_E + (k_j q_i + \cancel{q_j k_i})\Gamma_M \qquad \text{for} \qquad \mathbf{k}\cdot\mathbf{q} = 0, \quad \mathbf{k}\cdot\epsilon = 0$$

- Current conservation: $k_\mu \Gamma_{j\mu} = 0 \quad \Longrightarrow \quad \omega\Gamma_{j0} = k_j \Gamma_E$
- $B(E1)$ transition strength:

$$B(E1) = \frac{1}{4\pi}\left(\frac{\Gamma_E}{\omega}\right)^2 = \frac{Z_{eff}^2 e^2}{3\pi}\,\frac{\gamma_0}{-r_1}\left[\frac{2\gamma_1 + \gamma_0}{(\gamma_0+\gamma_1)^2}\right]^2 + \ldots$$

- No cutoff required: divergences cancel!
- Experiment: $B(E1) = 0.105 \ldots 0.116\ e^2$ fm^2

  (Summers et al., PLB **650** (2007) 124; Millener et al., PRC **28** (1983) 497)
- Strategy: determine $r_1 = -0.66$ fm^{-1} at LO

Universal Correlations

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- EFT gives correlations between different observables
- Example: $B(E1)$ for S-to-P transition and radius of P-wave state

$$B(E1) = \frac{2e^2 Q_c^2}{15\pi} \left(\langle r_c^2 \rangle_{^{11}\mathrm{Be}^*} - \langle r_c^2 \rangle_{^{10}\mathrm{Be}} \right) x \left[\frac{1+2x}{(1+x)^2} \right]^2 + \ldots \,,$$

where $x = \sqrt{B_1/B_0}$

- Adapt strategy to experimental situation
- P-wave radius relative to ^{10}Be core from $B(E1)$

$$\langle r_c^2 \rangle_{^{11}\mathrm{Be}^*} - \langle r_c^2 \rangle_{^{10}\mathrm{Be}} = 0.35 ... 0.39 \text{ fm}^2$$

Universality: can be applied to any one-neutron halo nucleus with shallow S- and/or P-Wave State