Finite-range corrections and Universality in Efimov physic

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Outline

Efimov Physics

- Efimov Effect
- Discrete Scale Invariance

Finite-range Effect

- 3-Body Bound States
- Scattering Length
- Recombination

N-body Universality

- N-Body States
- Universality

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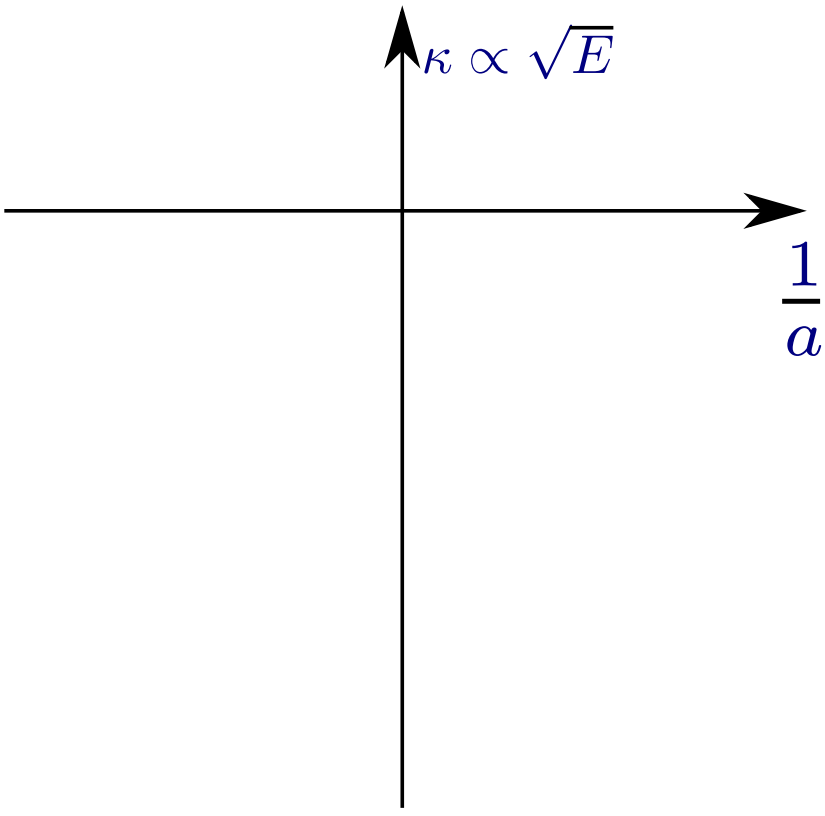
Efimov Effect



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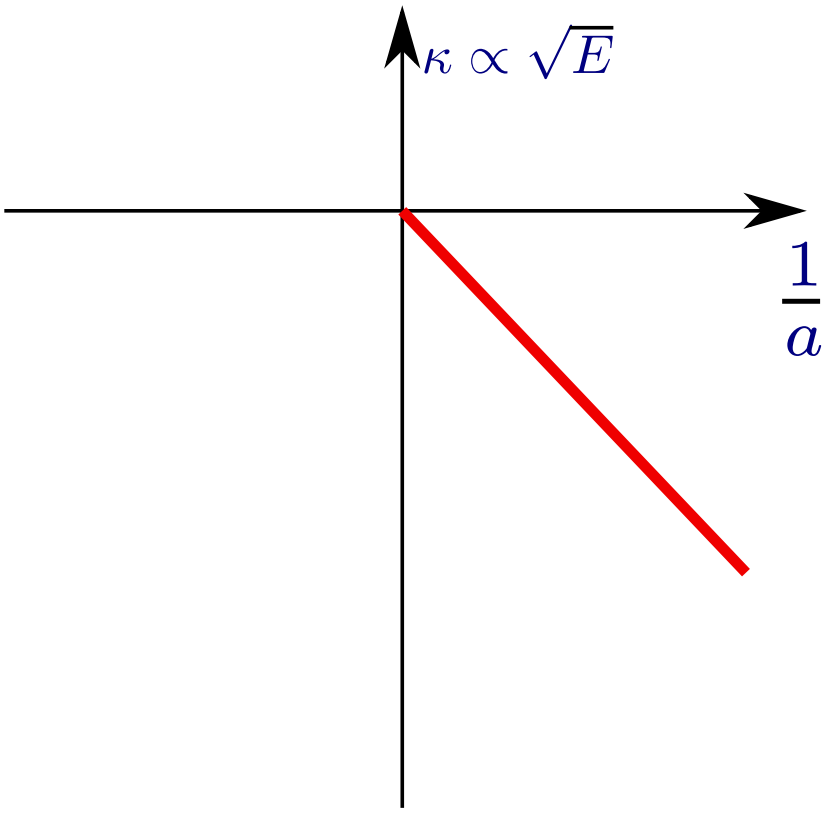

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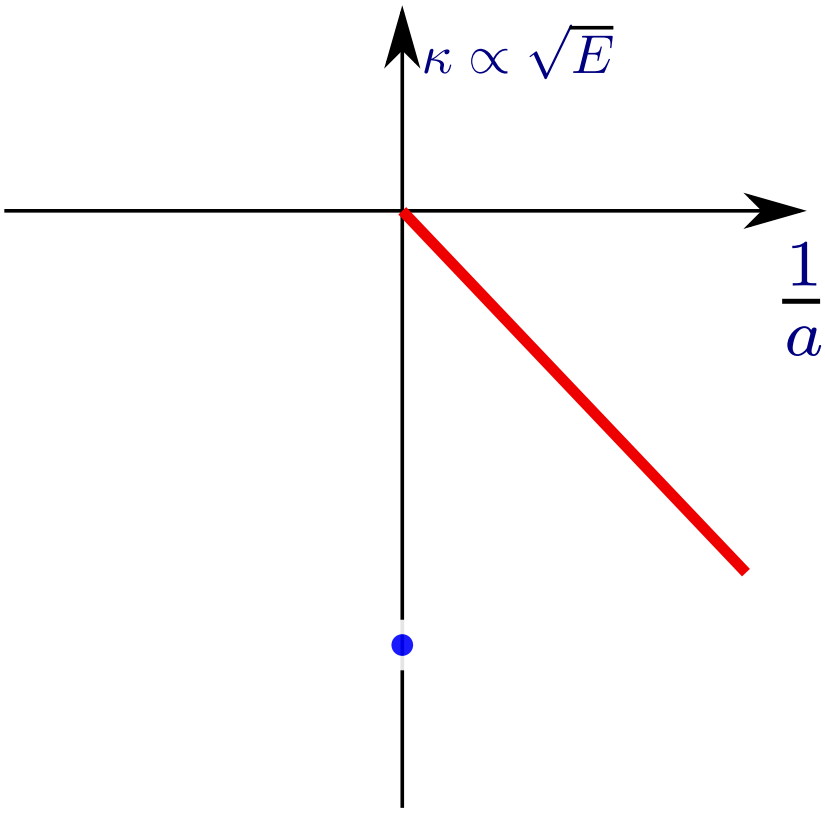

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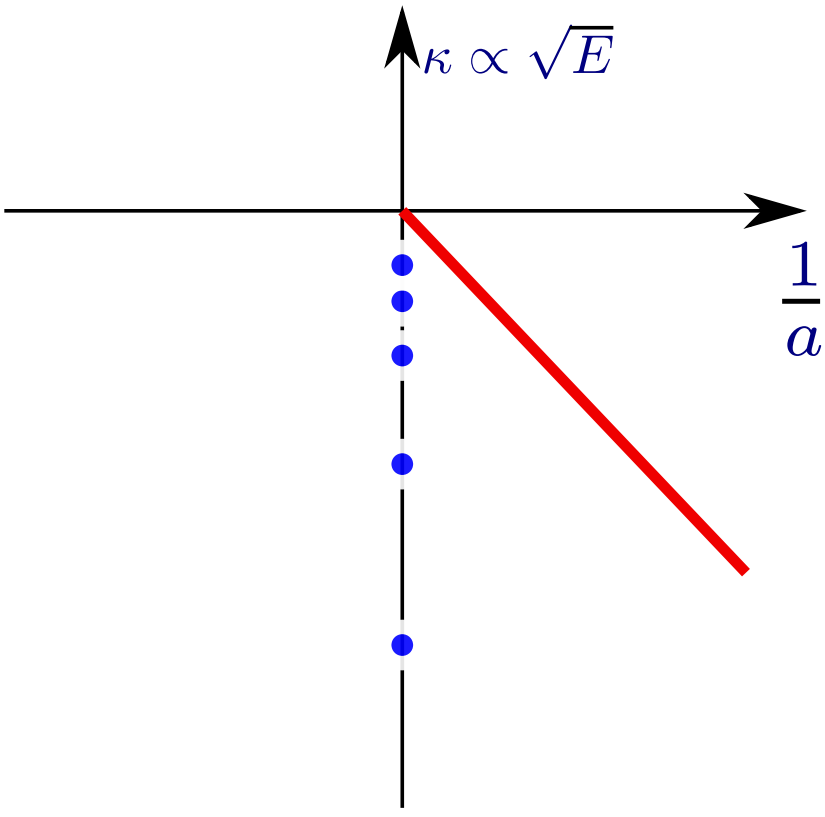

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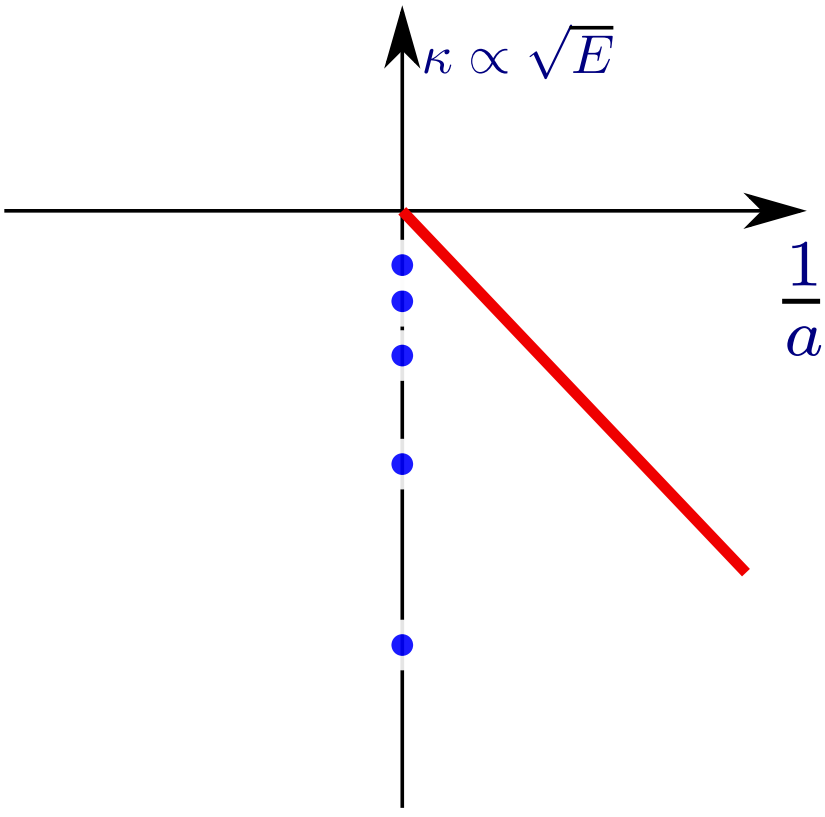

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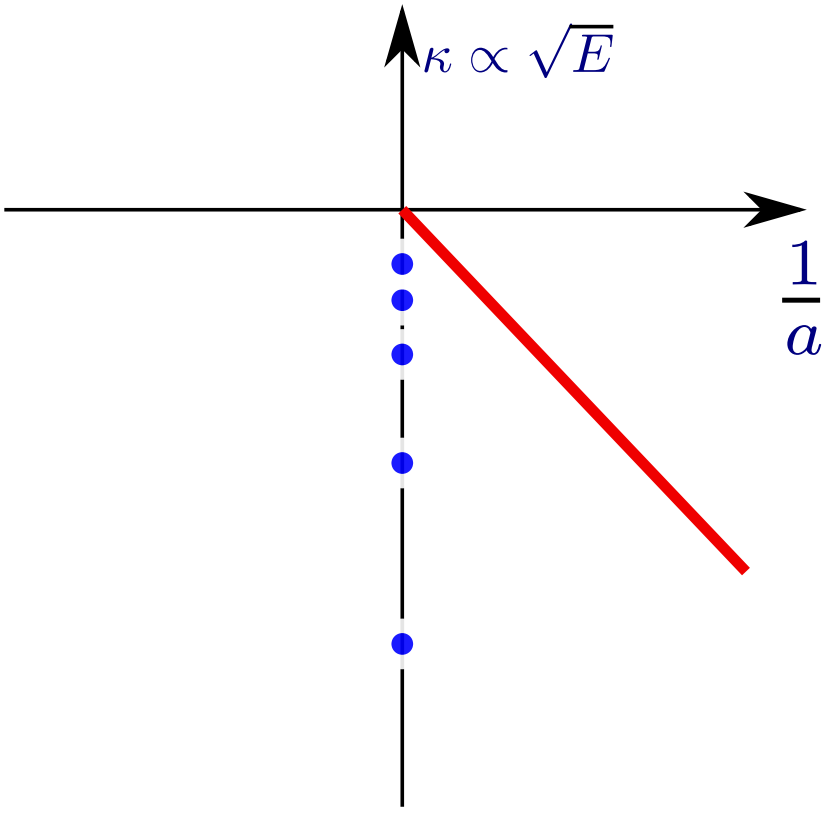

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Discrete Scale Invariance

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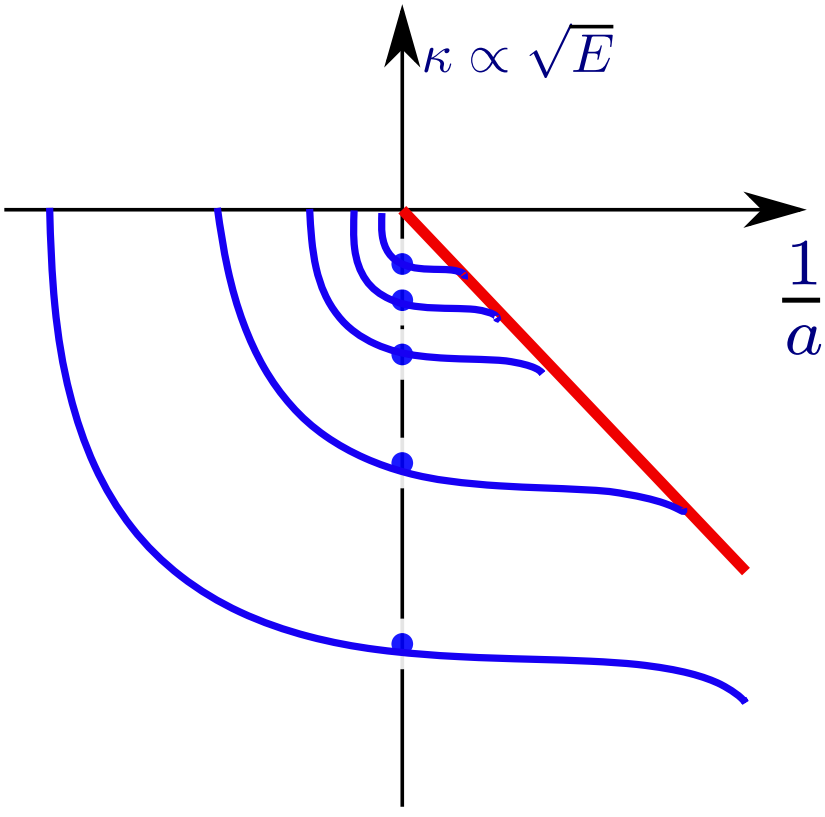

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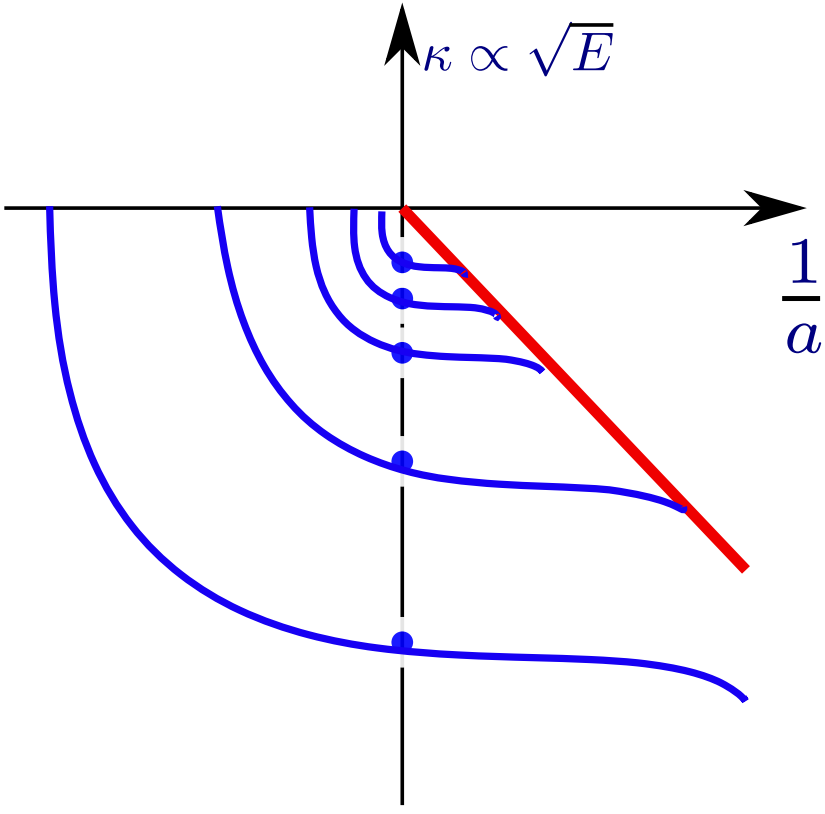

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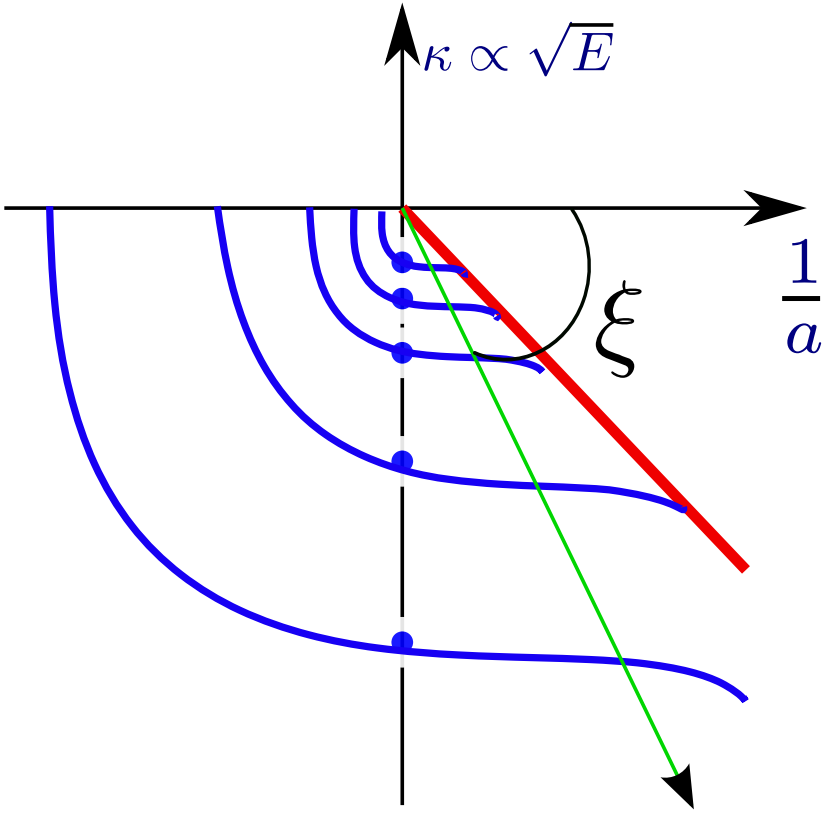

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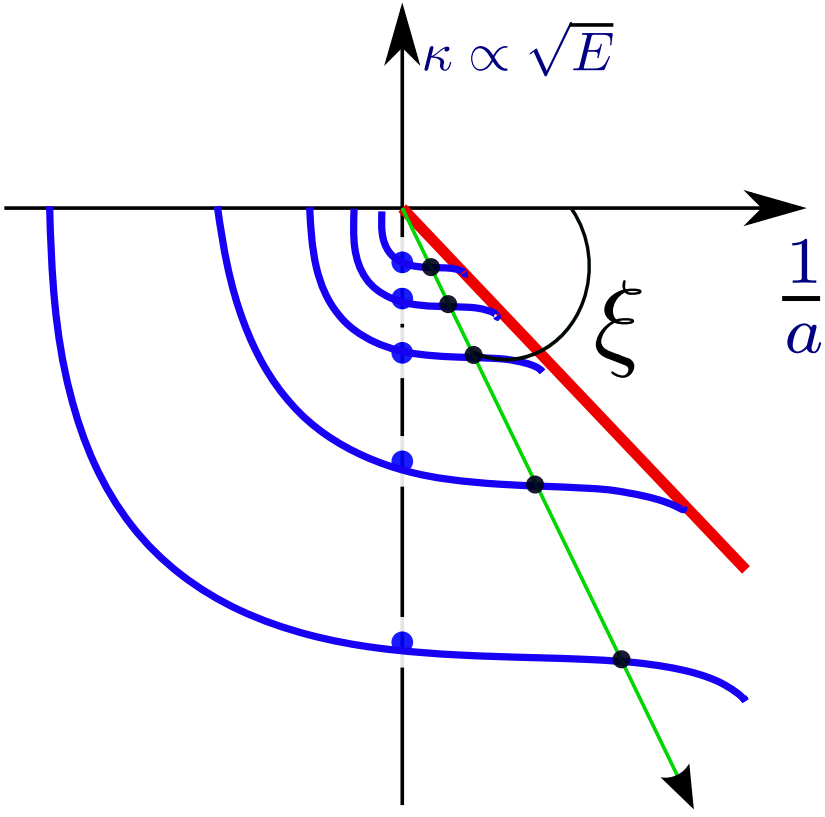

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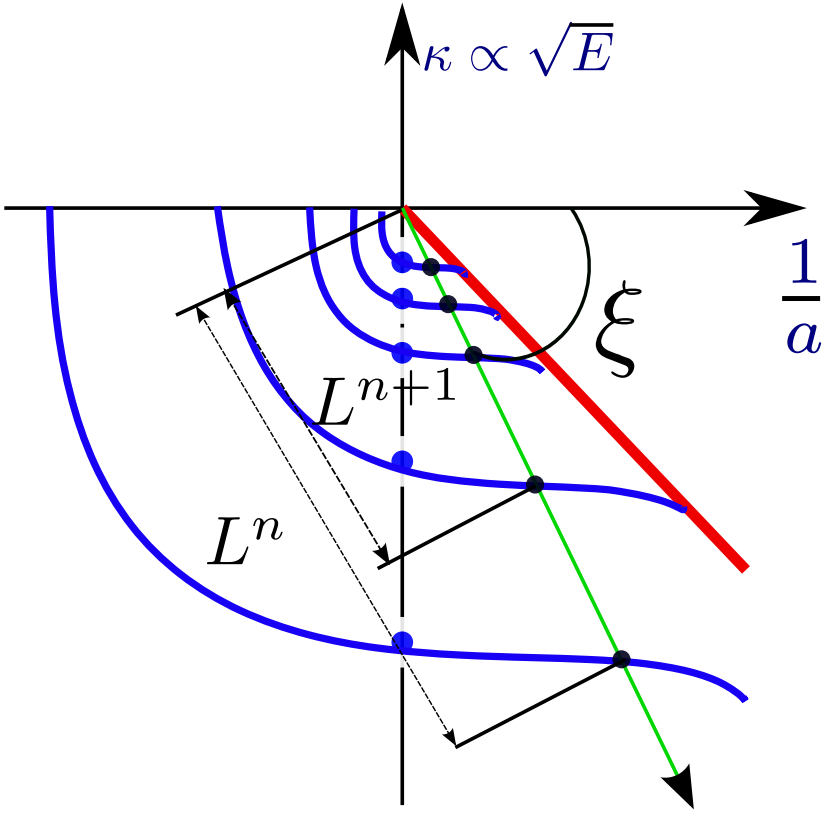

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$$L^{n+1}/L^n \to 1/22.7$$

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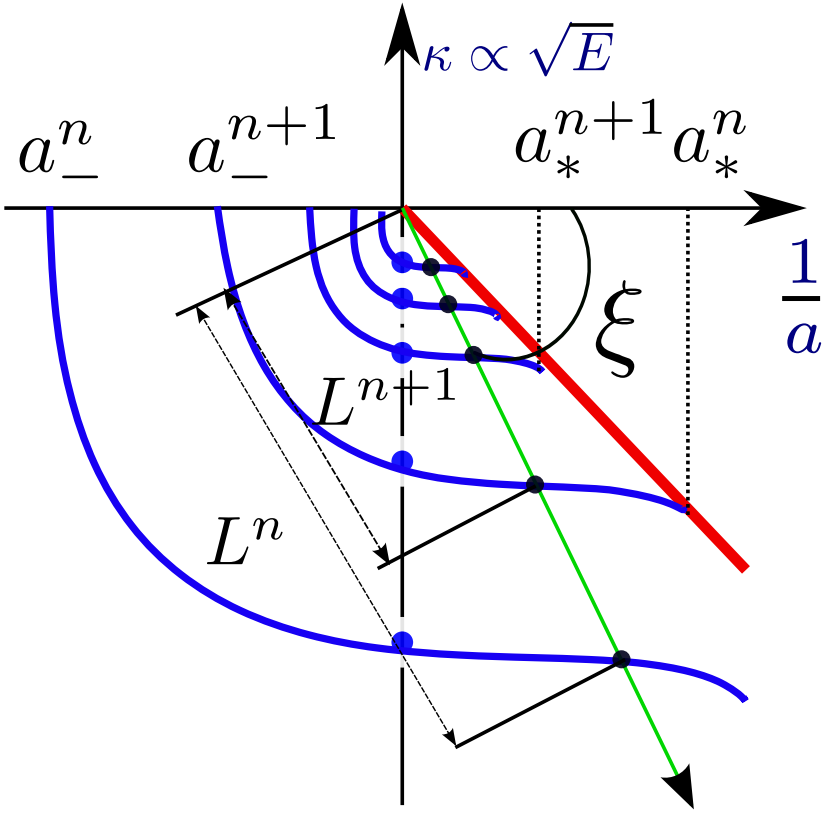

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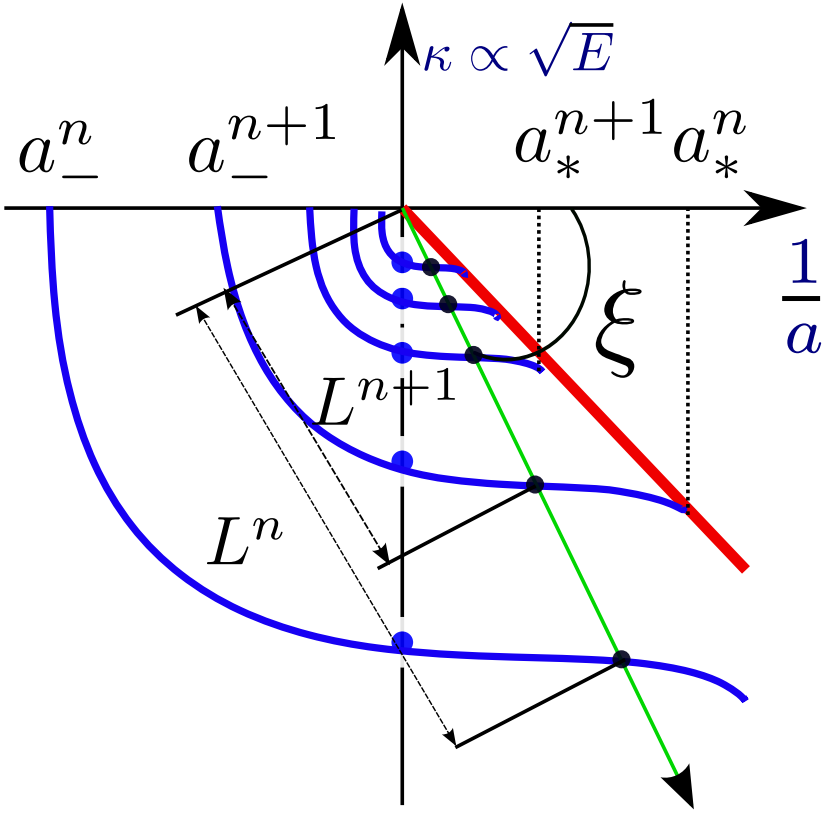

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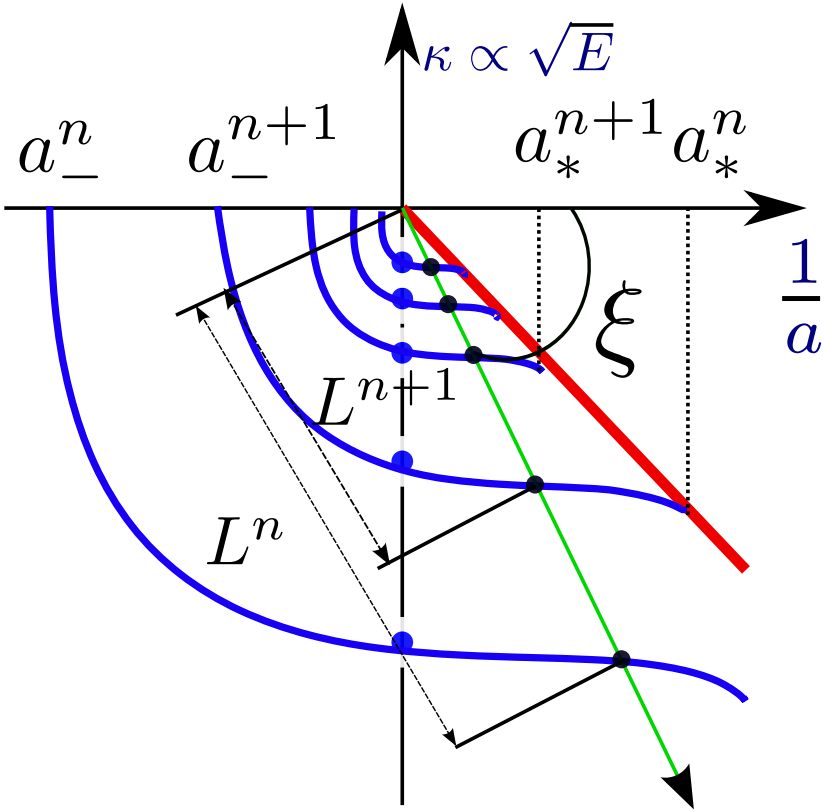

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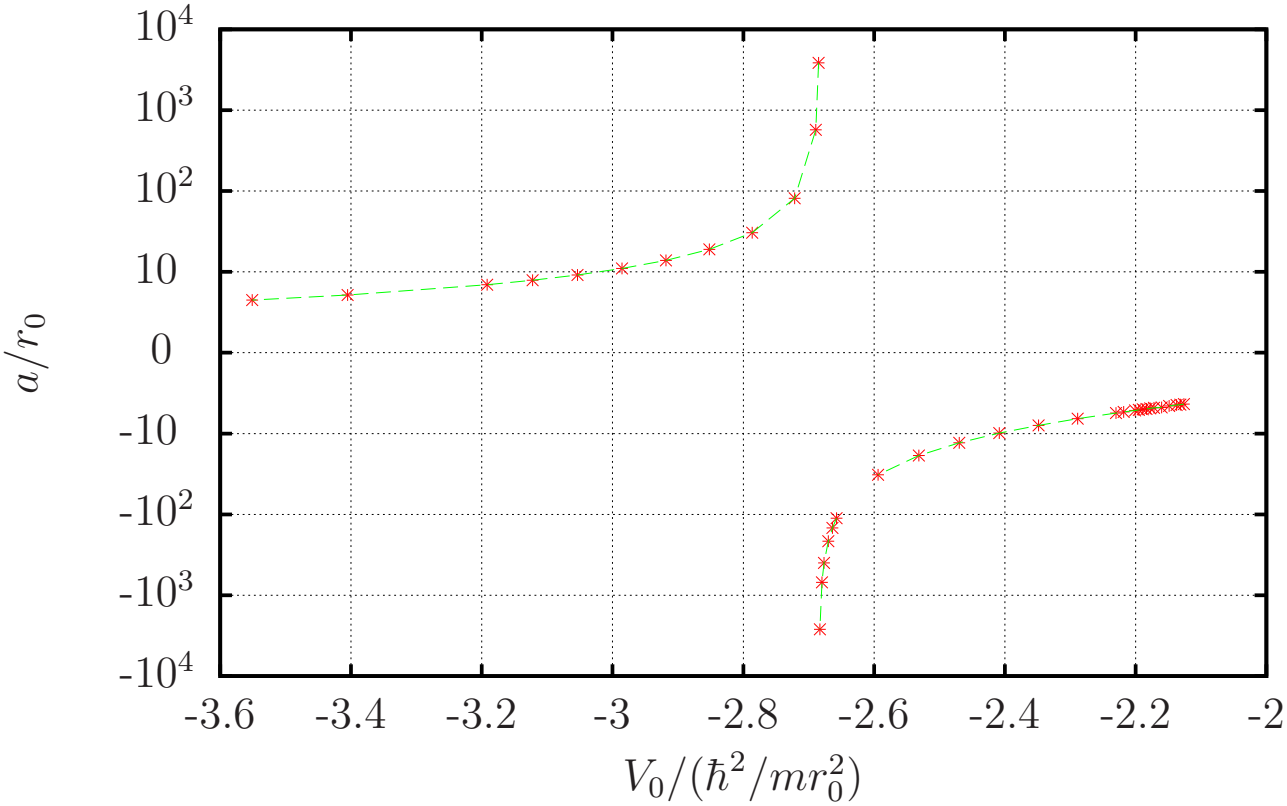

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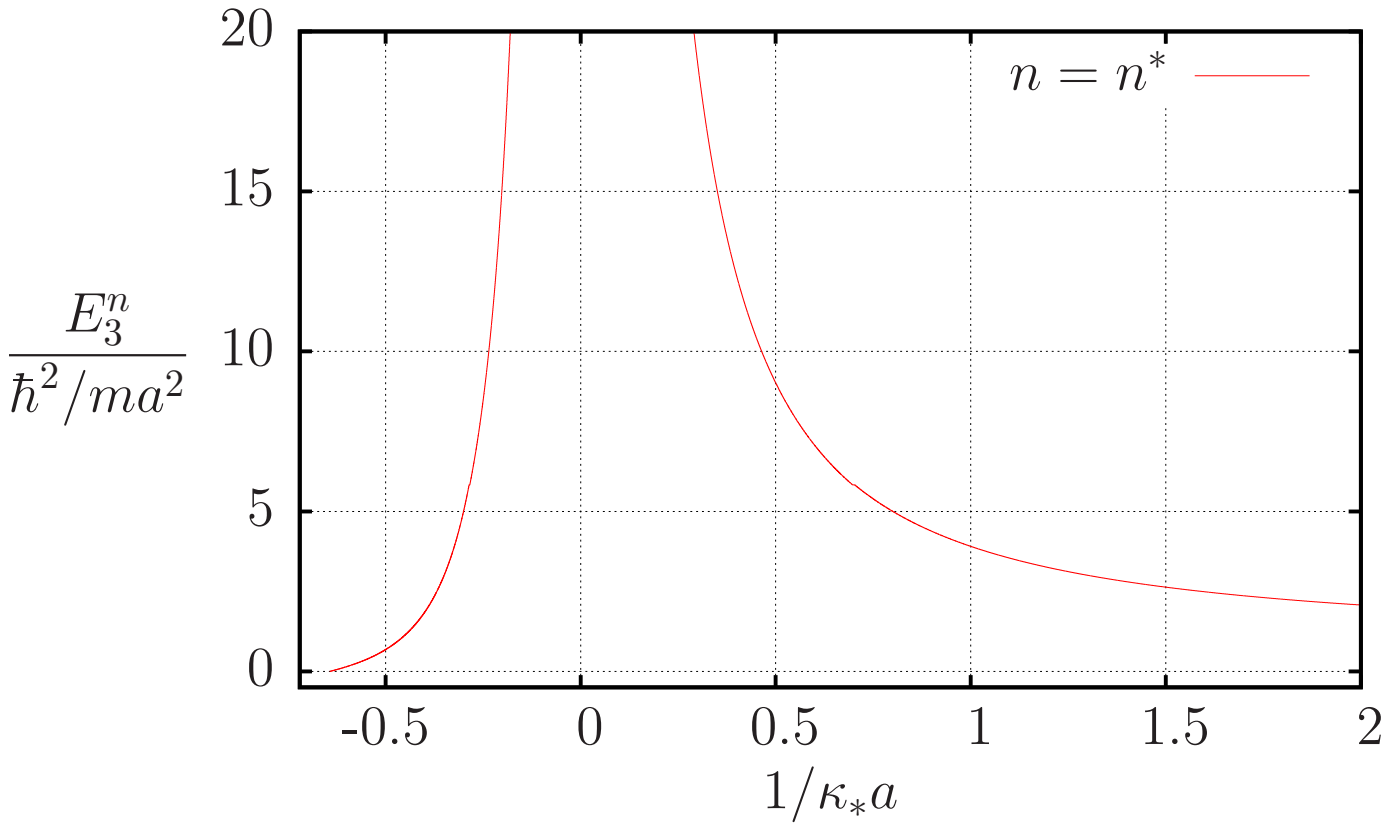

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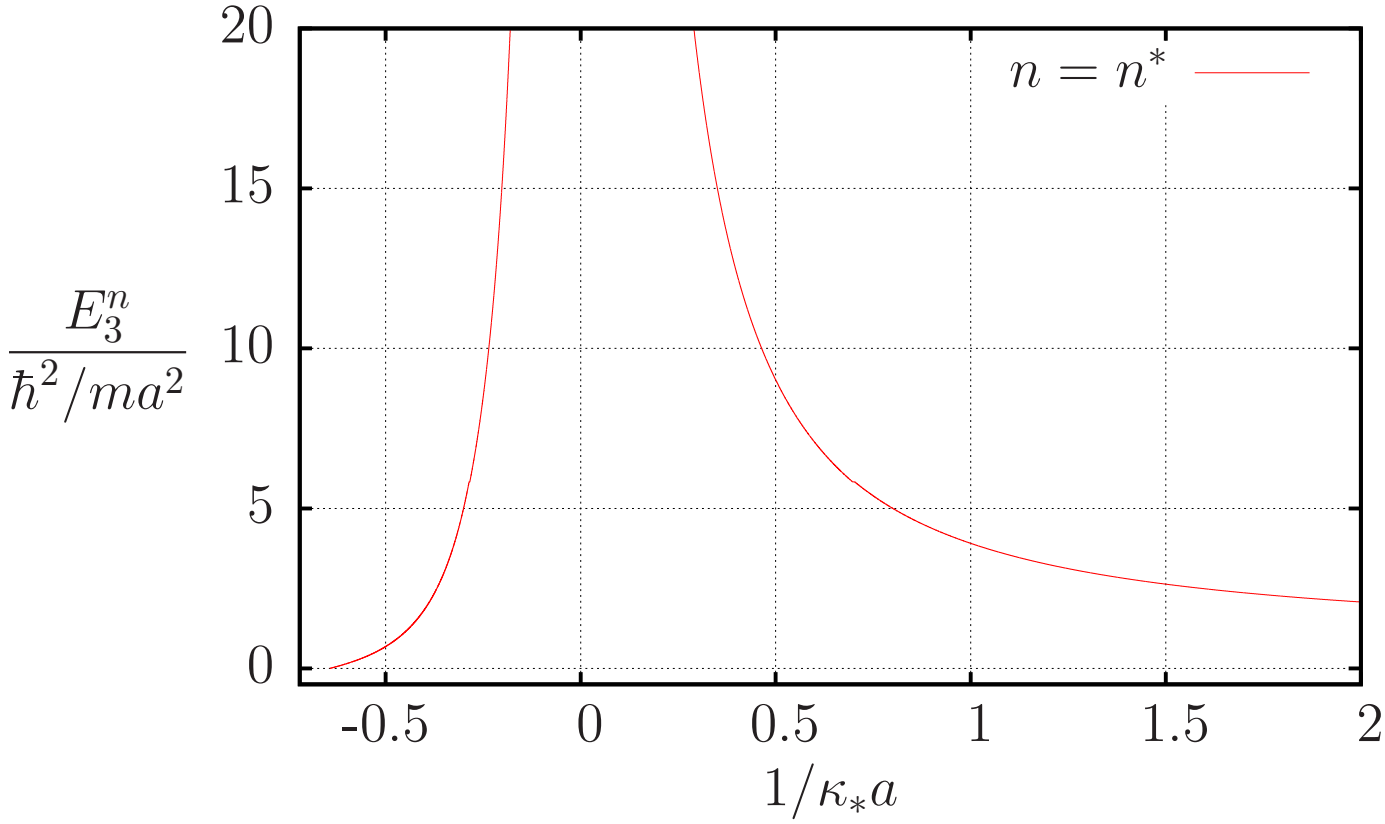

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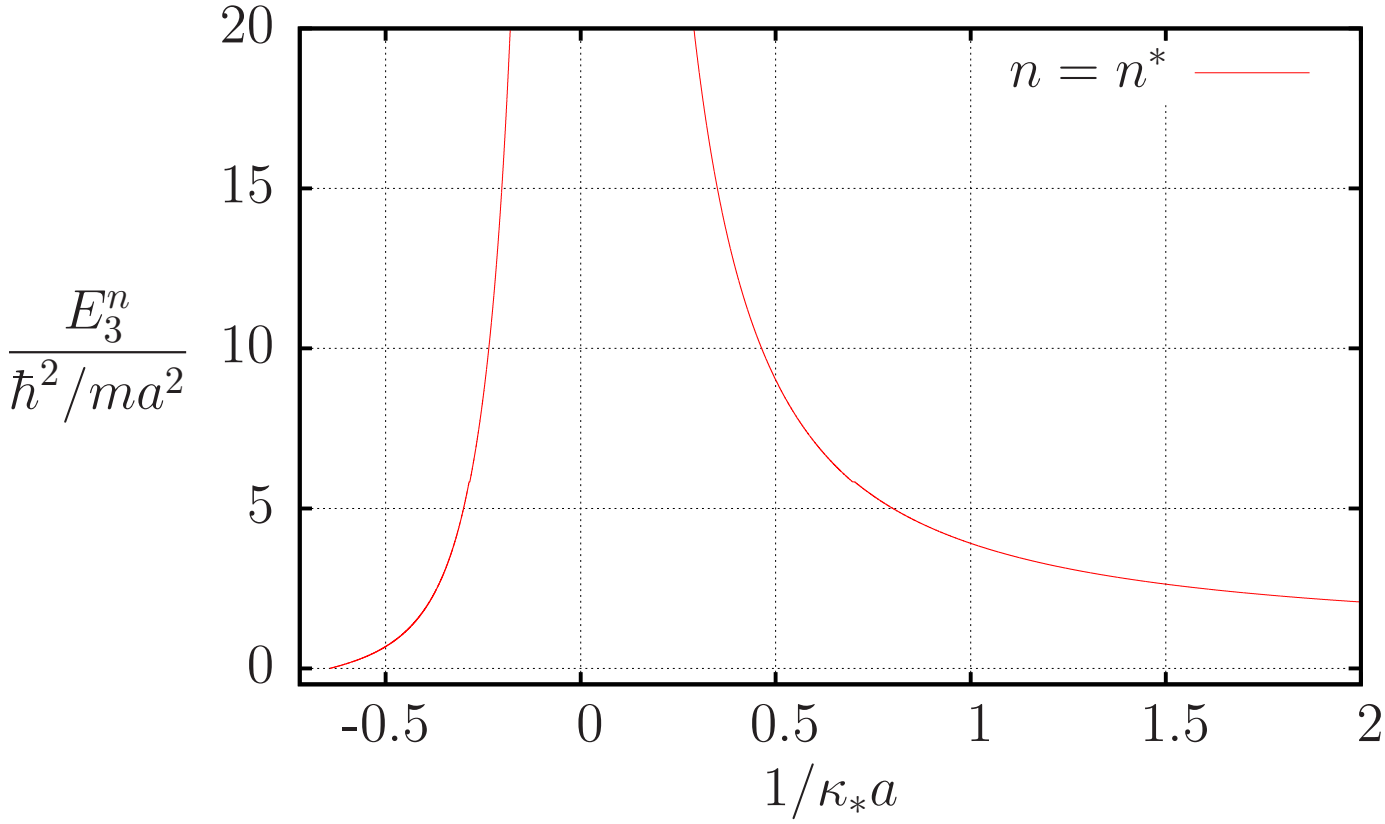

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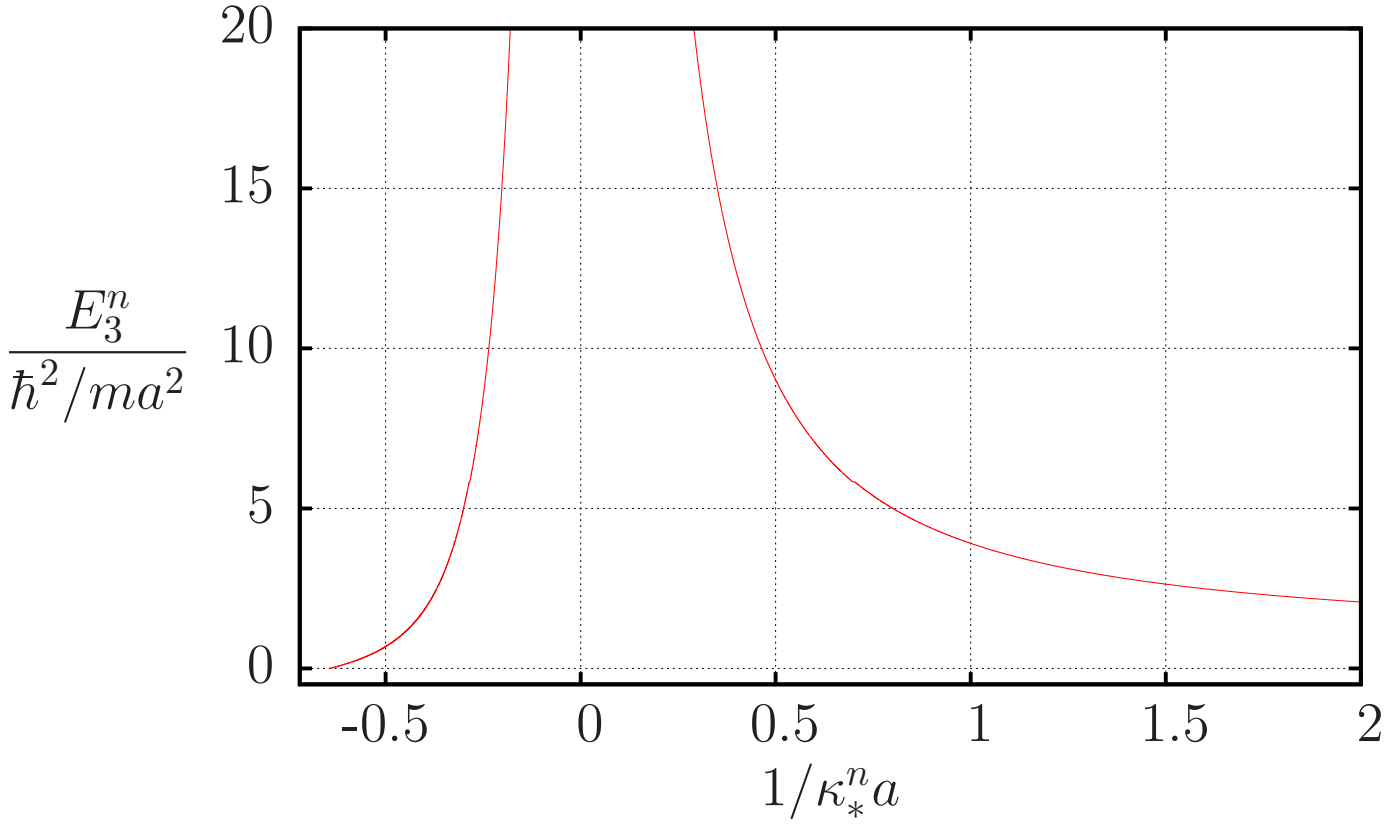

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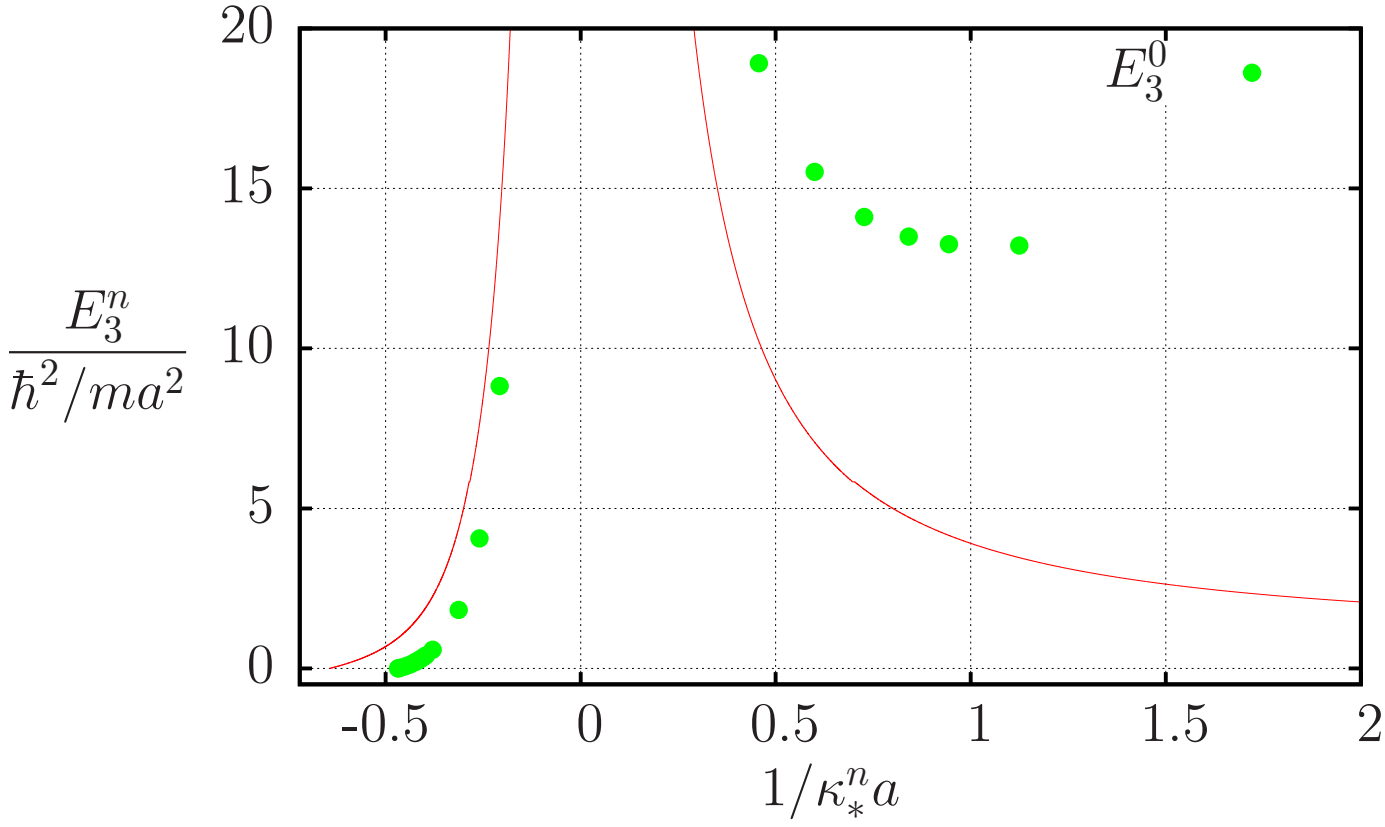

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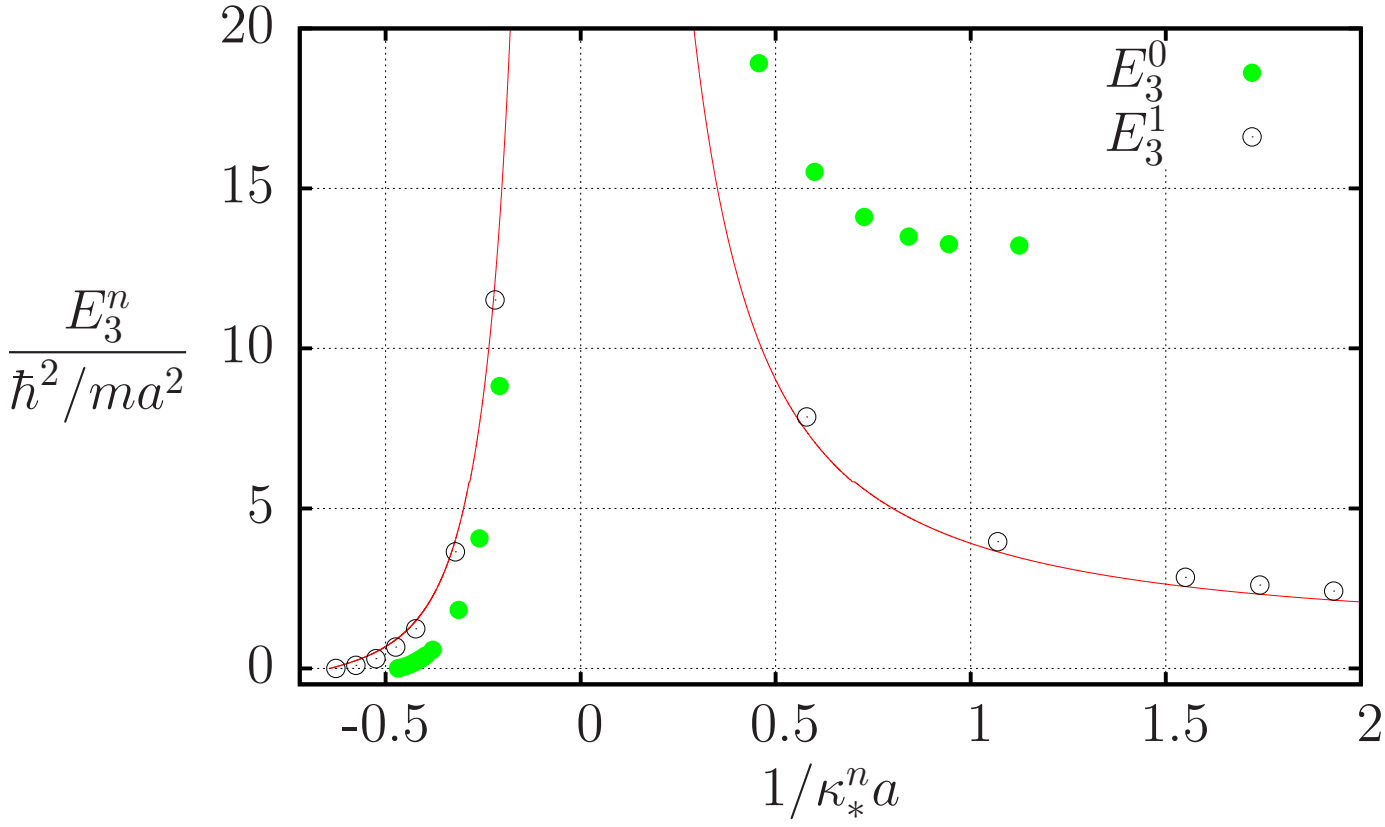

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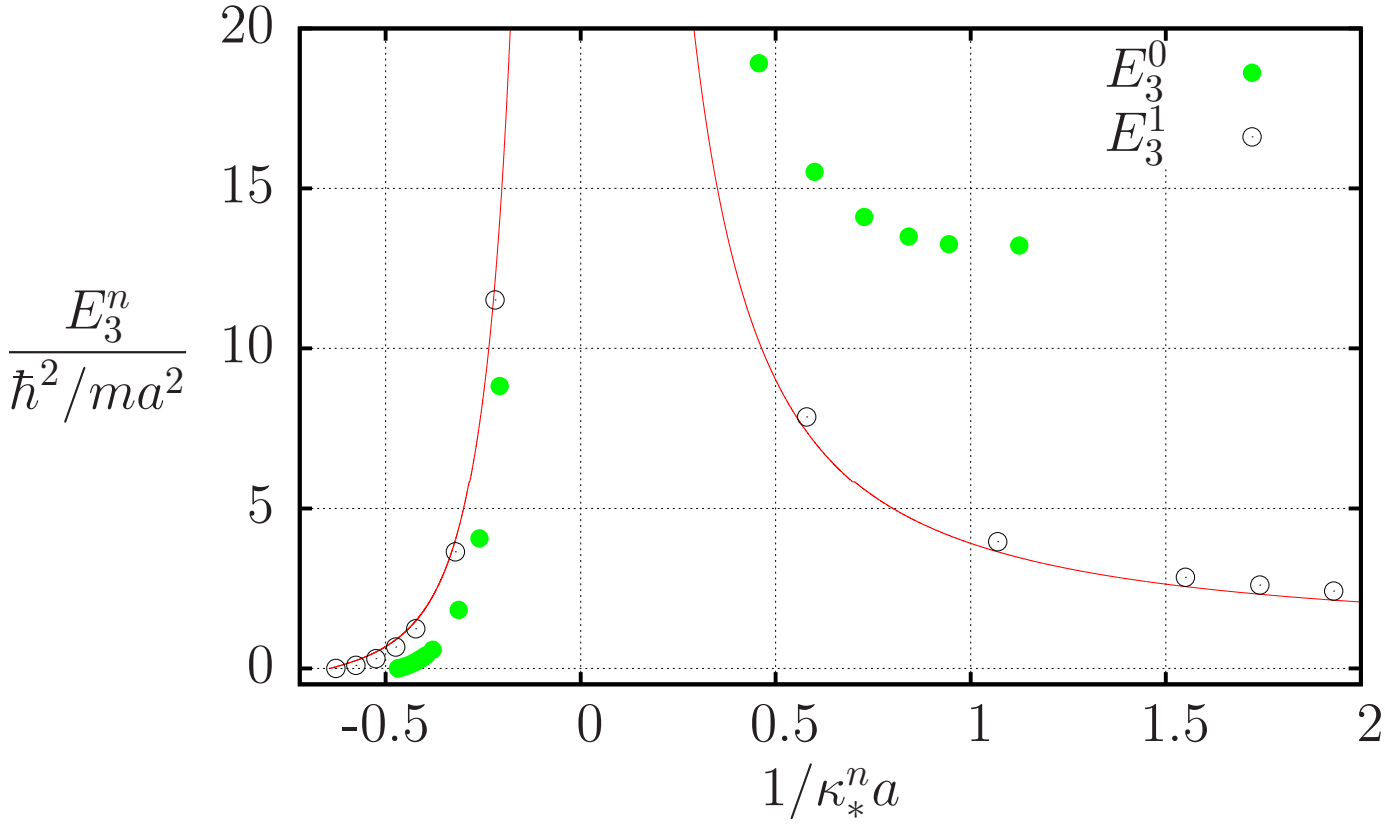

$$\begin{cases} E_3^n/(\hbar^2/m a_B{}^2) = \tan^2\xi \\ \kappa_*^n a_B = \dfrac{e^{-\Delta(\xi)/2s_0}}{\cos\xi} - \Gamma_n^3 \end{cases} \qquad \frac{\hbar^2}{m a_B{}^2} = \begin{cases} \text{Bound State} & a > 0 \\ \text{Virtual State} & a < 0 \end{cases}$$

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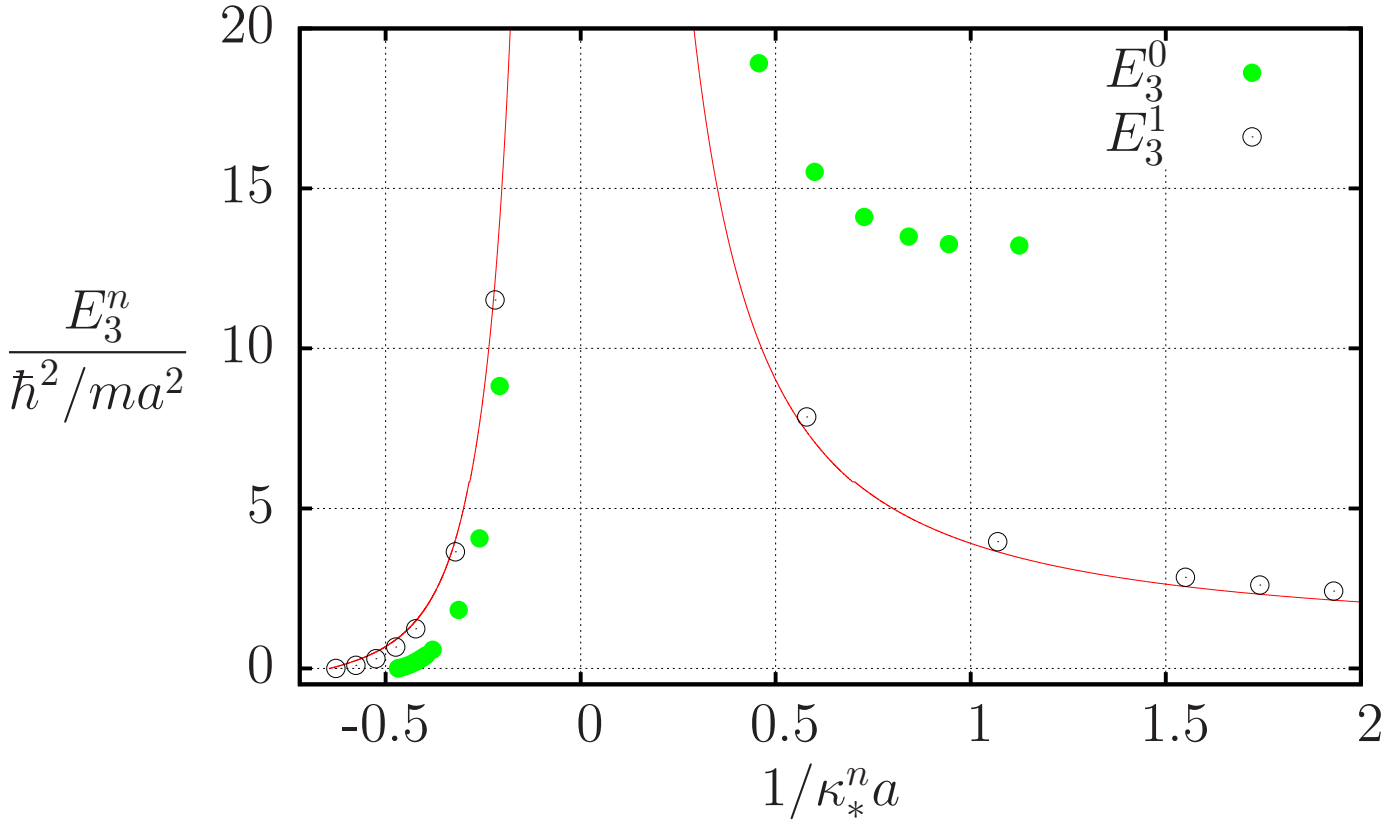

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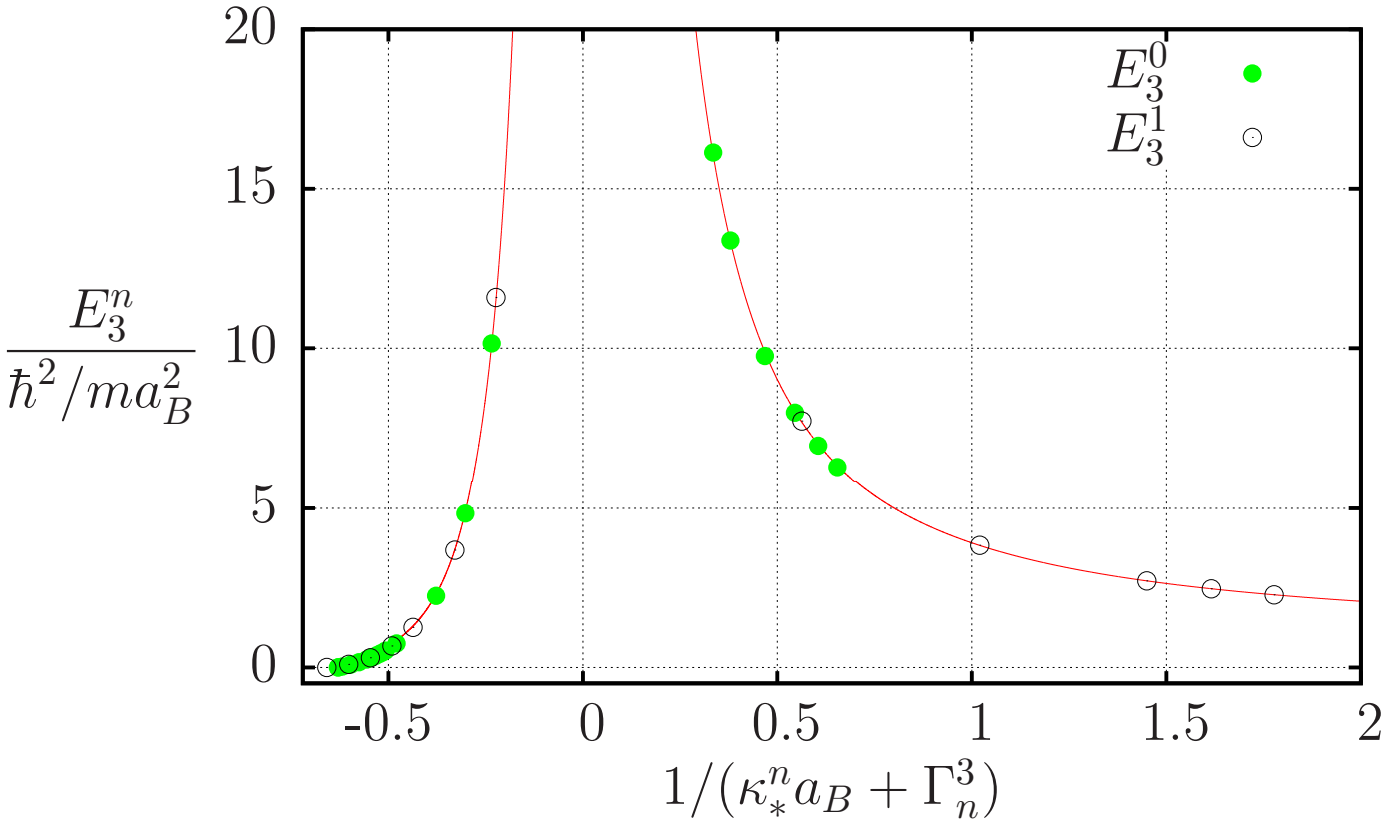

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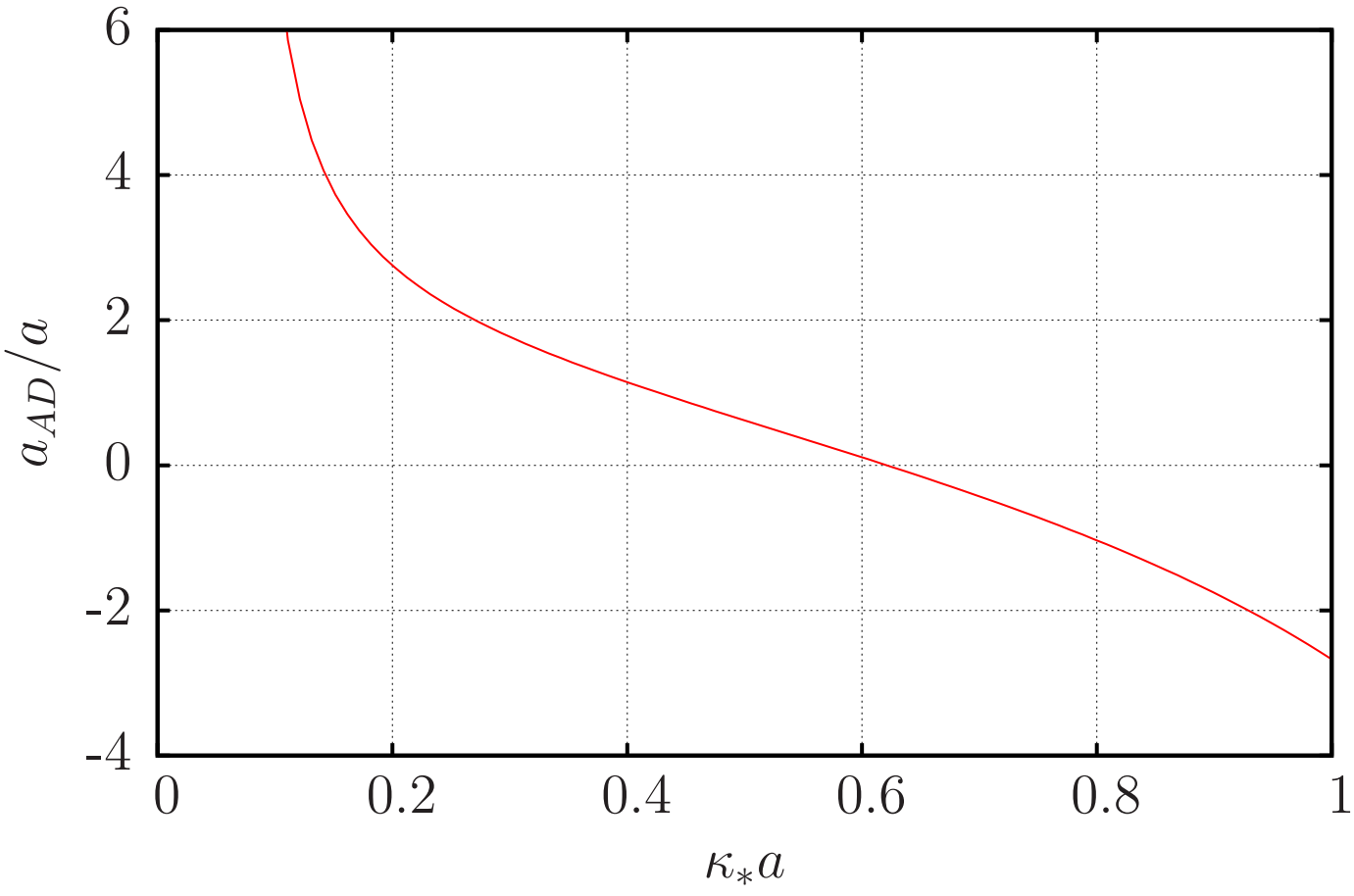

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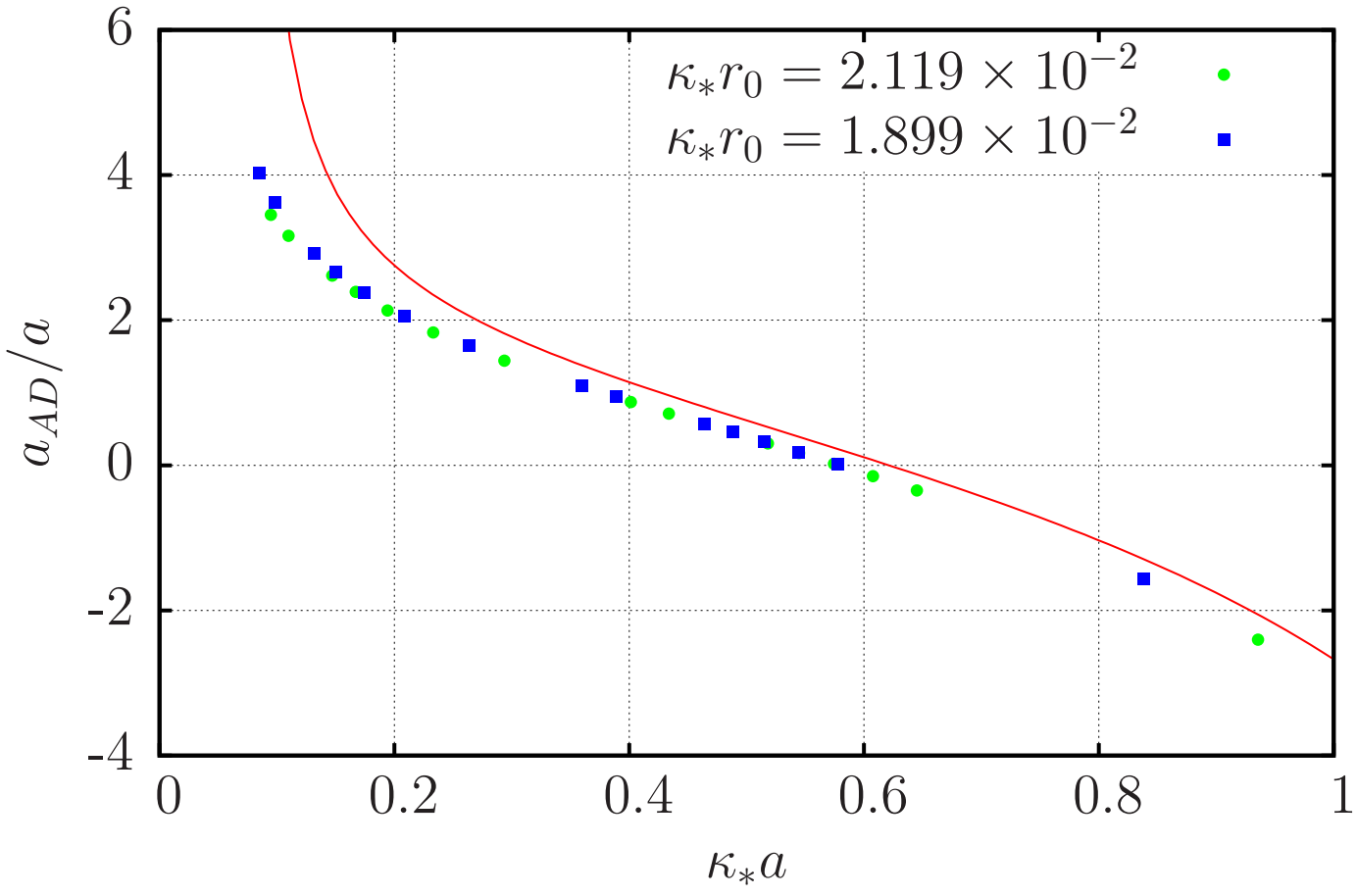

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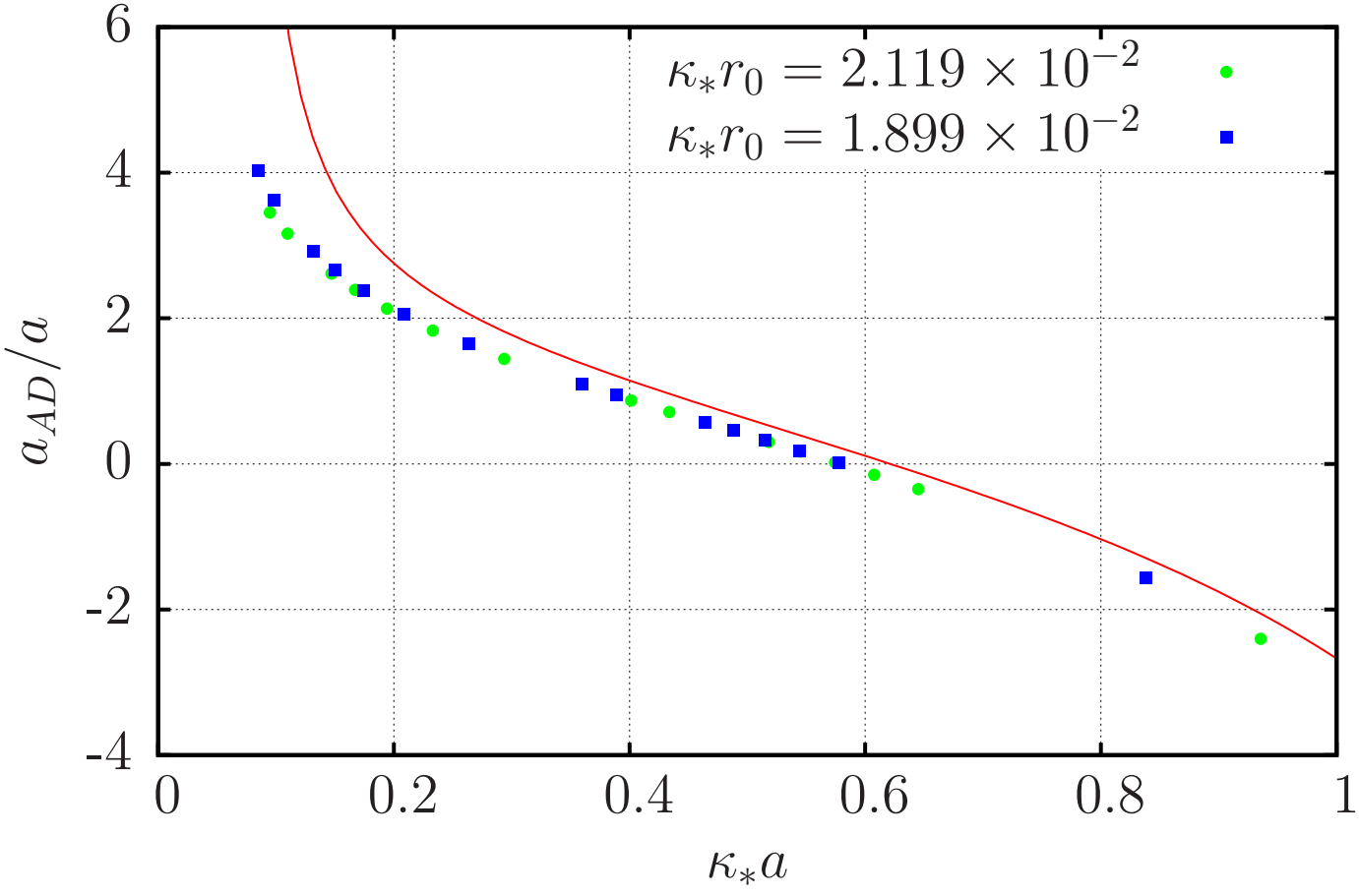

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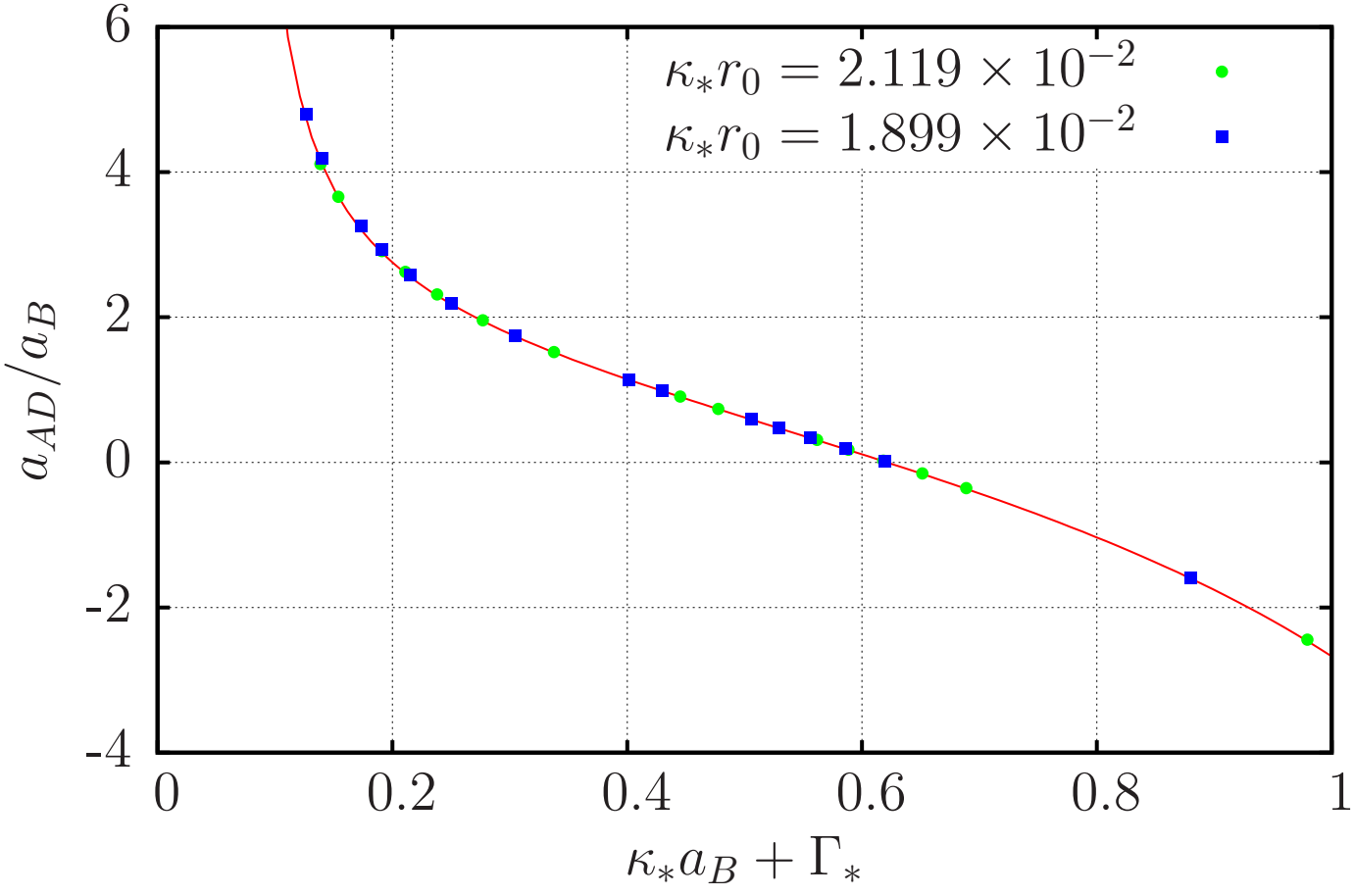

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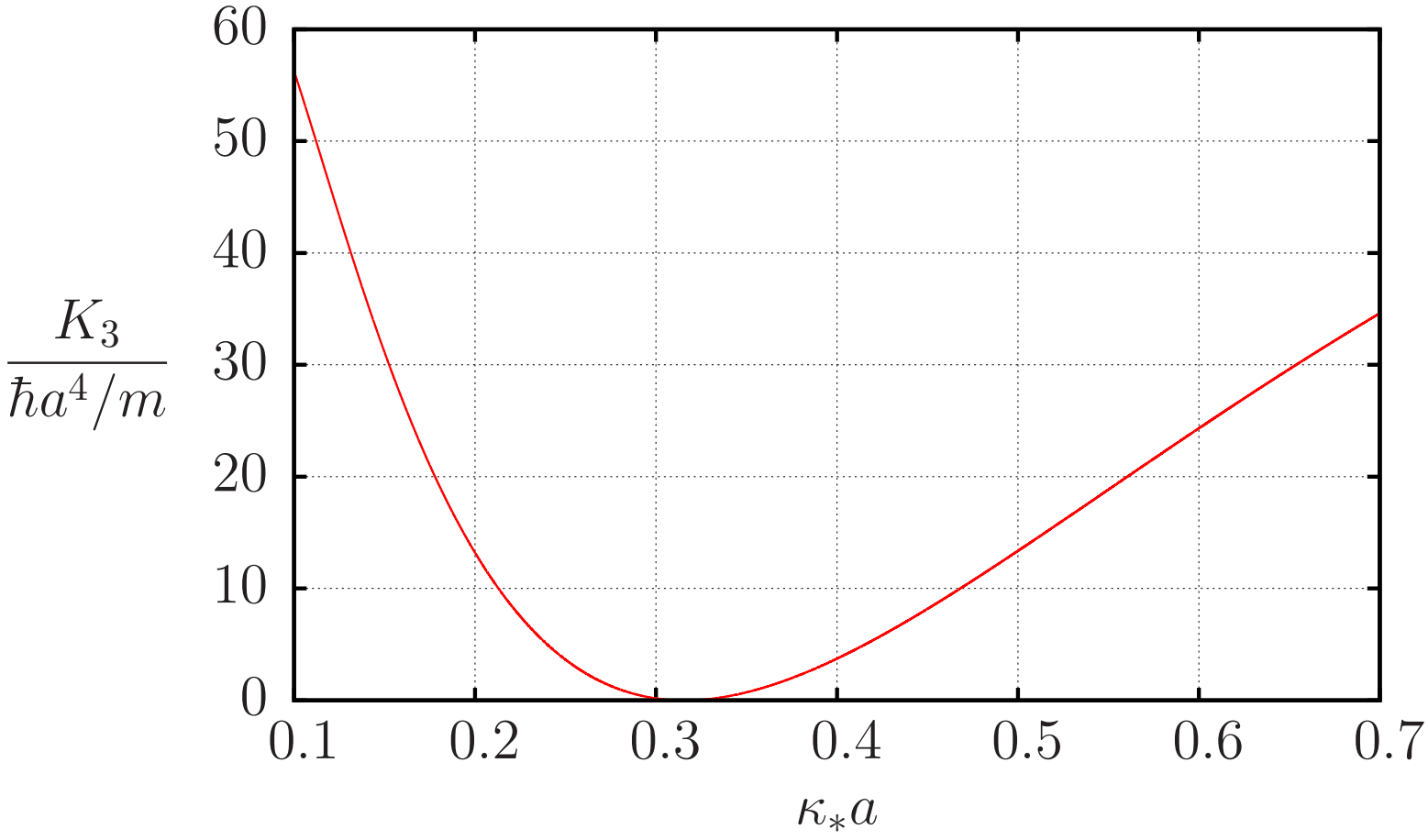

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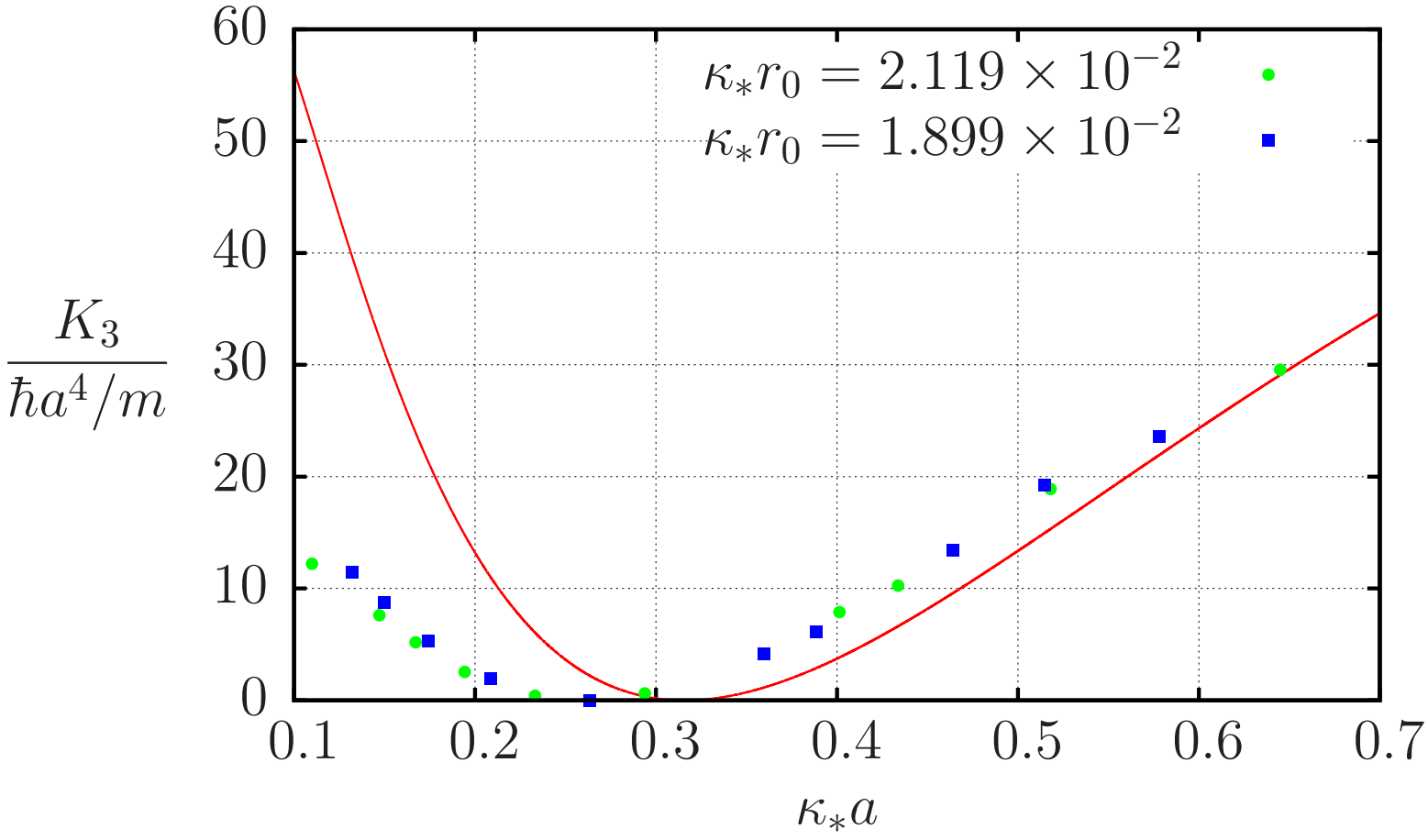

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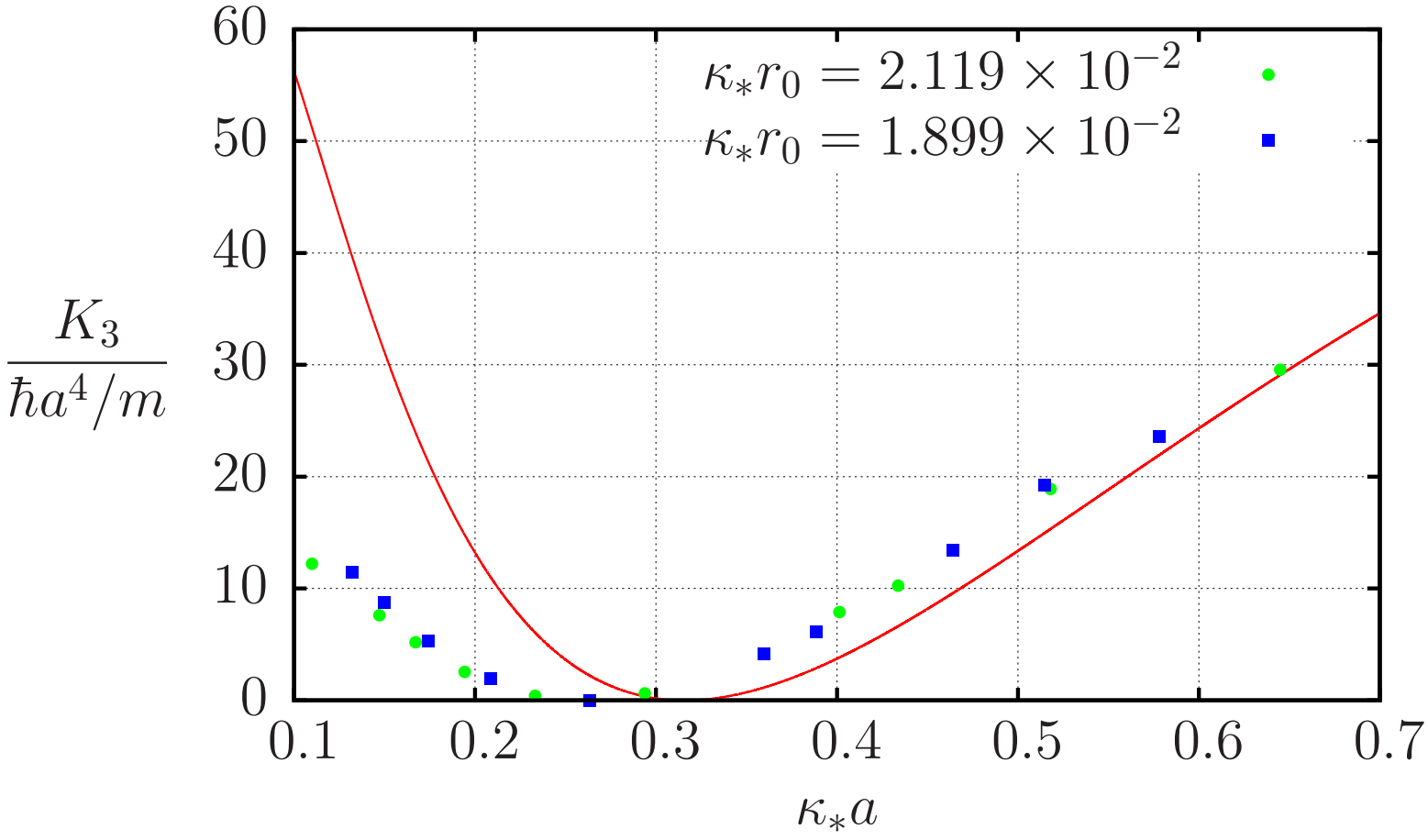

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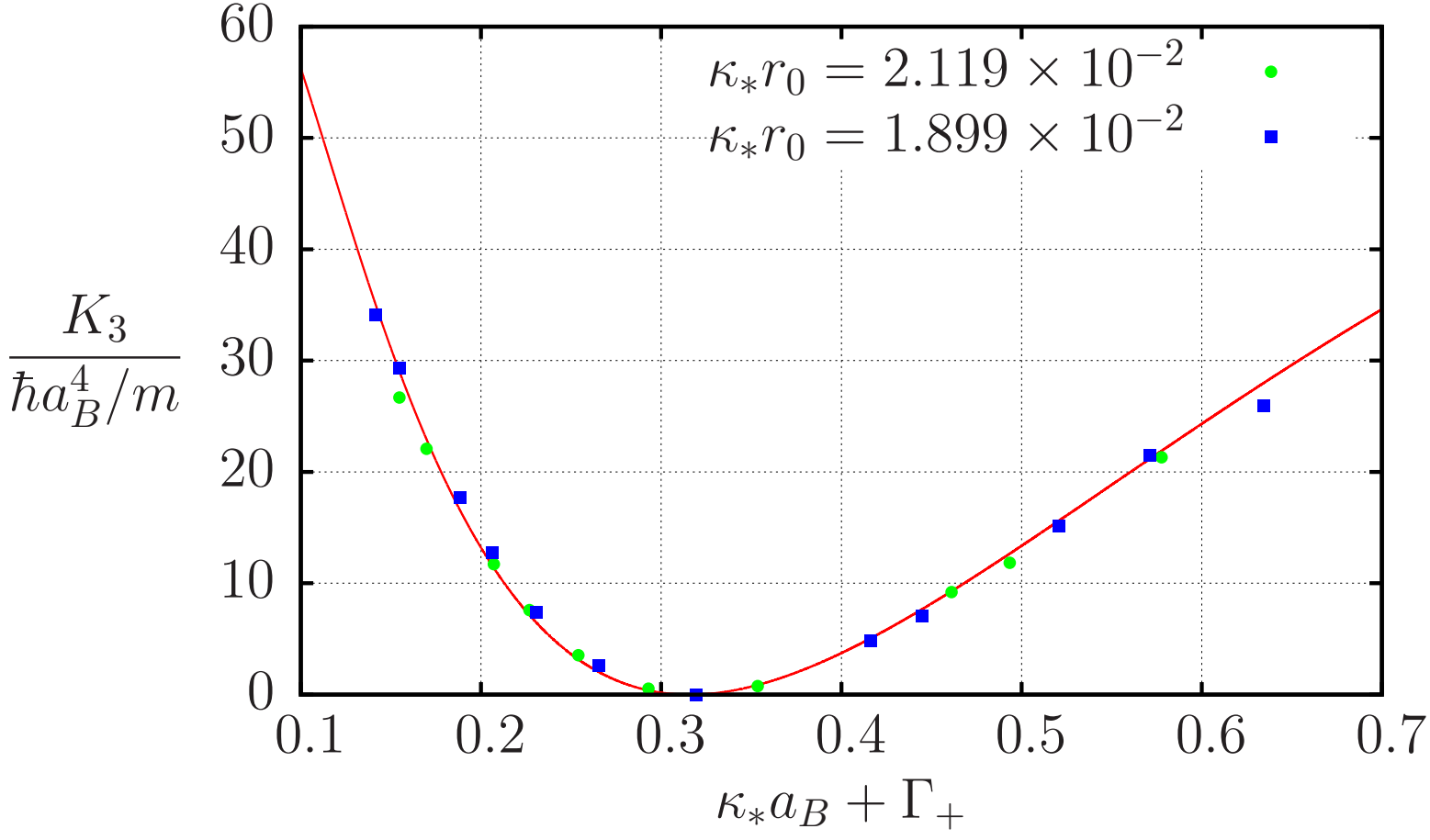

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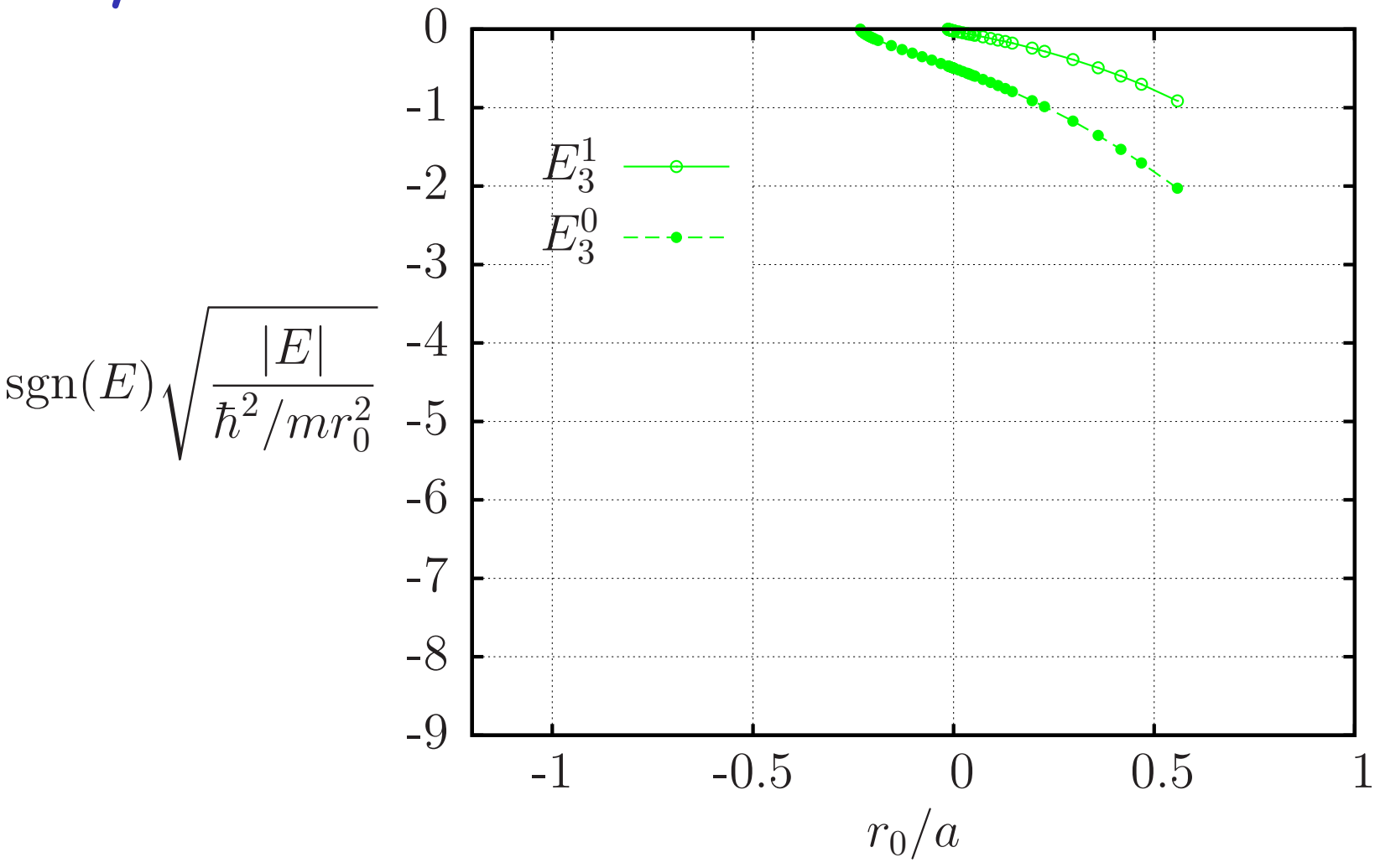
N-body Efimov Plot



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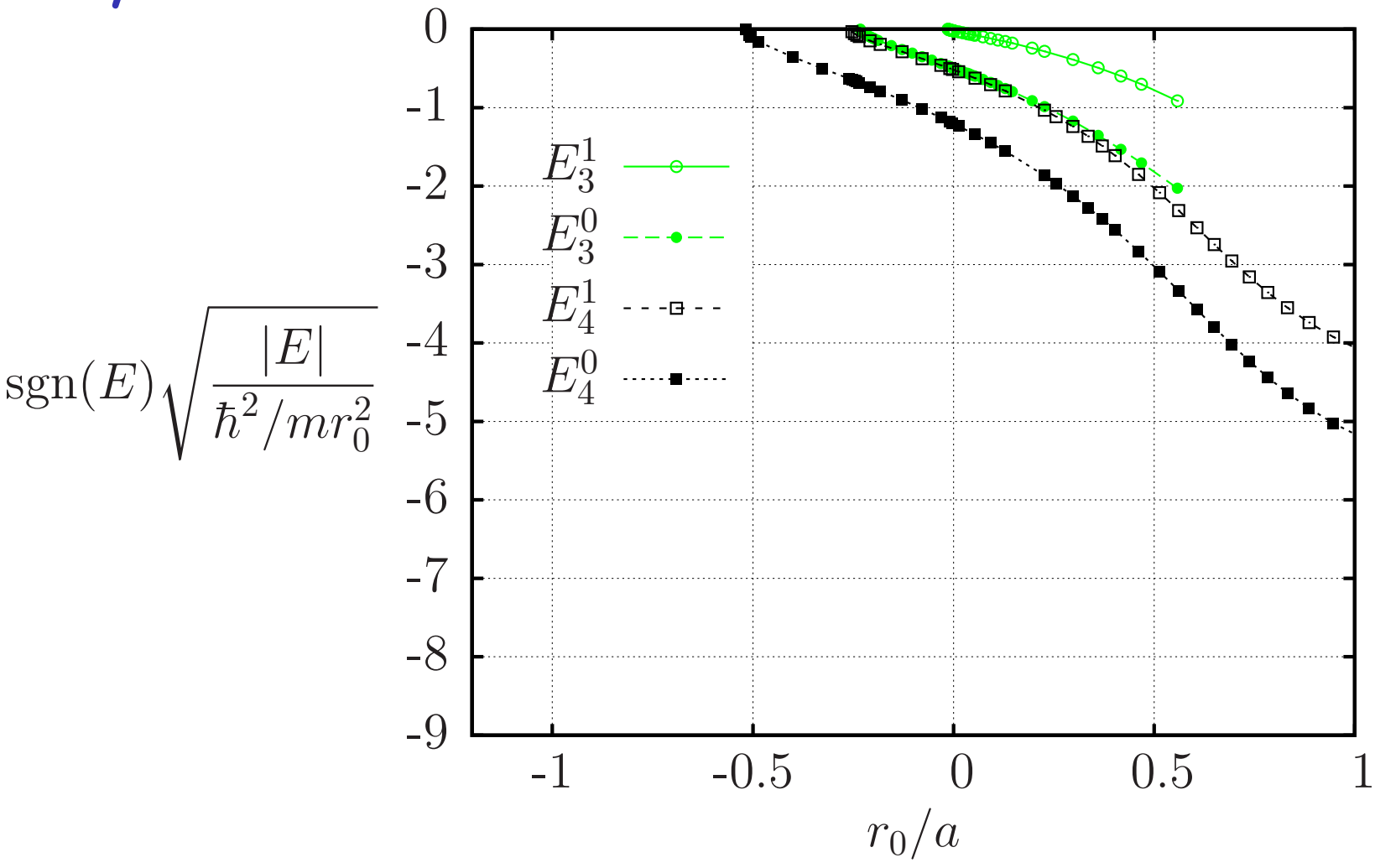

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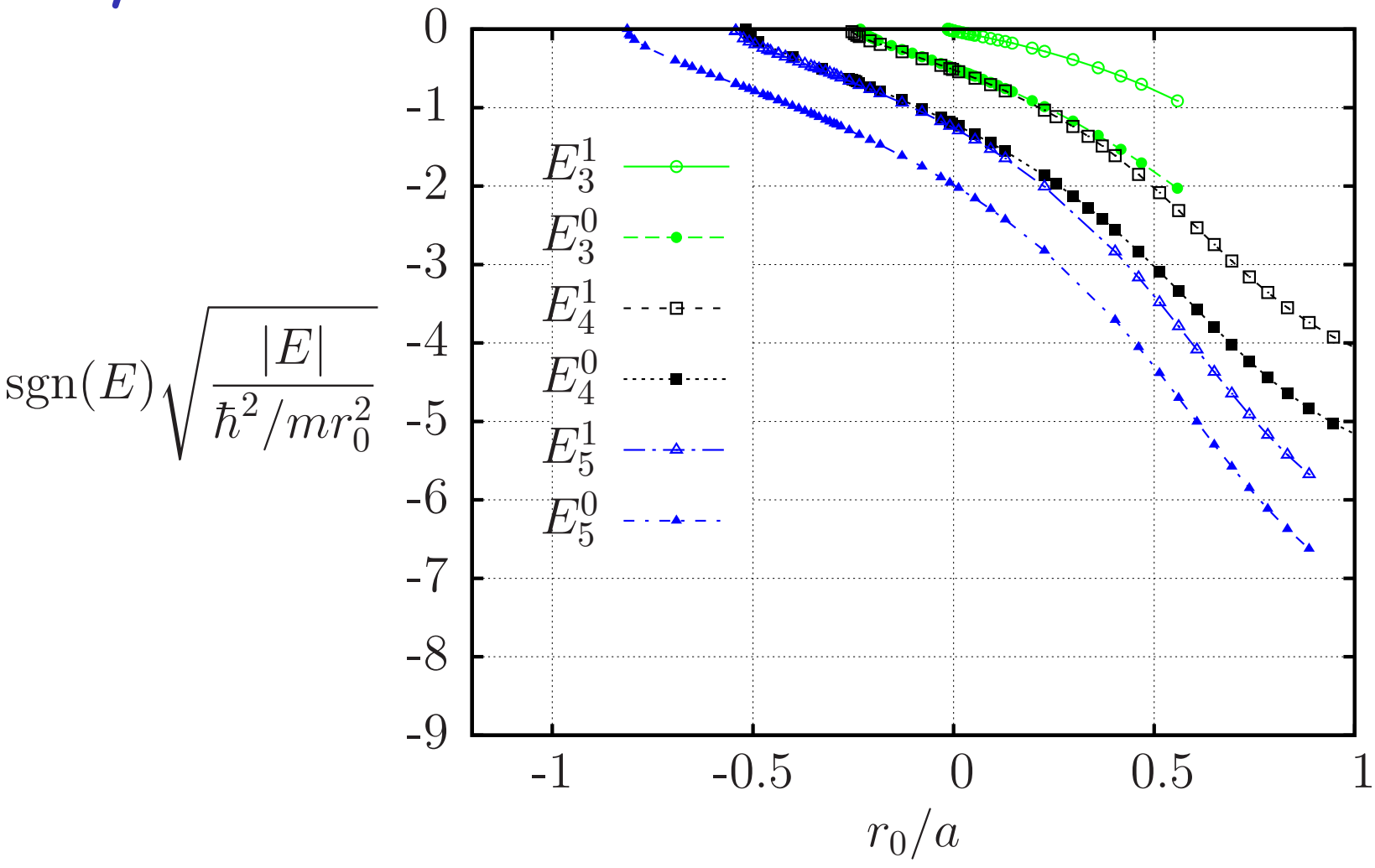

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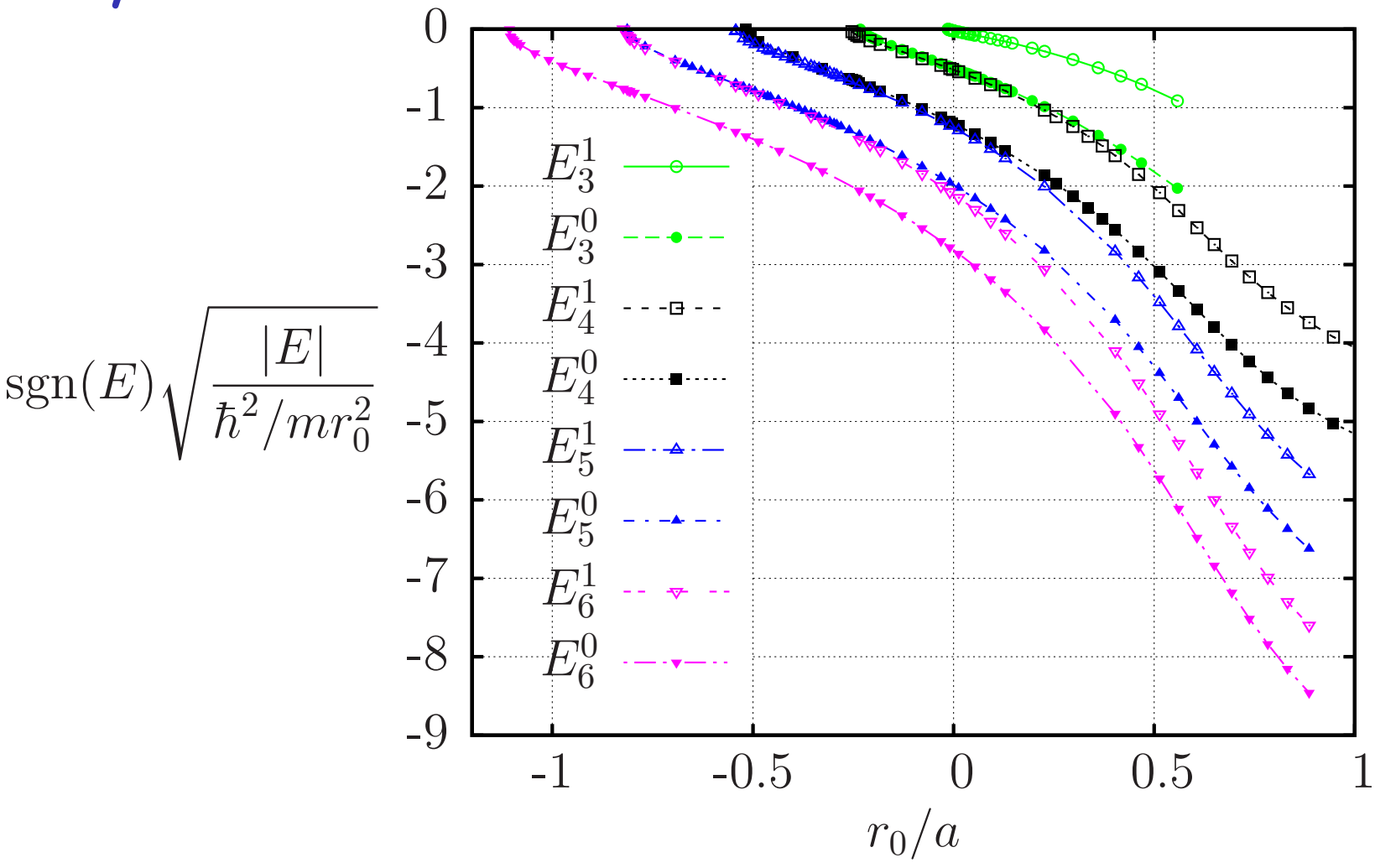

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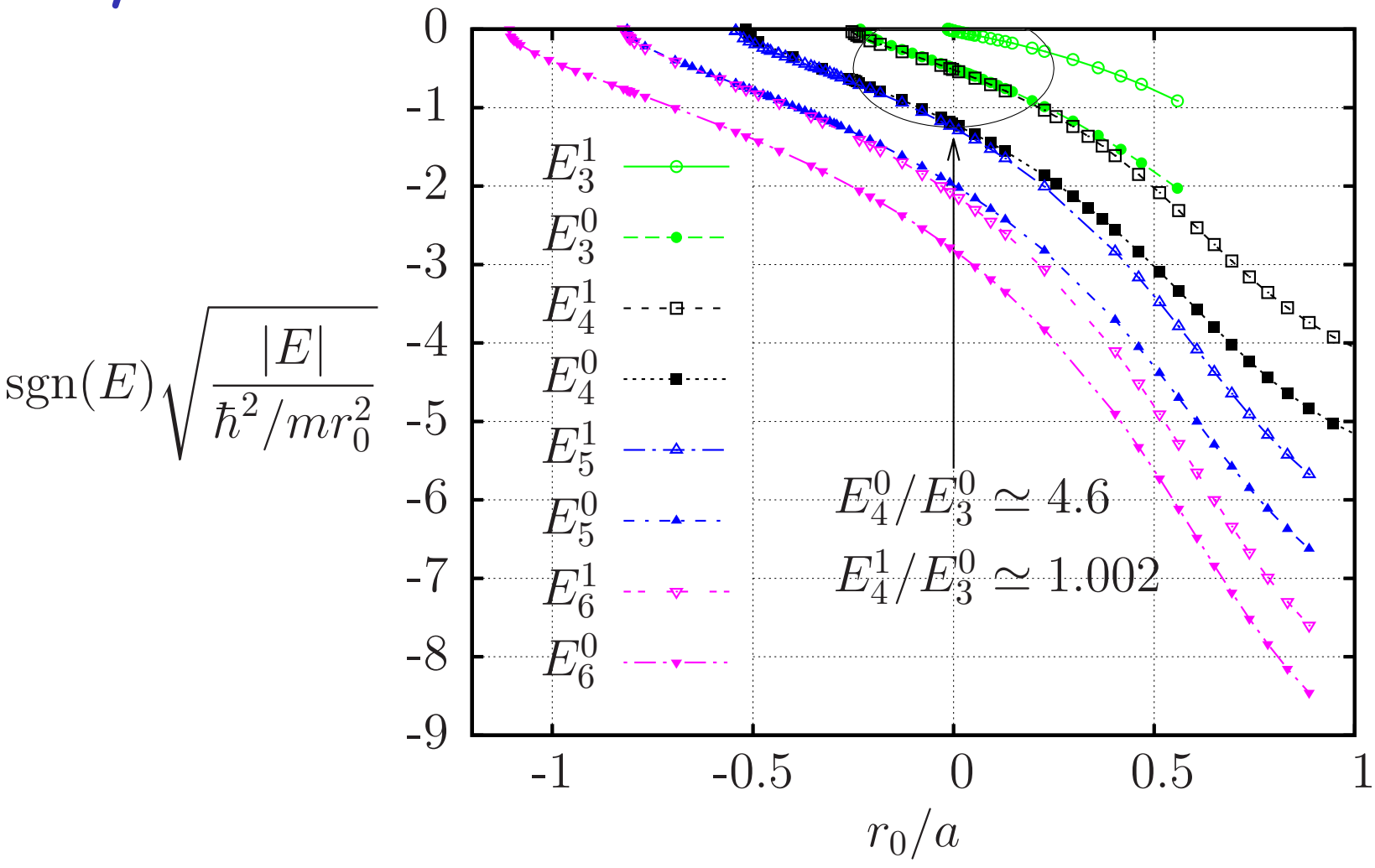

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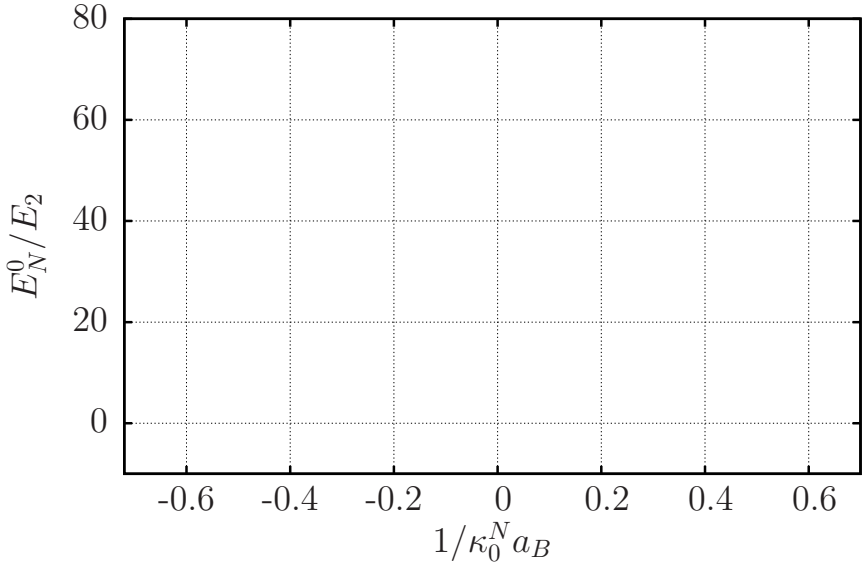
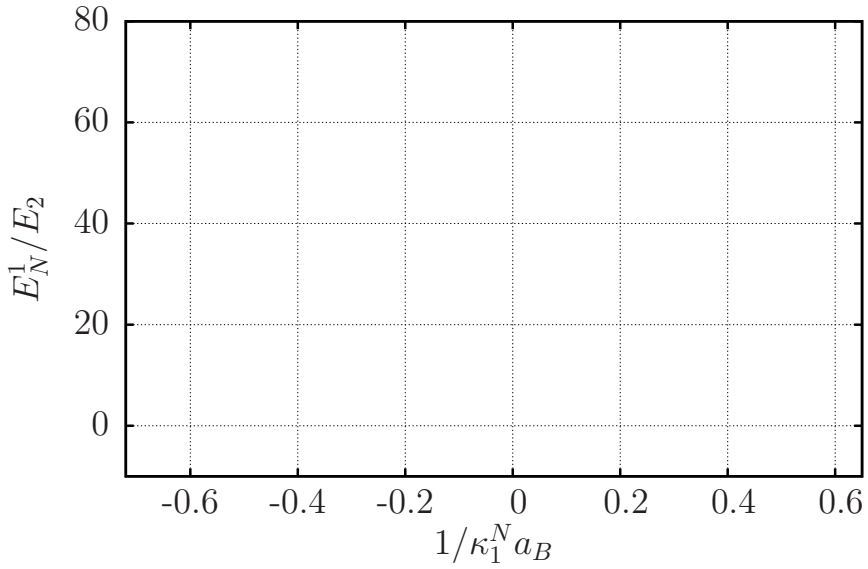
Universality





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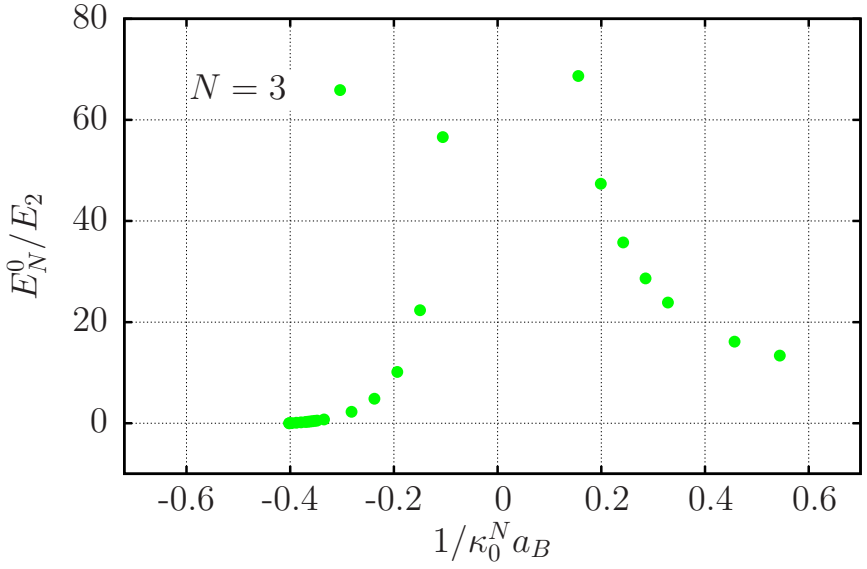

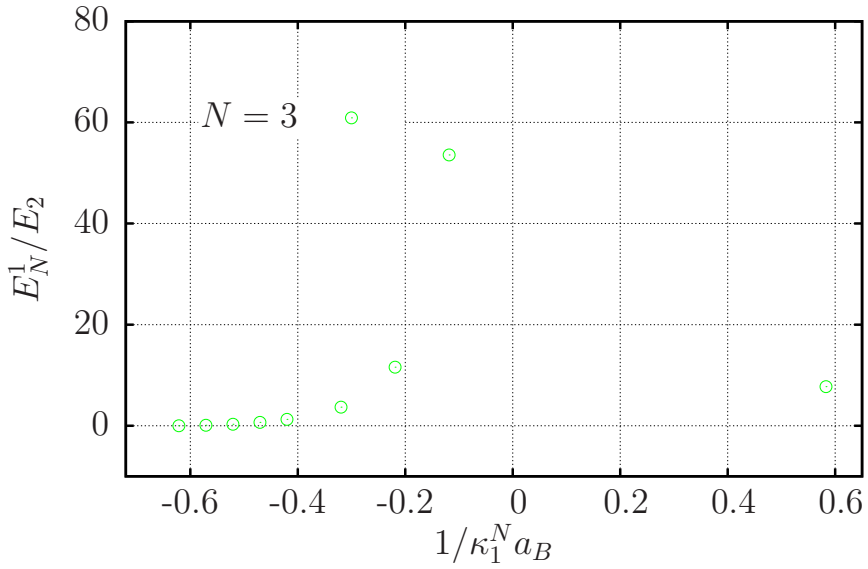

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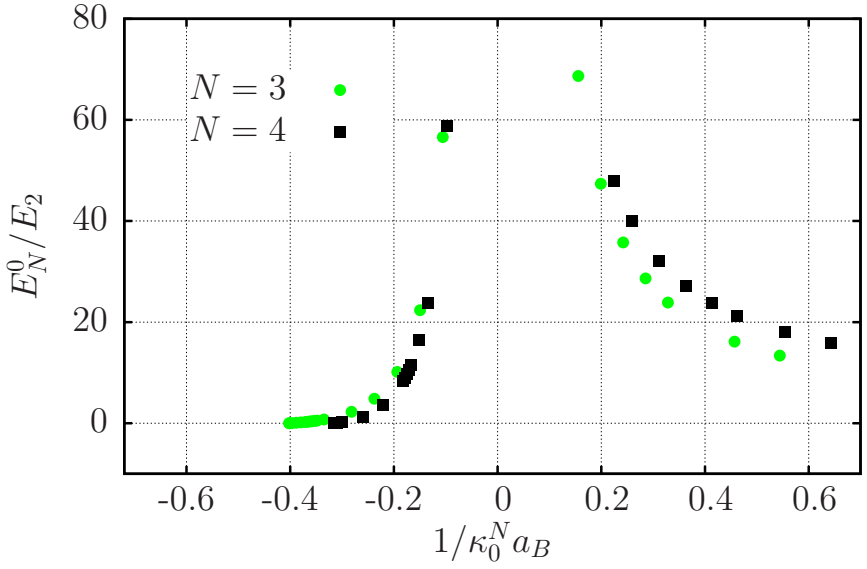

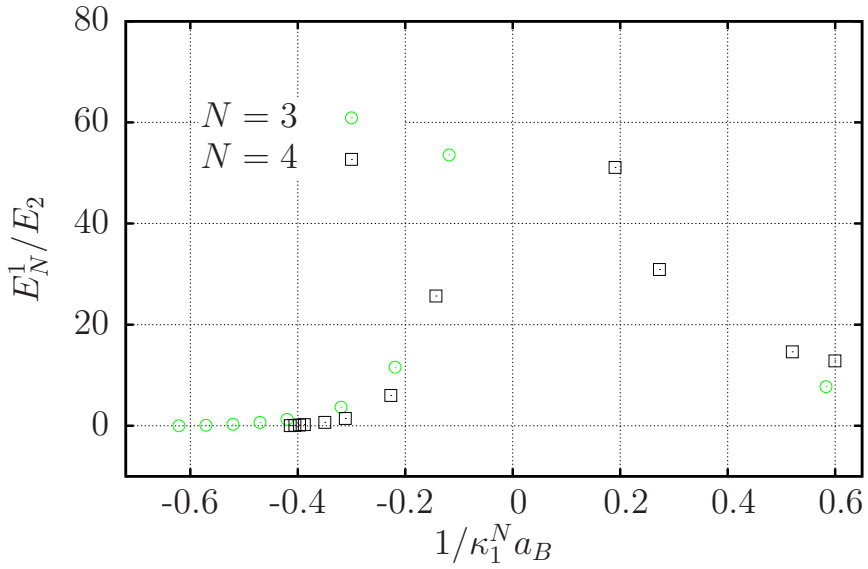

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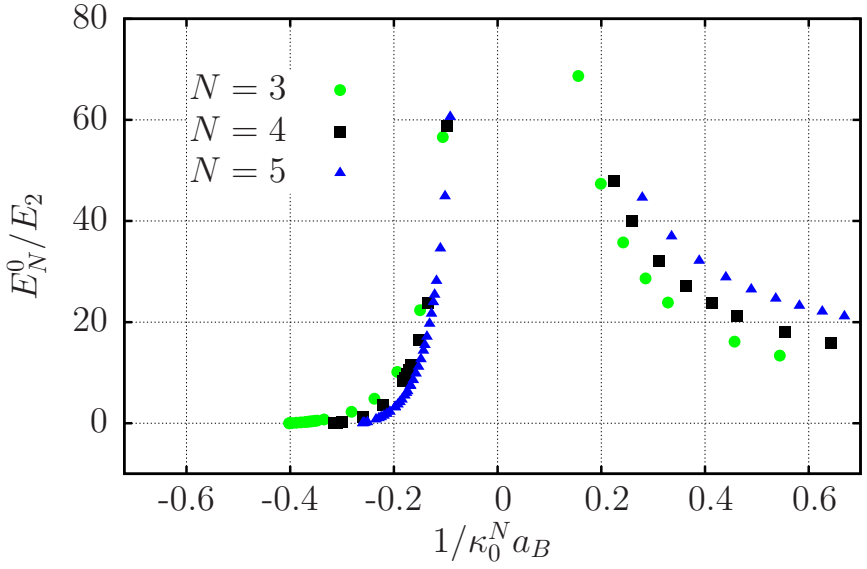

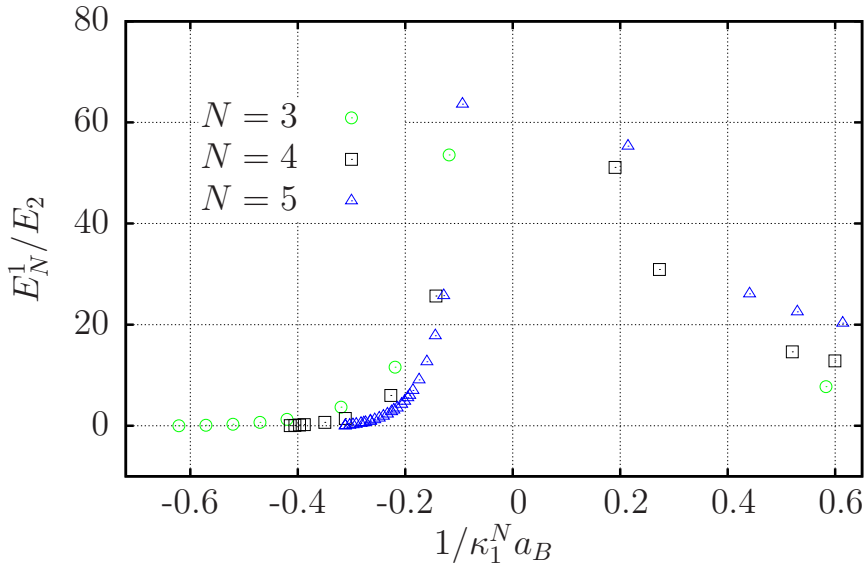

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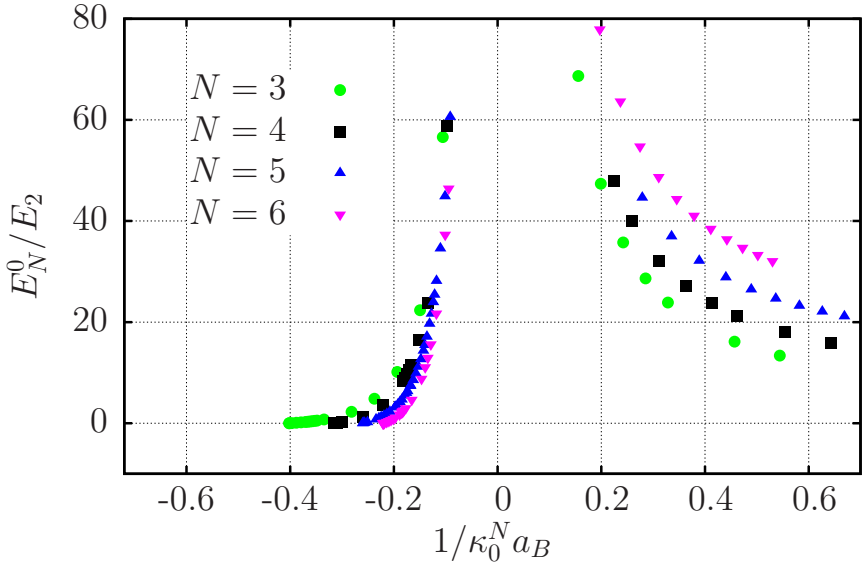

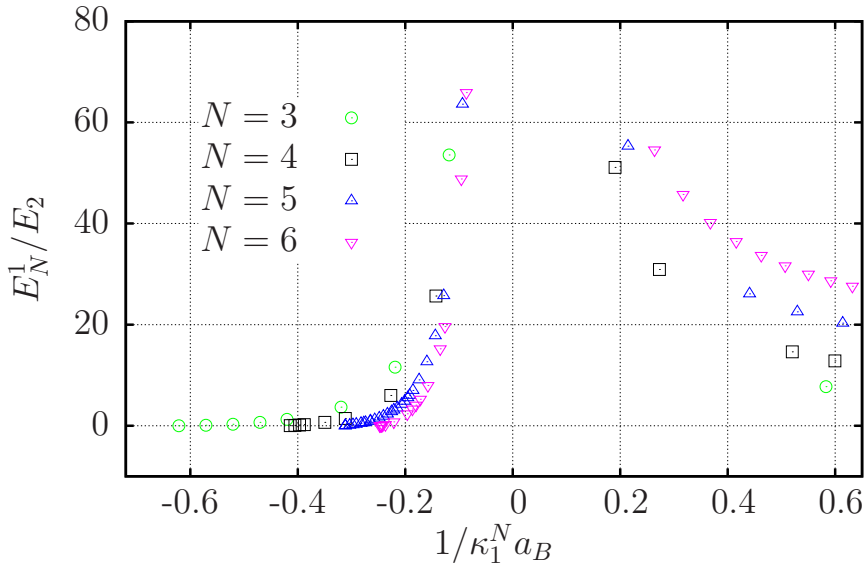

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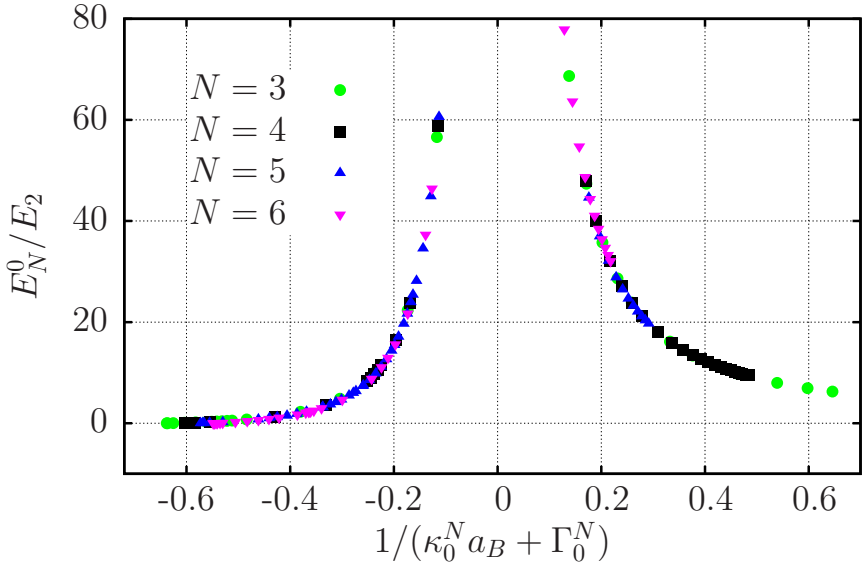

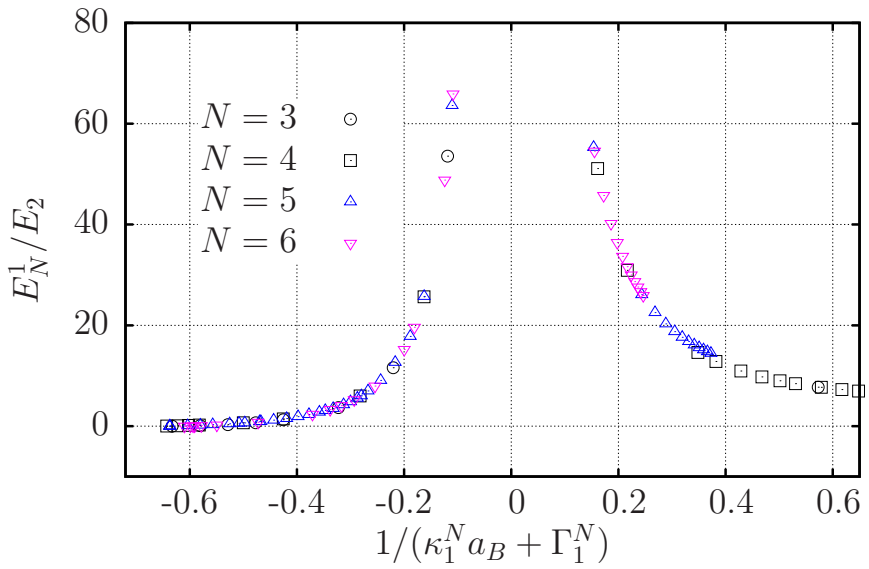

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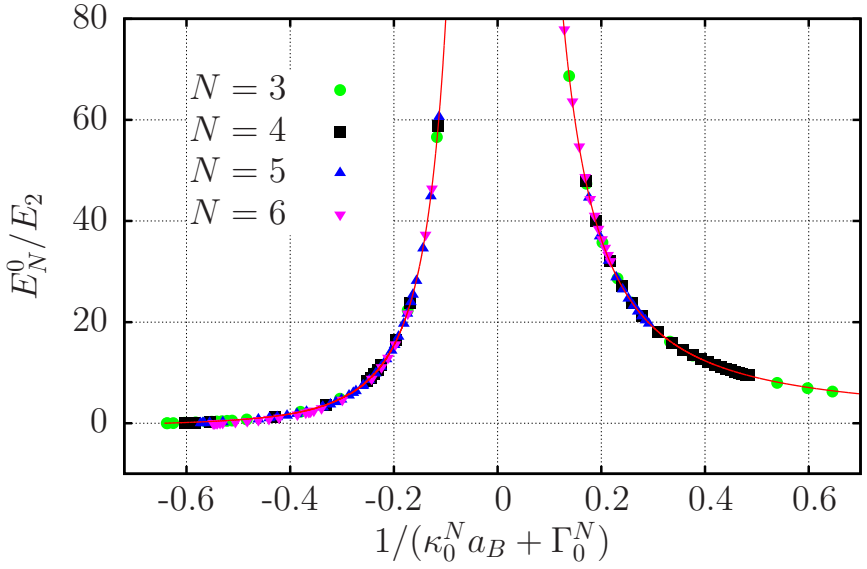
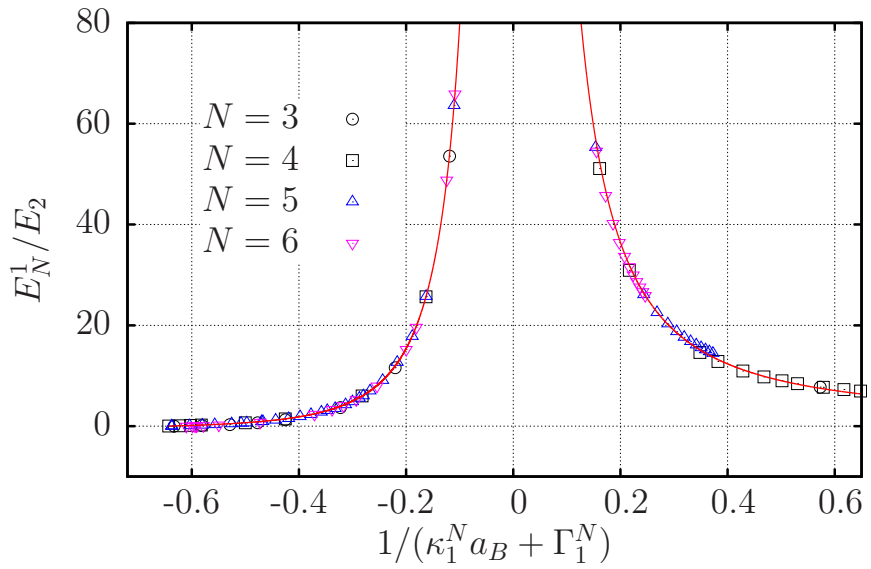


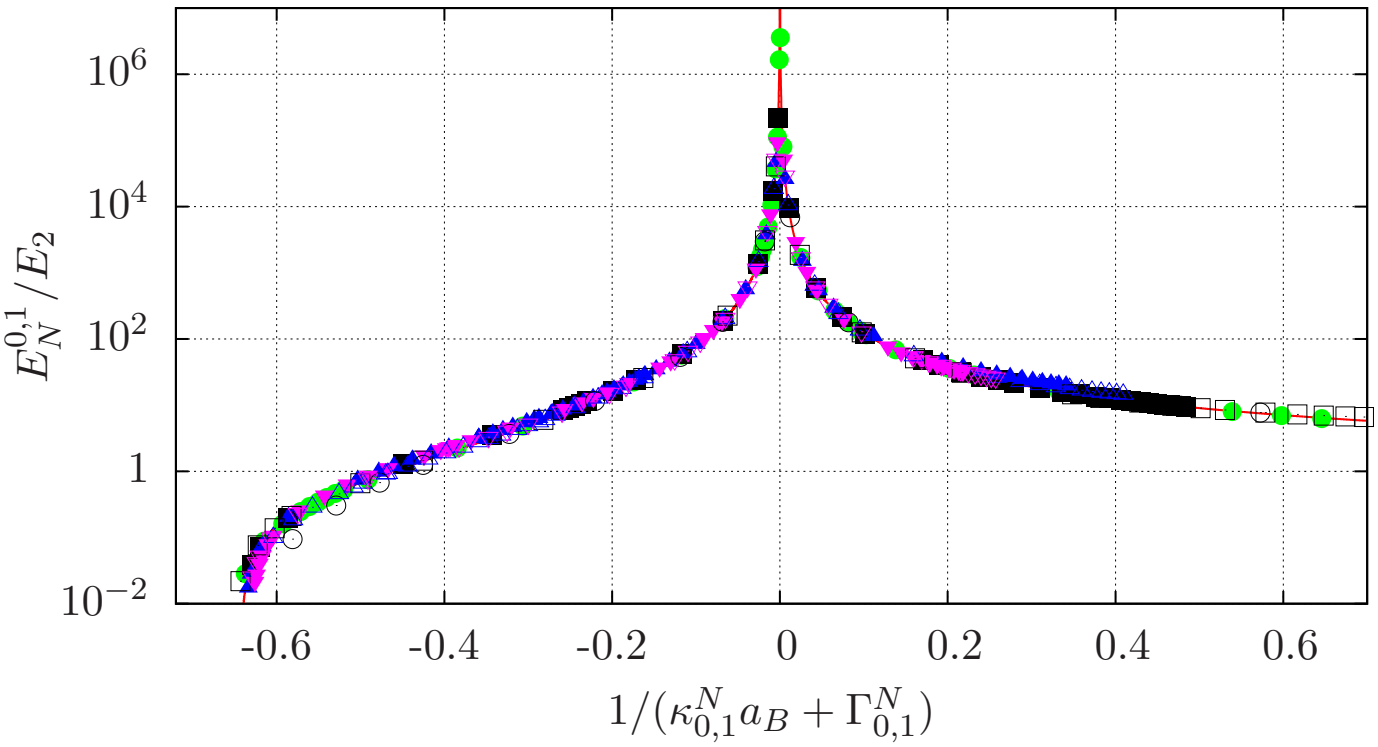


Universal Formula

$$E_N^n/E_2 = \tan^2 \xi$$

$$\kappa_n^N a_B + \Gamma_n^N = \frac{e^{-\Delta(\xi)/2s_0}}{\cos \xi}$$

Universality



Universal Formula

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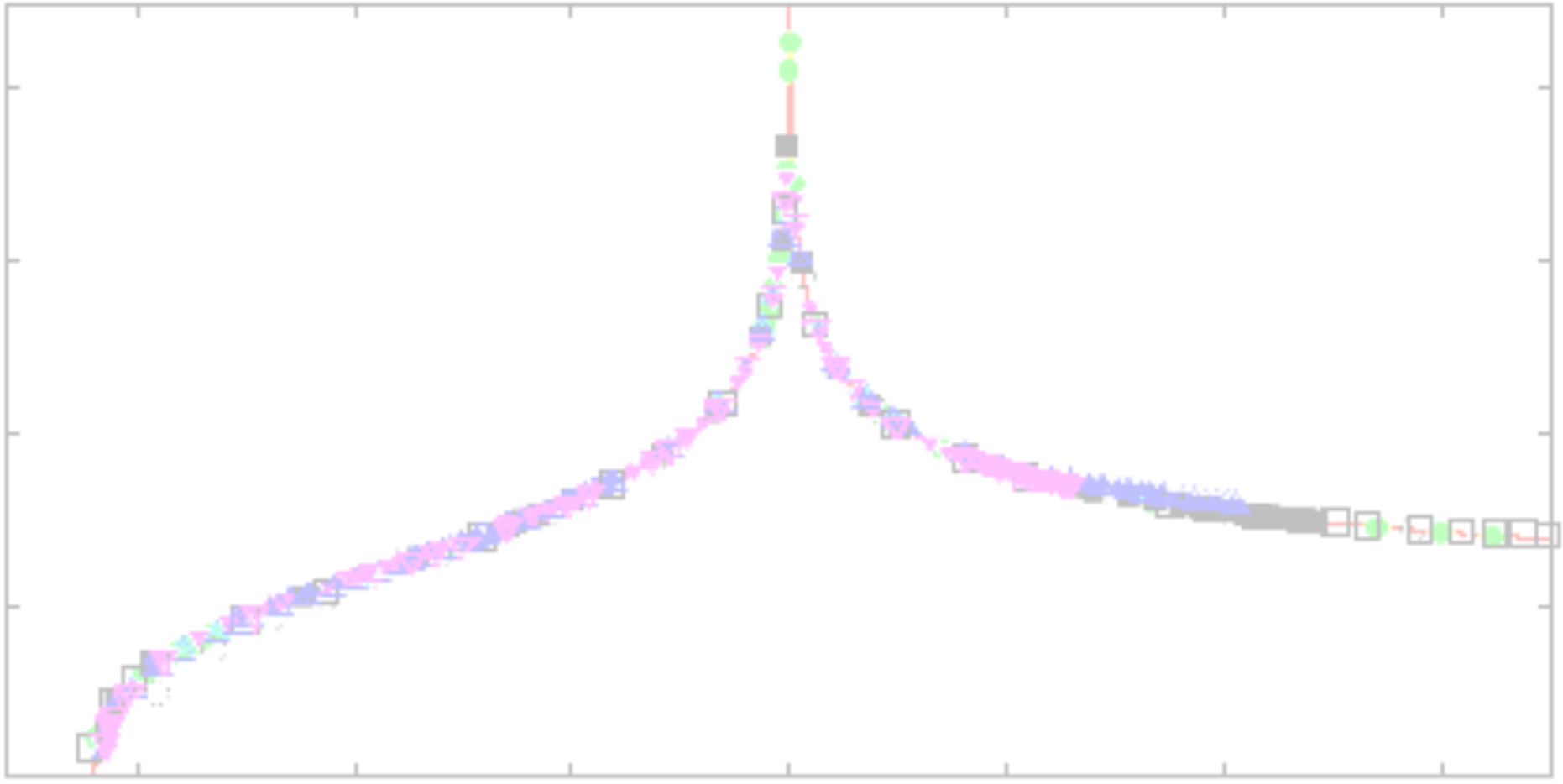

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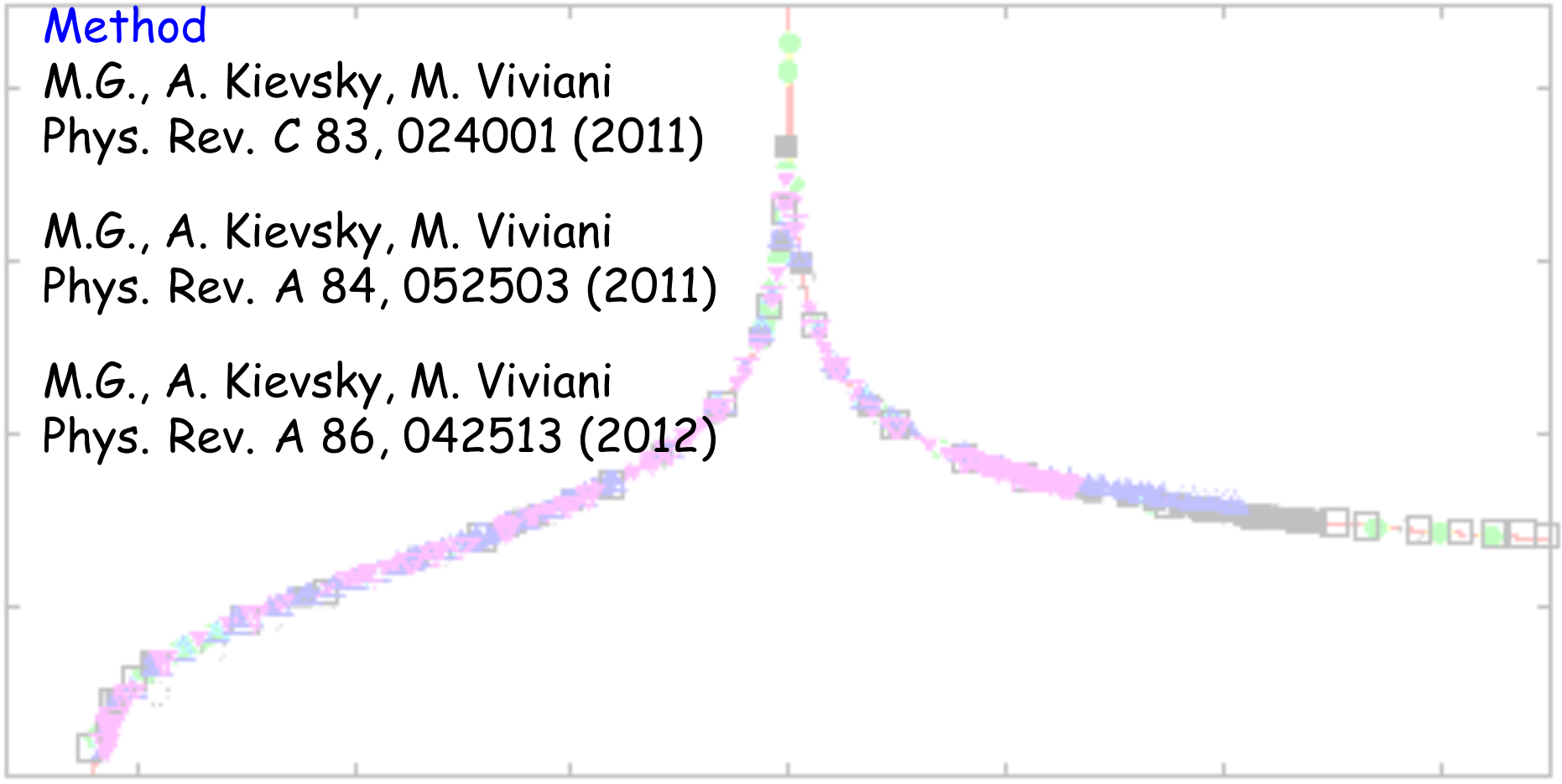

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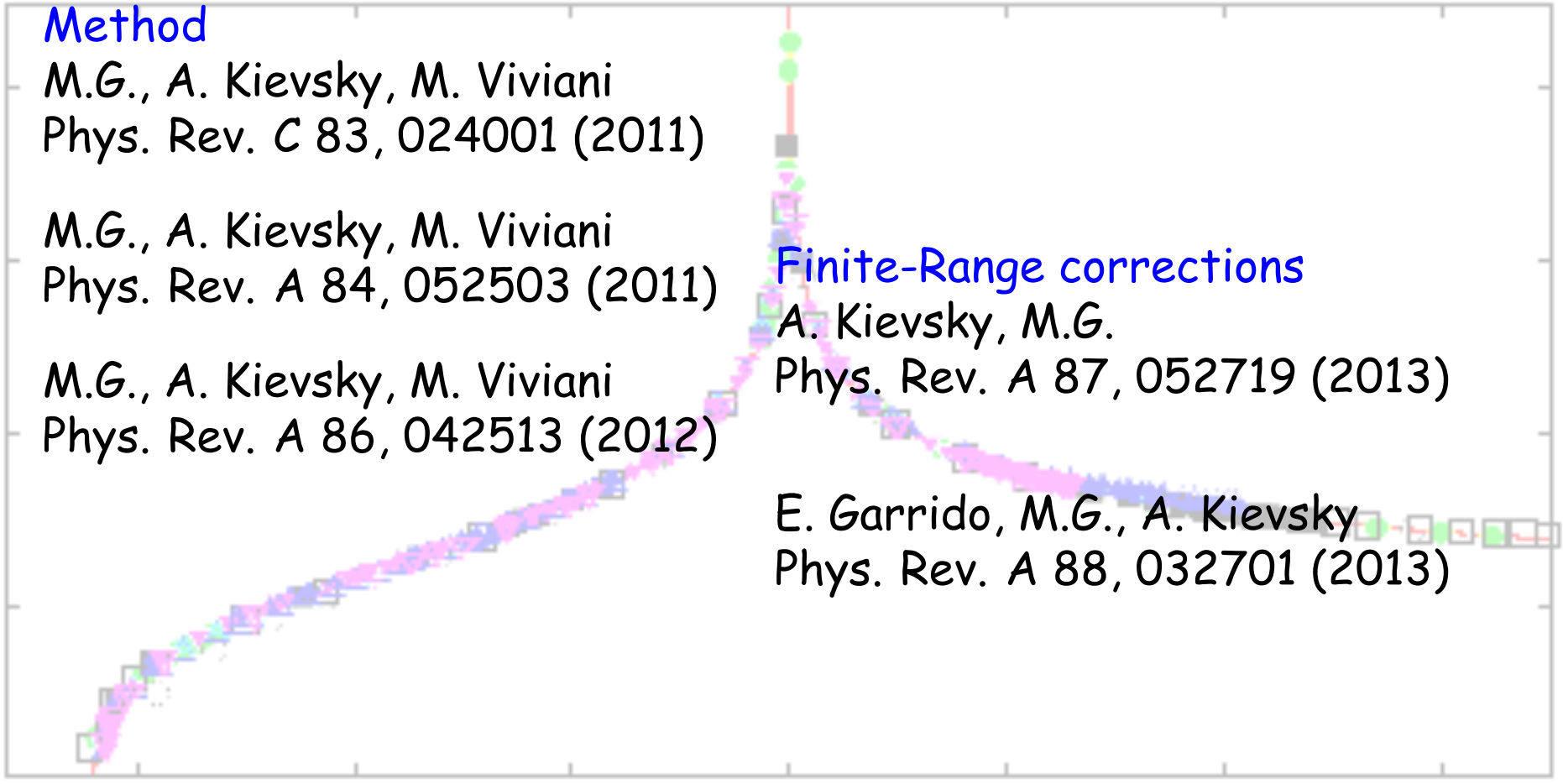

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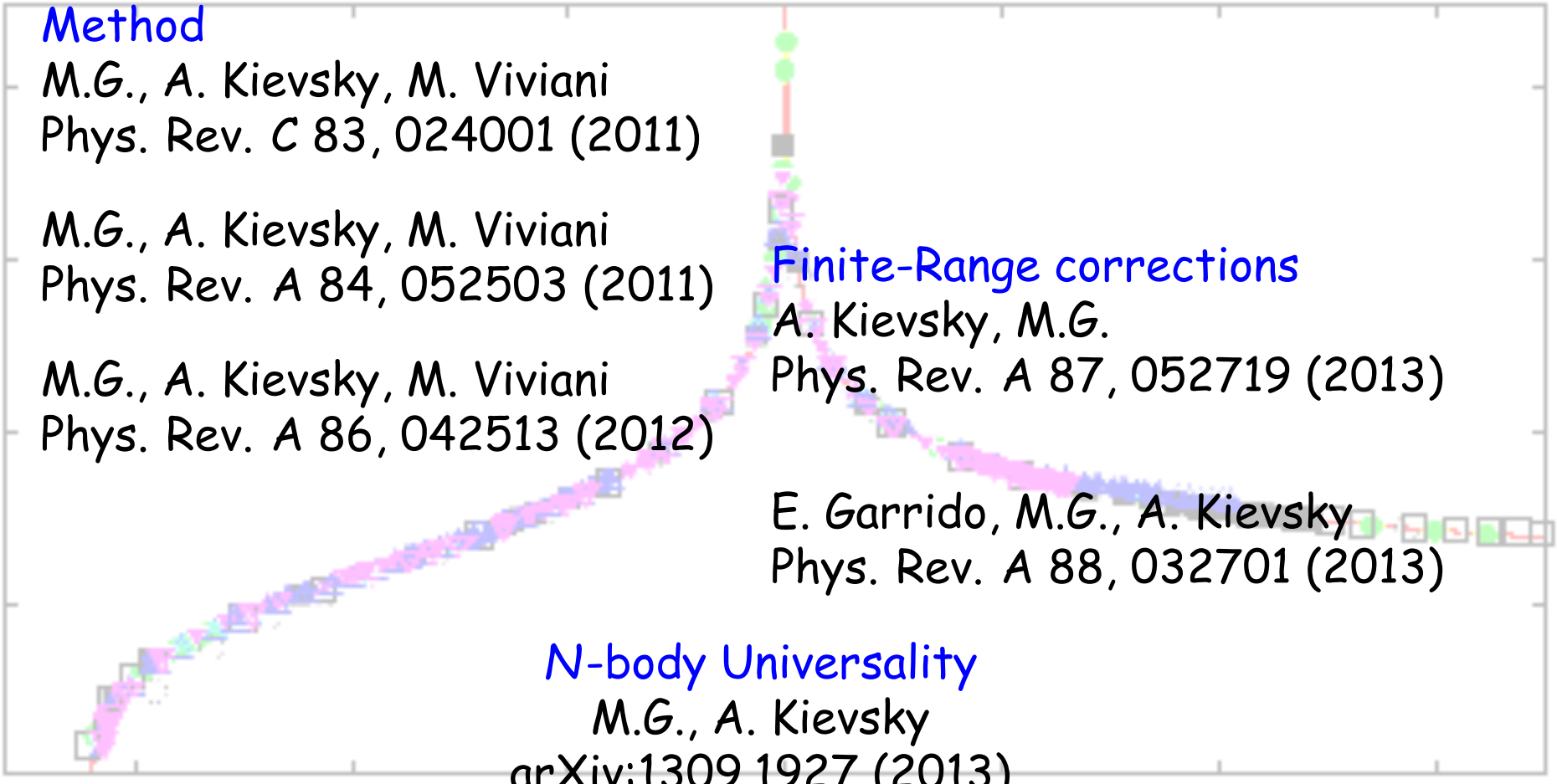

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arXiv:1309.1927 (2013)

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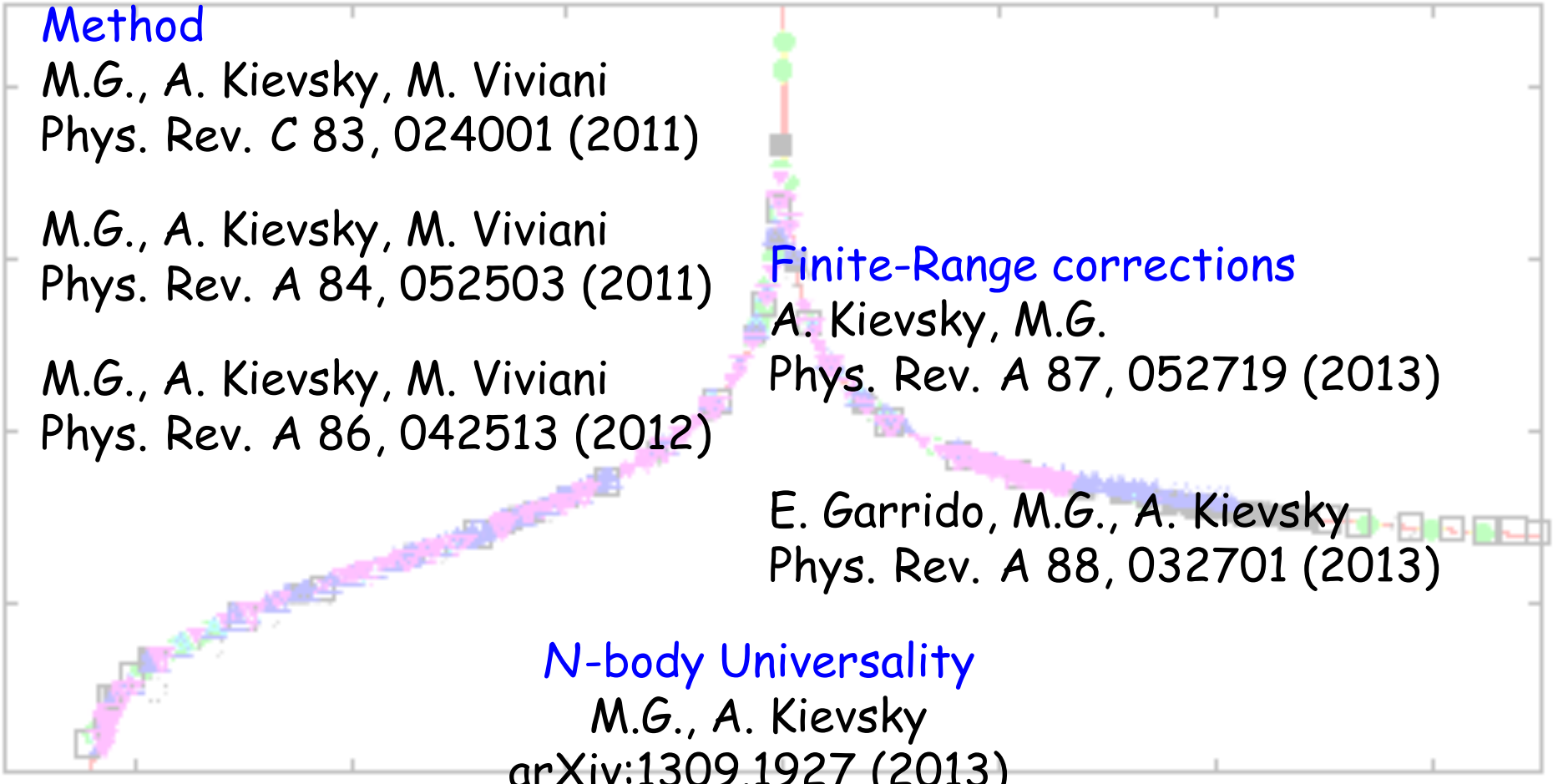

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Thanks!