

Transitions between rotational nuclear few-body states in the continuum



A.S. Jensen, D.V. Fedorov, Aarhus University, *Denmark*



E. Garrido, *IEM-CSIC, Madrid, Spain*

Krakow, September 2013

Cross sections after continuum to continuum γ -transitions:
Properties of the ${}^8\text{Be}$ spectrum.

- ✓ Rotational character of the ${}^8\text{Be}$ spectrum
- ✓ Dependence on the α - α potential



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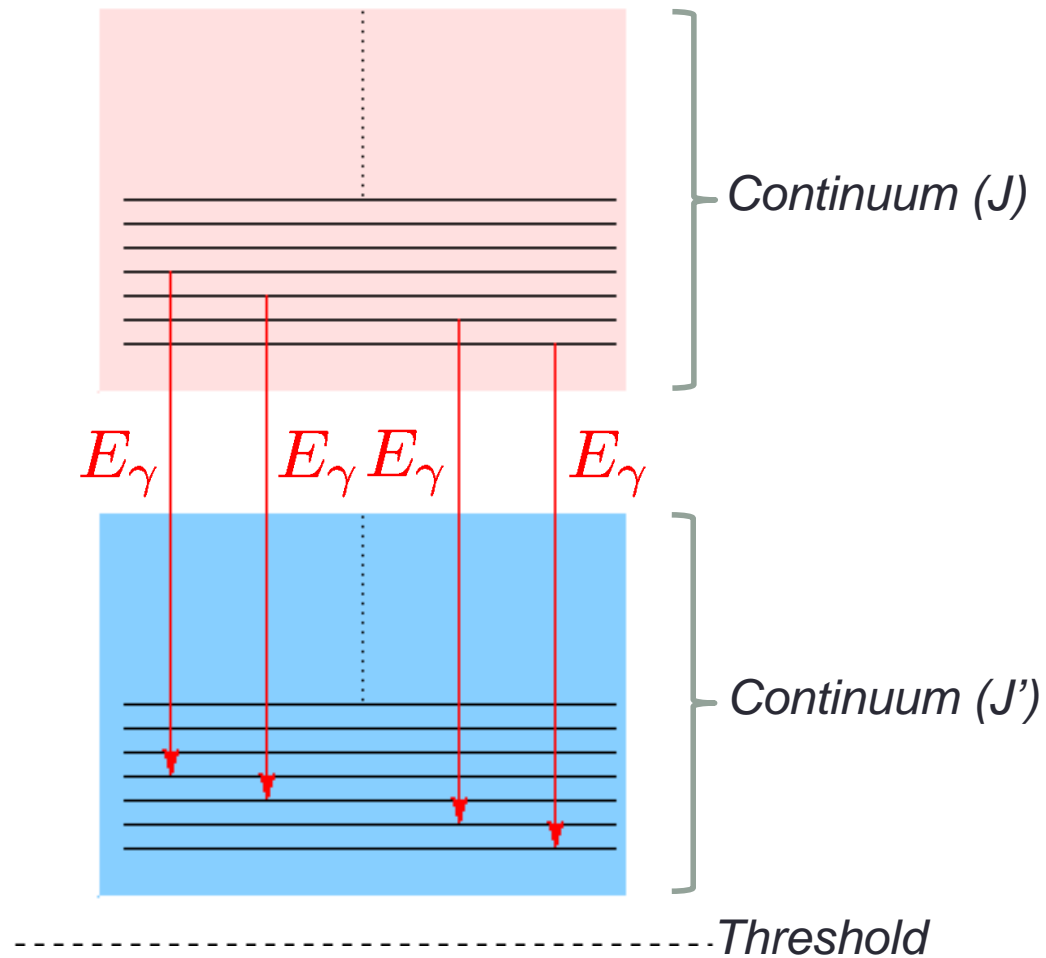


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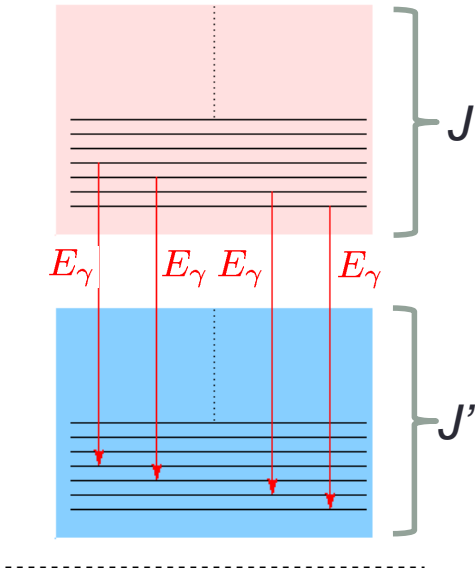
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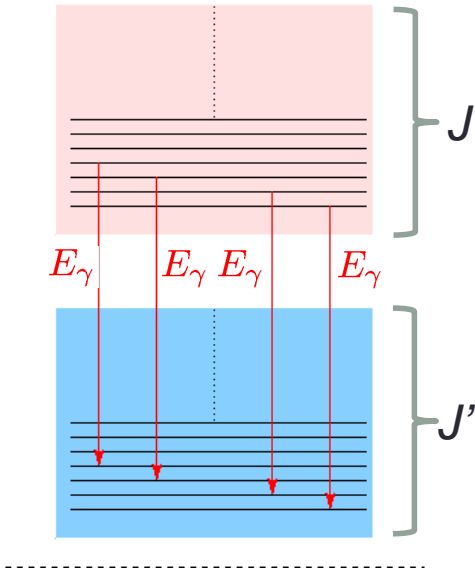
$$\int \Phi_J(E) \Phi_{J'}(\tilde{E}) d\tau = \delta(E - \tilde{E})$$

$$\left. \frac{d\mathcal{B}^{(\lambda)}}{dE dE'} \right|_{J \rightarrow J'} = \frac{1}{2J + 1} \left| \langle \Phi_J(E) | \hat{O}_\lambda | \Phi_{J'}(E') \rangle \right|^2$$

$$\frac{d\sigma_{r.c.}^{(\lambda)}}{dE'}(E) = \frac{2(2J + 1)}{(2J_a + 1)(2J_b + 1)} \frac{(2\pi)^3 (\lambda + 1)}{\lambda [(2\lambda + 1)!!]^2} \frac{1}{k^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2\lambda + 1} \left. \frac{d\mathcal{B}^{(\lambda)}}{dE dE'} \right|_{J \rightarrow J'}$$

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✓ A given value of E_γ can be associated to infinitely many continuum to continuum transitions.

✓ All final energies E' below the incident energy E are allowed.

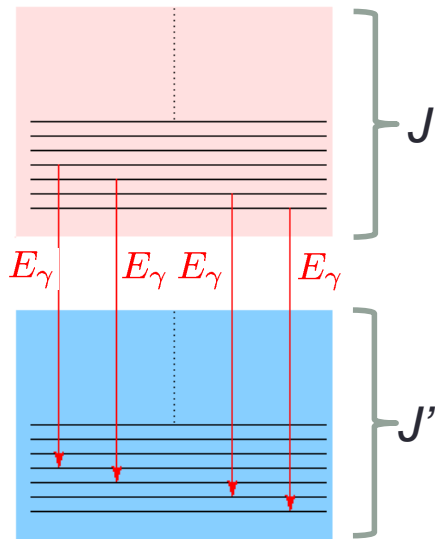
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^8Be : two-body α - α states

α - α interaction: Ali-Bodmer potential

Two-body Schrödinger Eq. + Complex scaling ($r \rightarrow re^{i\theta}$)



^8Be two-body resonances

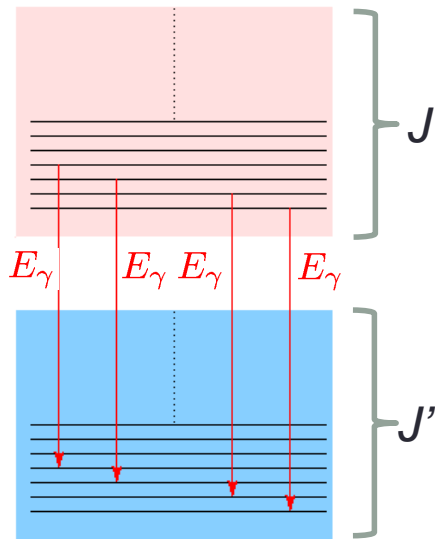
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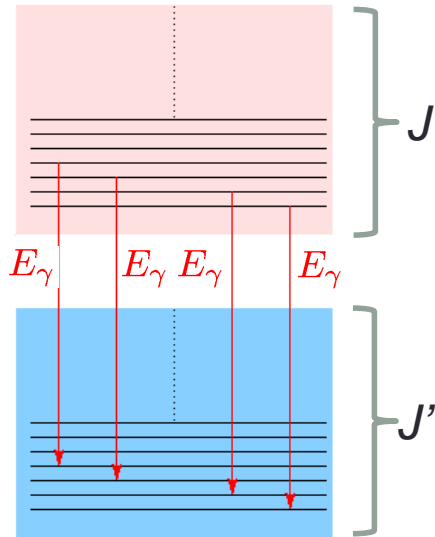
J^π	0^+	2^+	4^+	6^+	8^+
E_r (Exp.)	0.0918	2.94 ± 0.01	11.35 ± 0.15	—	—
Γ_r (Exp.)	$(5.57 \pm 0.25)10^{-6}$	1.51 ± 0.02	~ 3.5	—	—
E_r (MeV)	0.092	2.90	11.70	34.38	53.65
Γ_r (MeV)	$3.1 \cdot 10^{-6}$	1.27	3.07	37.19	93.74

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^8Be : two-body α - α states

$4^+ \rightarrow 2^+$ transition for given E and E'

$$\hat{O}_{E2} = \frac{Ze}{2} r^2 Y_2(\Omega) \longrightarrow \langle u_{4^+}(E, r) | r^2 | u_{2^+}(E', r) \rangle$$

$$\langle u_{4^+}(E, r) | r^2 | u_{2^+}(E', r) \rangle = \lim_{\eta \rightarrow 0} \langle u_{4^+}(E, r) | e^{-\eta^2 r^2} r^2 | u_{2^+}(E', r) \rangle$$

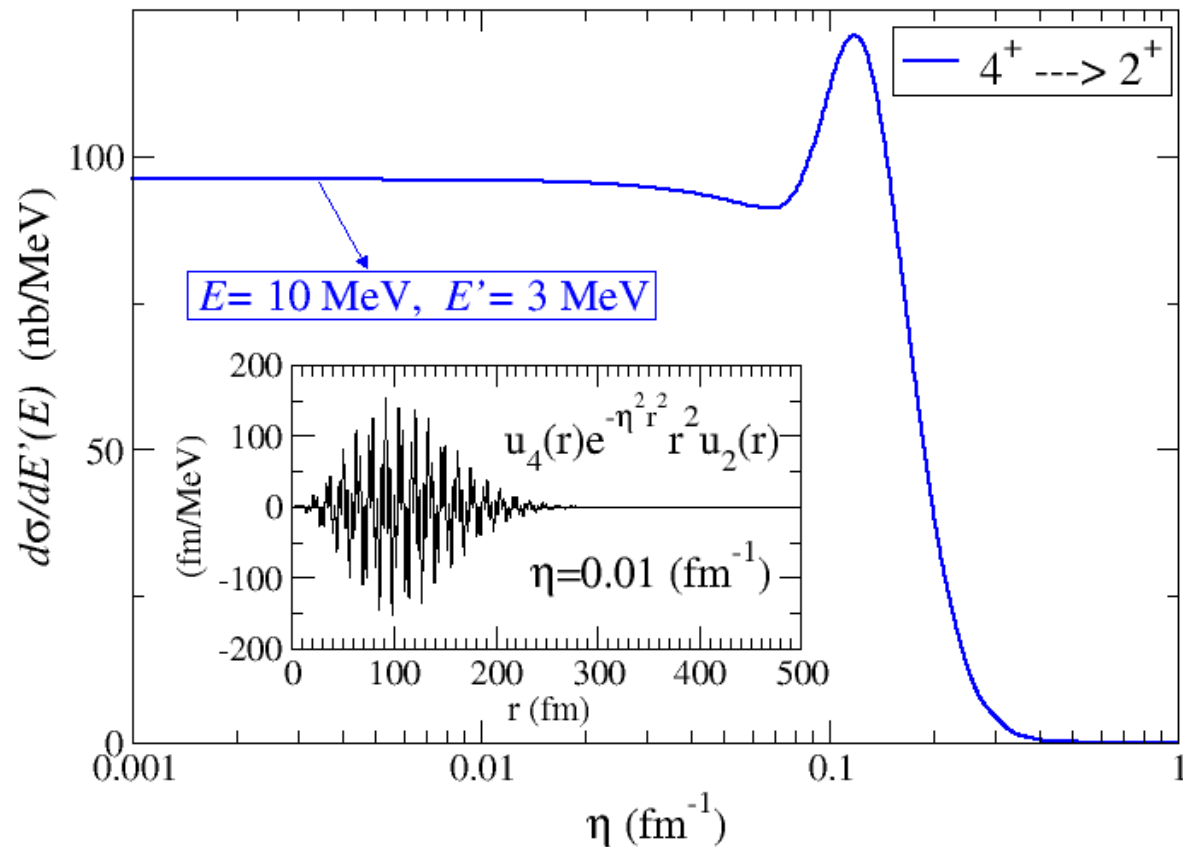
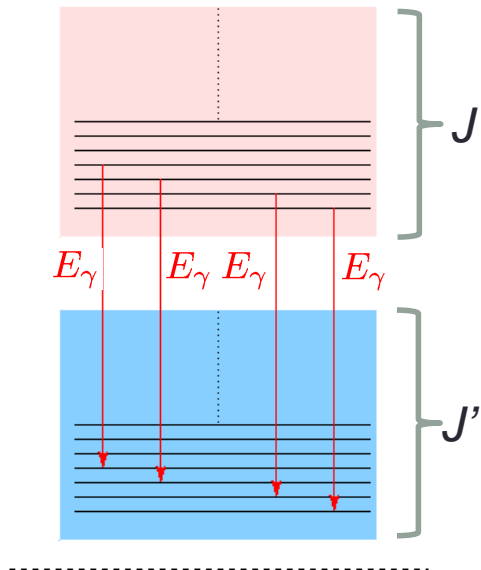
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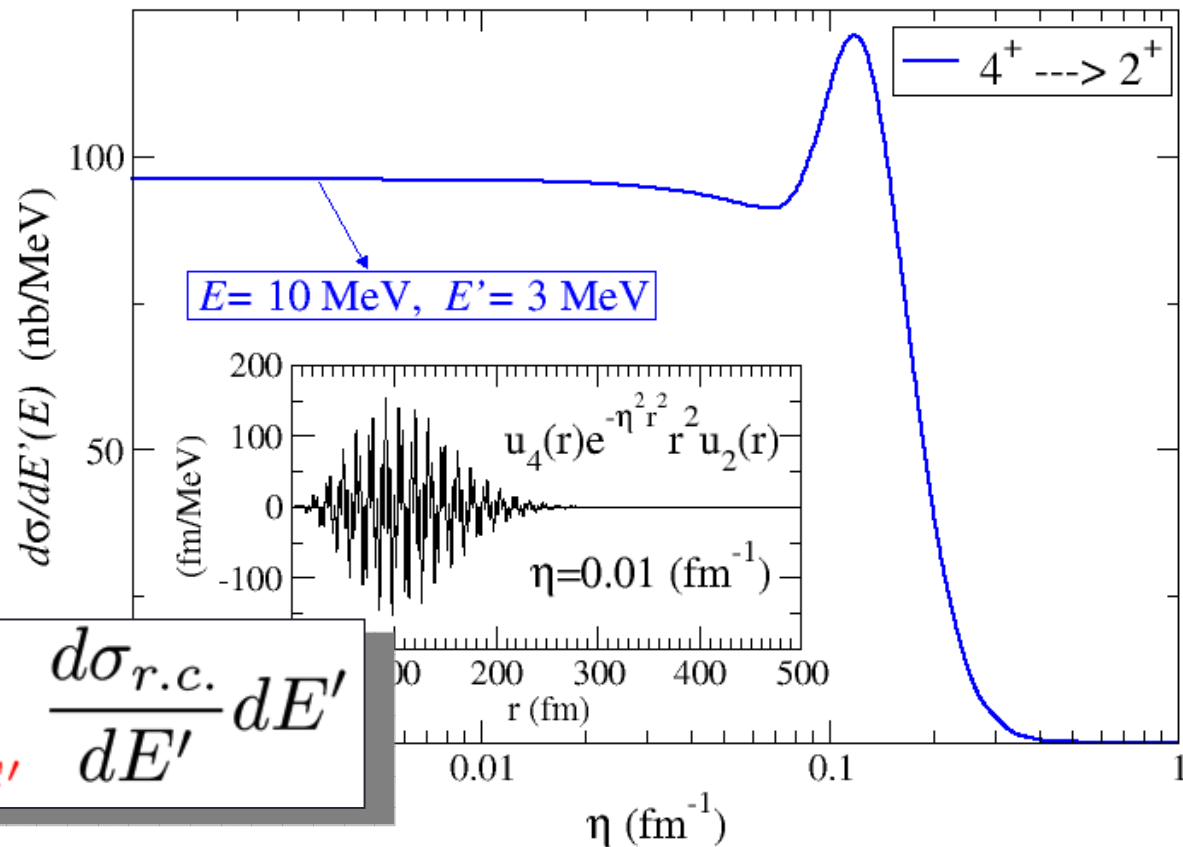
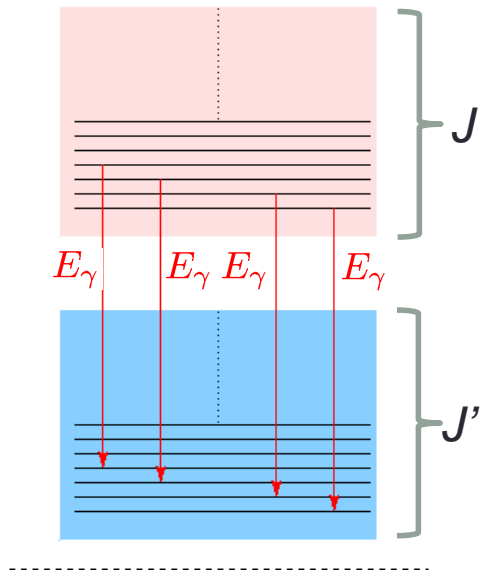


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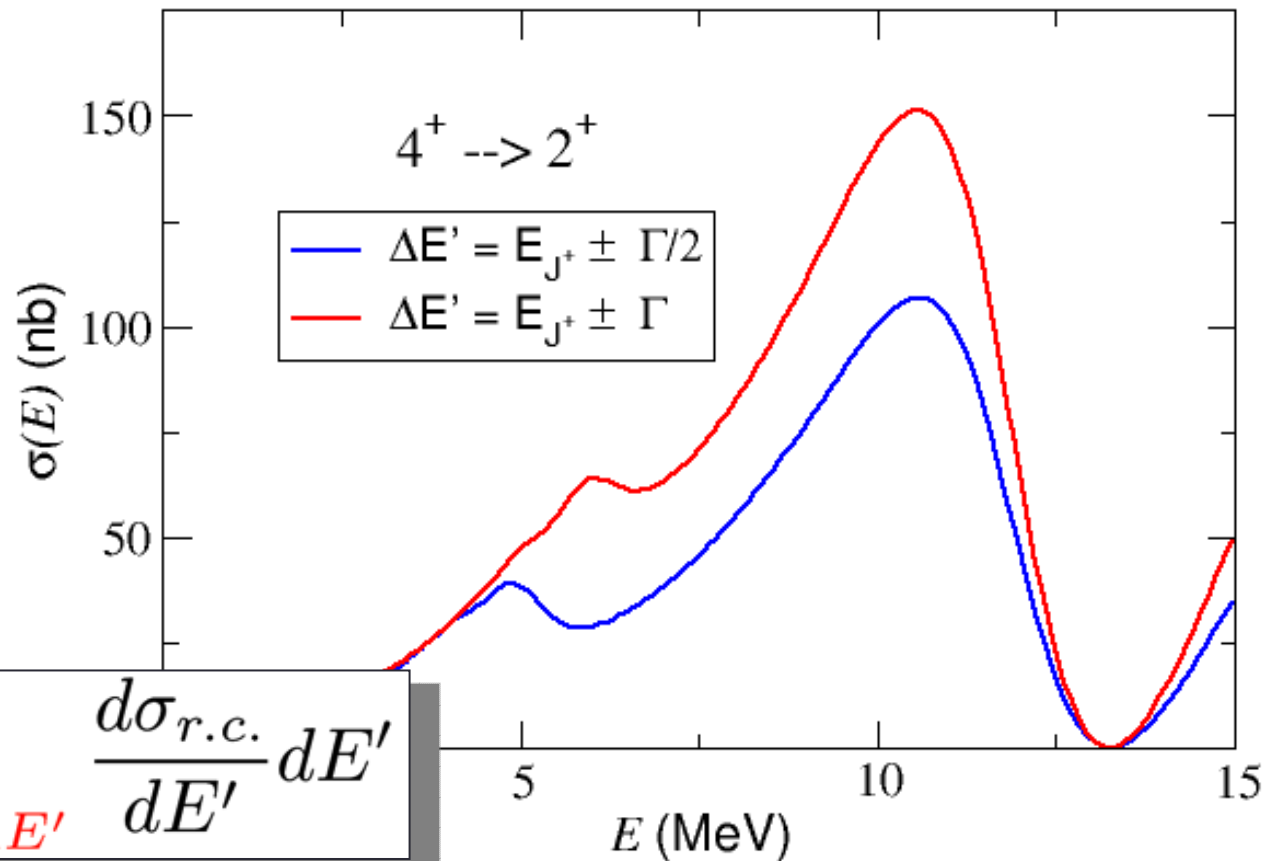
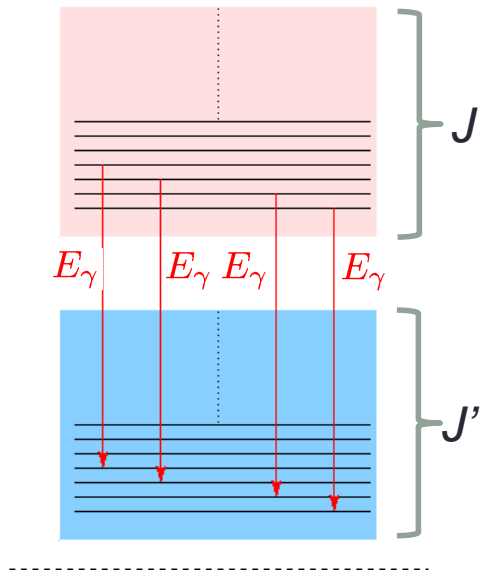
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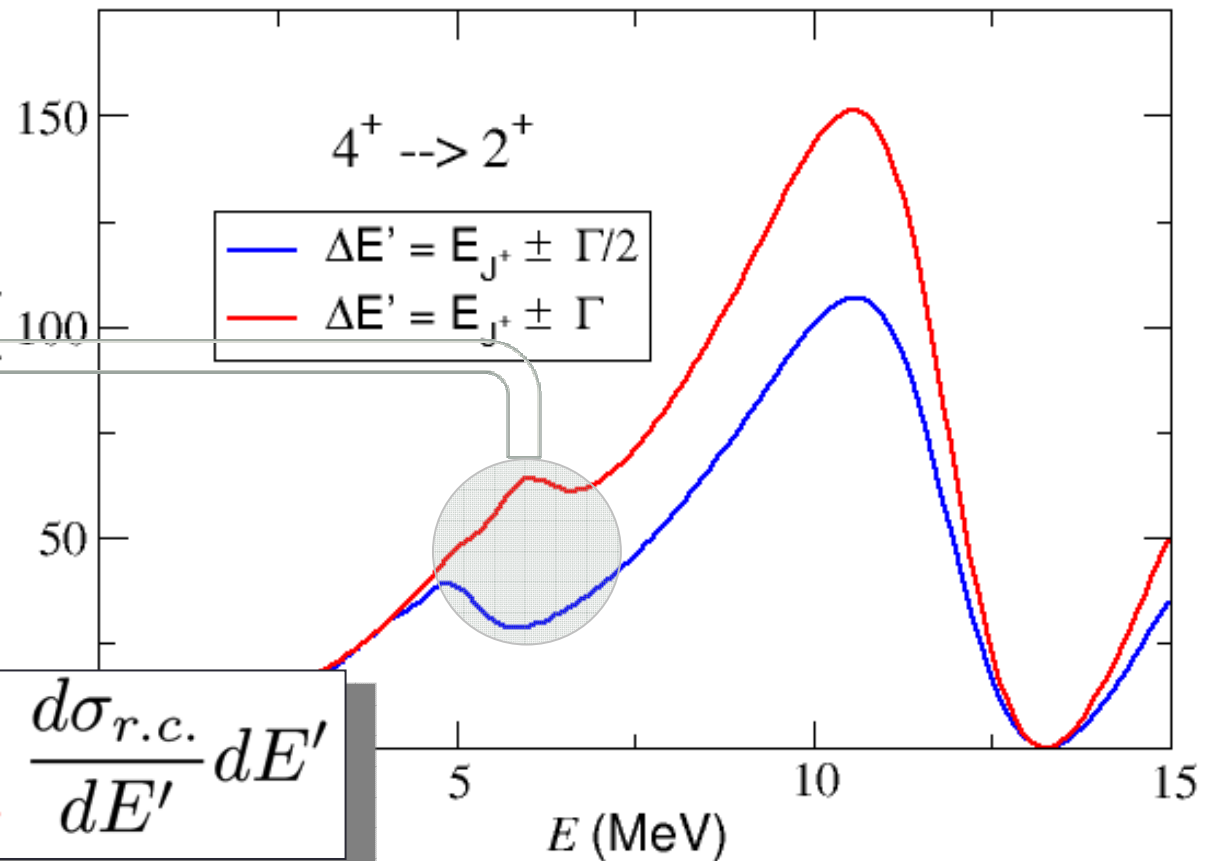
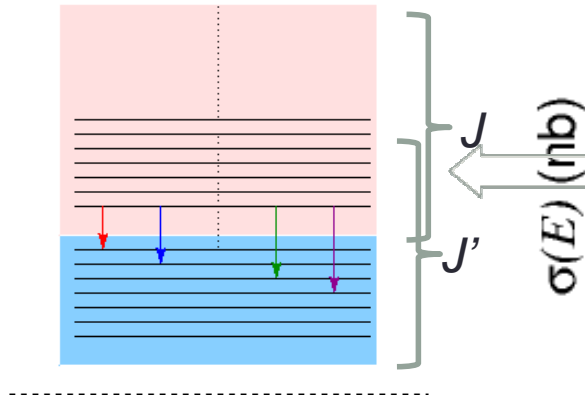
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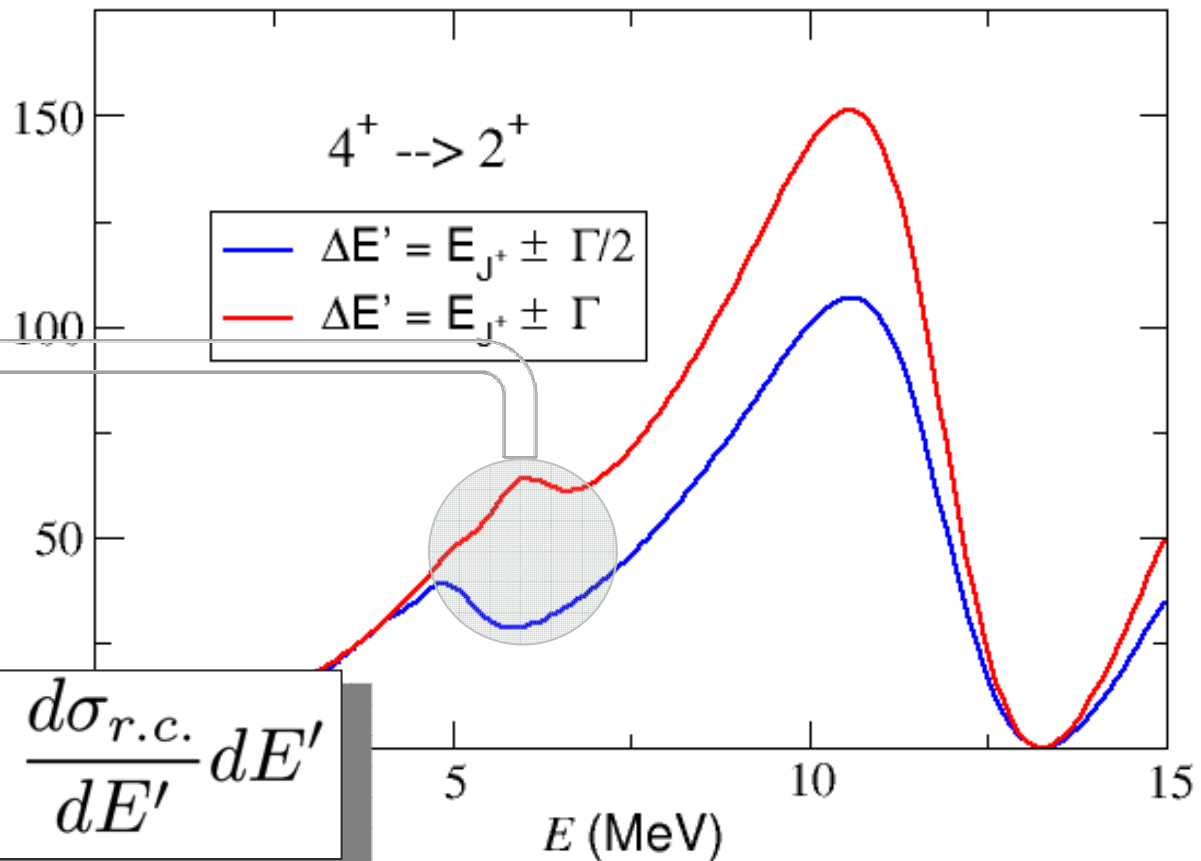
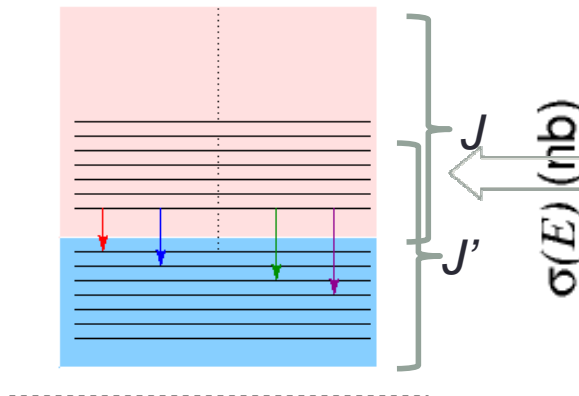
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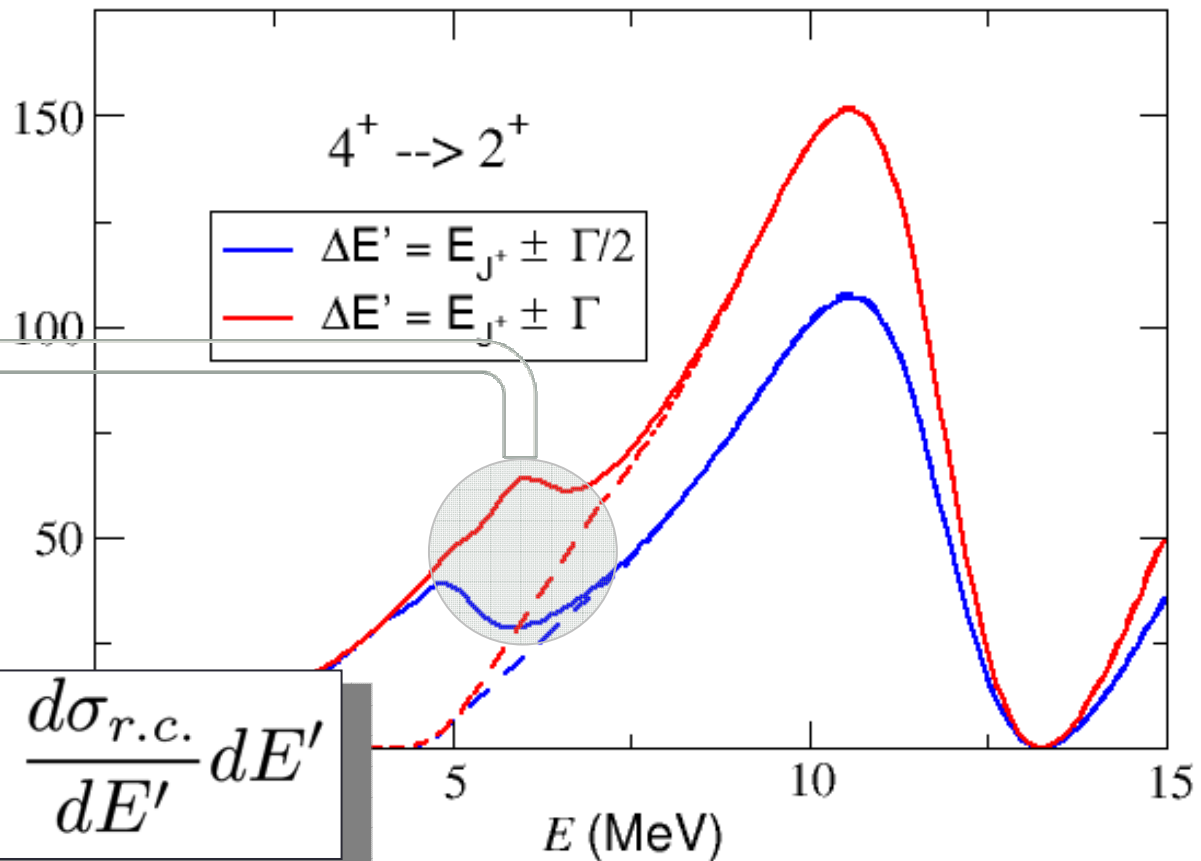
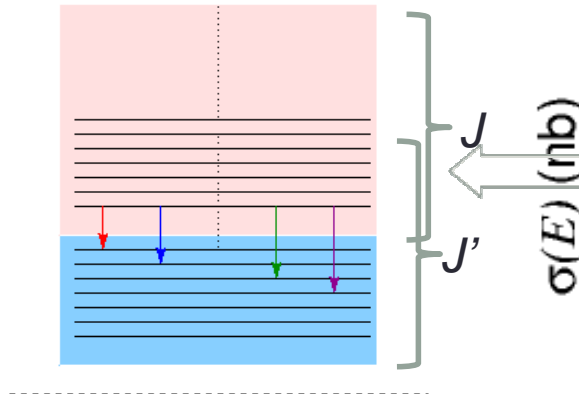
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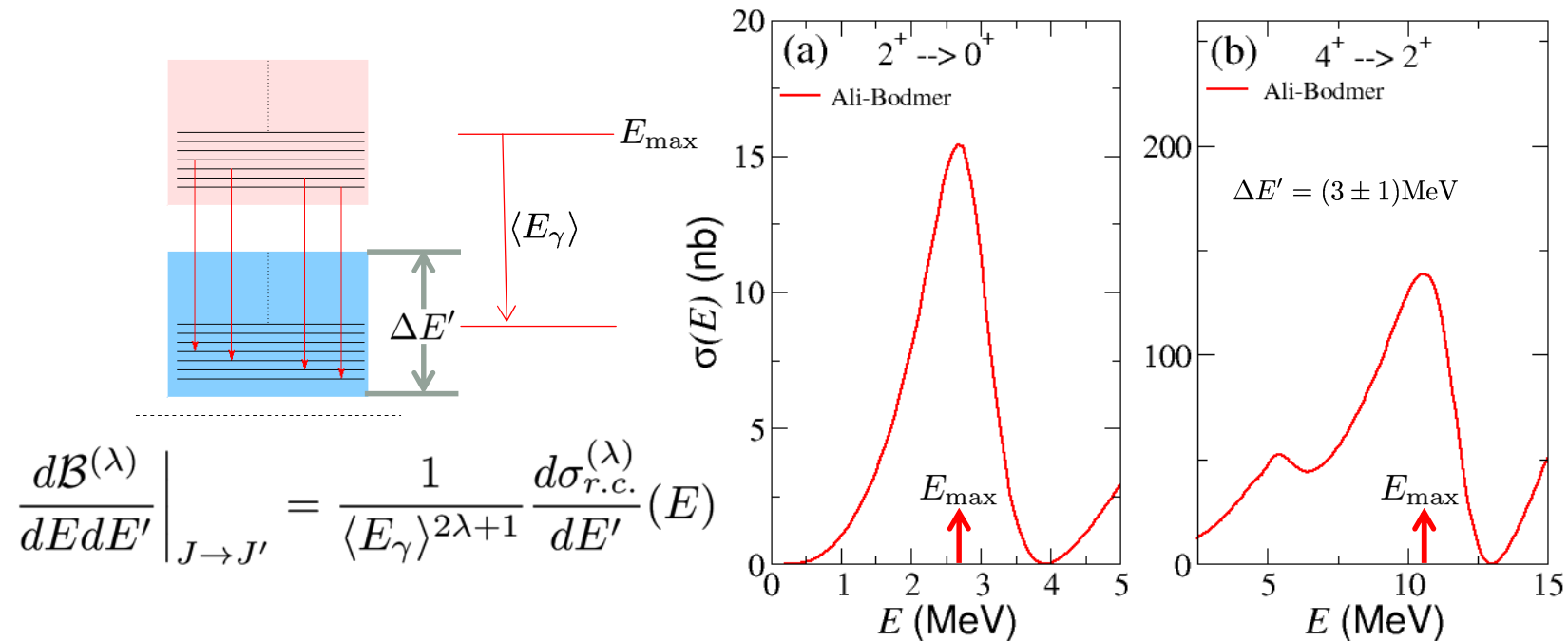
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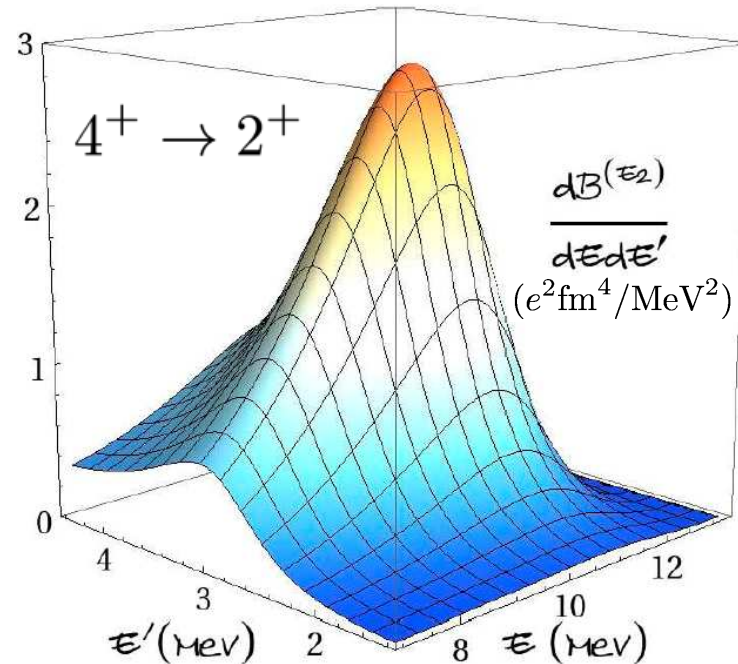
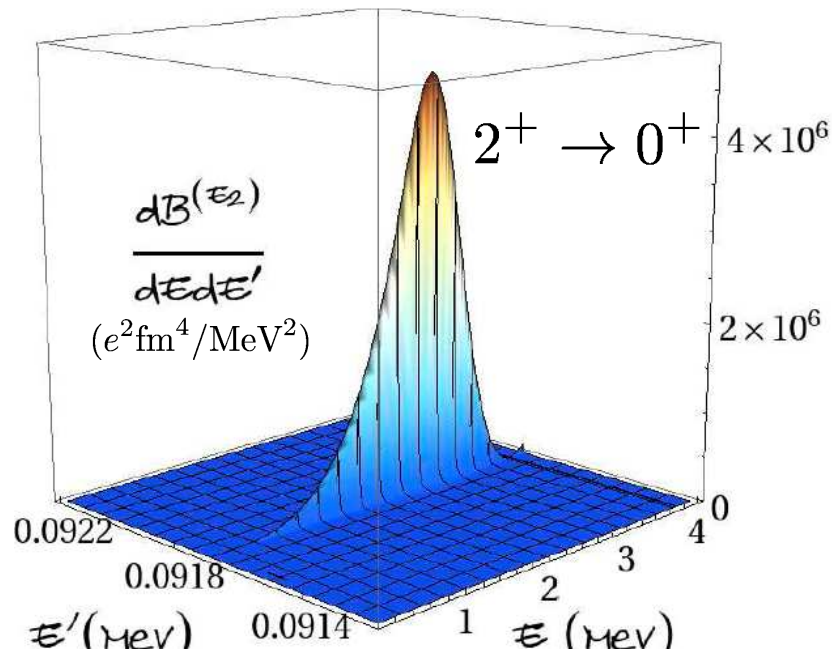
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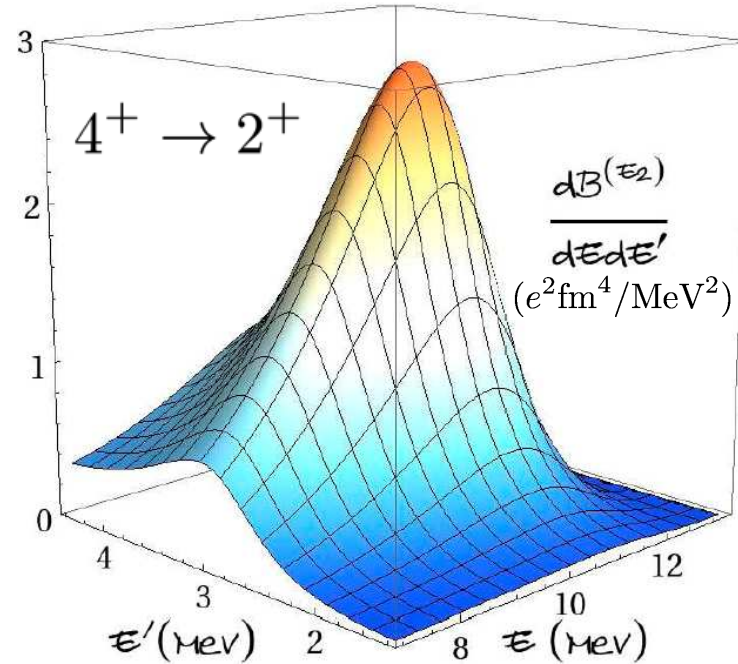
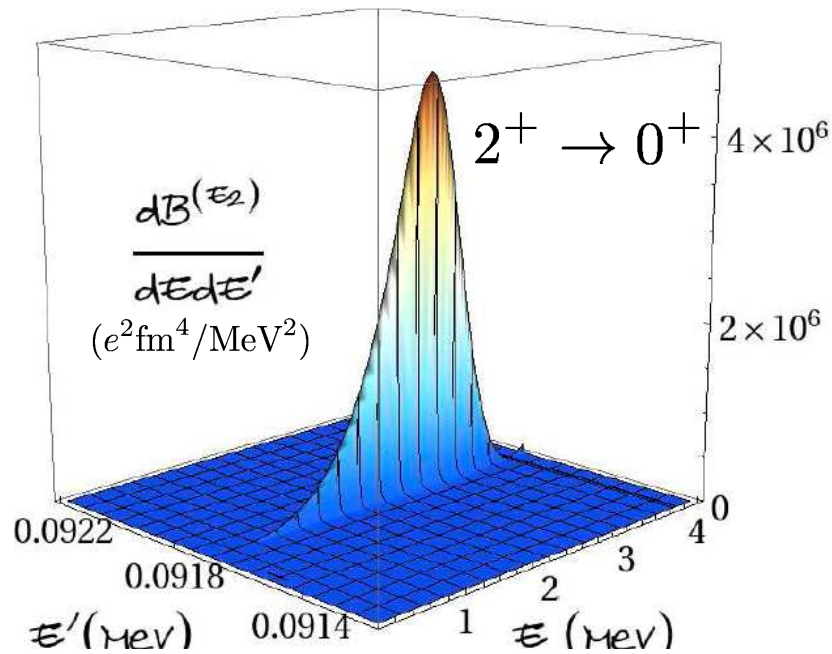
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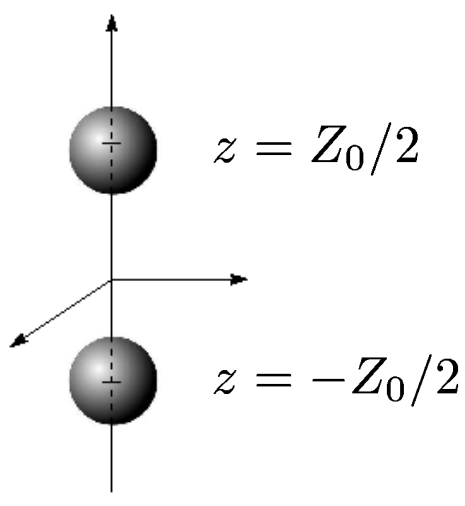
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$B^{(E2)}$ ($e^2 \text{fm}^4$)	$E'_r \pm \Gamma'_r / 2$ $B^{(E2)}$	$E'_r \pm \Gamma'_r$ $B^{(E2)}$	[K. Langanke] (Cluster)	[R.B. Wiringa] (QMC)	[V.M. Datar <i>et al.</i>] (PRL 111 (2013) 062502)
$2^+ \rightarrow 0^+$	53.4	79.1	71.3	14.8	—
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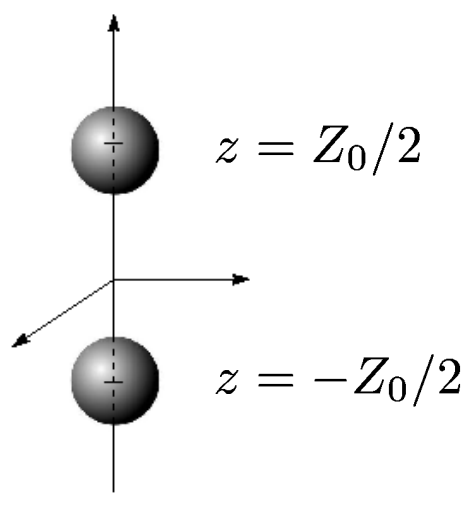
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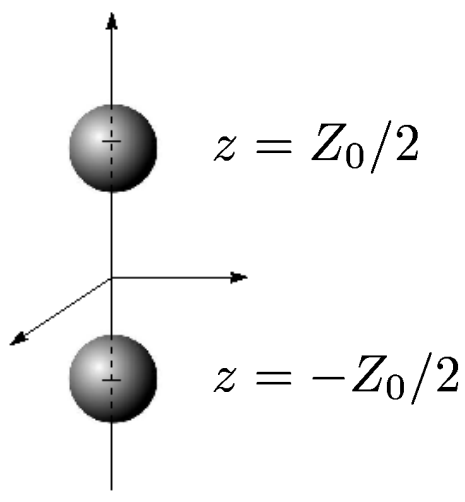
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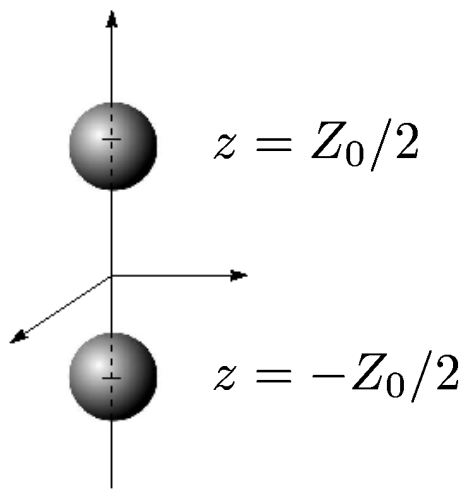


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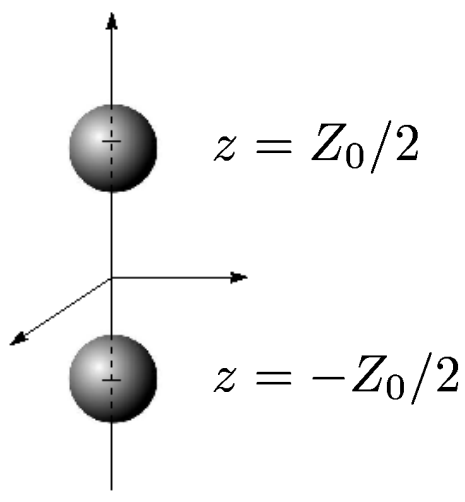


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$4^+ \rightarrow 2^+$	15.5 (0.29)	22.1 (0.28)	18.0 (0.25)	9.2 (1.43)
$6^+ \rightarrow 4^+$	6.7 (0.13)	10.1 (0.13)	–	10.1 (1.57)
$8^+ \rightarrow 6^+$	6.6 (0.12)	13.0 (0.16)	–	10.6 (1.65)

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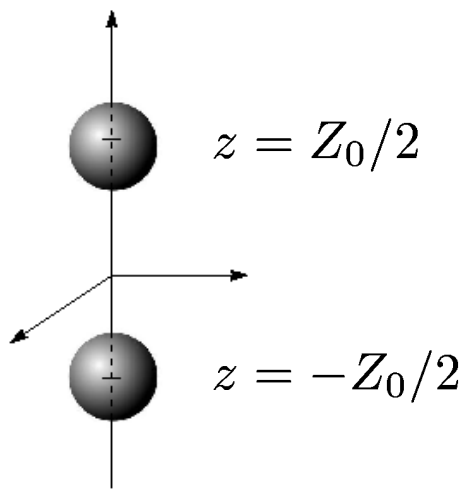
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J^π	0^+	2^+	4^+	6^+	8^+
$\text{Re}\sqrt{\langle r^2 \rangle}$ (fm)	5.80	3.58	2.91	2.70	2.73
$\text{Im}\sqrt{\langle r^2 \rangle}$ (fm)	0.001	1.24	0.76	1.40	1.73

$\mathcal{B}^{(E2)}$ ($e^2\text{fm}^4$)	$E_r' \pm \Gamma_r'/2$ $\mathcal{B}^{(E2)}$	$E_r' \pm \Gamma_r'$ $\mathcal{B}^{(E2)}$	[K. Langanke] (Cluster)	Rotational model $Z_0 = 3$ fm
$2^+ \rightarrow 0^+$	53.4 (1)	79.1 (1)	71.3 (1)	6.4 (1)
$4^+ \rightarrow 2^+$	15.5 (0.29)	22.1 (0.28)	18.0 (0.25)	9.2 (1.43)
$6^+ \rightarrow 4^+$	6.7 (0.13)	10.1 (0.13)	–	10.1 (1.57)
$8^+ \rightarrow 6^+$	6.6 (0.12)	13.0 (0.16)	–	10.6 (1.65)

Transition strengths: Rotational band in ${}^8\text{Be}$?



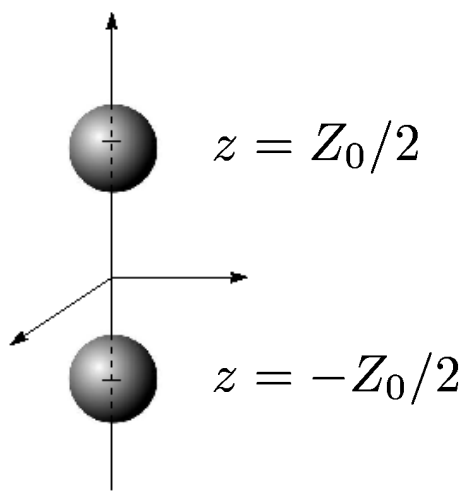
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	$\mathcal{B}^{(E2)}$	$\mathcal{B}^{(E2)}$		$Z_0 = 3 \text{ fm}$	$Z_0 = \text{Re}\sqrt{\langle r^2 \rangle}$
$2^+ \rightarrow 0^+$	53.4 (1)	79.1 (1)	71.3 (1)	6.4 (1)	84.0 (1)
$4^+ \rightarrow 2^+$	15.5 (0.29)	22.1 (0.28)	18.0 (0.25)	9.2 (1.43)	18.1 (0.22)
$6^+ \rightarrow 4^+$	6.7 (0.13)	10.1 (0.13)	–	10.1 (1.57)	9.1 (0.11)
$8^+ \rightarrow 6^+$	6.6 (0.12)	13.0 (0.16)	–	10.6 (1.65)	7.6 (0.09)

Transition strengths: Rotational band in ${}^8\text{Be}$?



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- ✓ The idea of ${}^8\text{Be}$ as a rigid rotor of two α -particles at a given distance is questionable.
- ✓ The intrinsic α -particle structure is maintained, although the particle separation changes.

$\mathcal{B}^{(E2)}$ ($e^2\text{fm}^4$)	$E_r' \pm \Gamma_r'/2$	$E_r' \pm \Gamma_r'$	[K. Langanke] (Cluster)	Rotational model	
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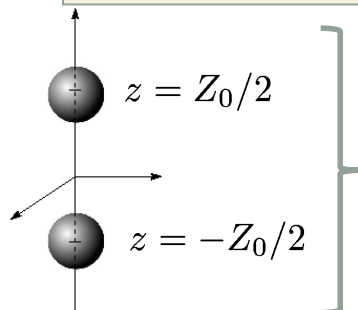
Transitions between rotational nuclear few-body states in the continuum

Rotational band in ${}^8\text{Be}$?

$$E_J = E_0 + \frac{\hbar^2}{2\mathcal{I}} J(J+1)?$$

J^π	0^+	2^+	4^+	6^+	8^+
E_r (Exp.)	0.0918	2.94 ± 0.01	11.35 ± 0.15	—	—
Γ_r (Exp.)	$(5.57 \pm 0.25)10^{-6}$	1.51 ± 0.02	~ 3.5	—	—
E_r (Buck)	0.091	2.88	11.78	33.55	51.56
Γ_r (Buck)	$3.6 \cdot 10^{-5}$	1.24	3.57	37.38	92.38
E_r (A.-B.)	0.092	2.90	11.70	34.38	53.65
Γ_r (A.-B.)	$3.1 \cdot 10^{-6}$	1.27	3.07	37.19	93.74
$E_r^{(1)} = E_0 + B_1 J(J+1)$	$E_0 = 0.091$	3.8	12.5	26.2	44.8

- ✓ The idea of ${}^8\text{Be}$ as a rigid rotor of two α -particles at a given distance is questionable.
- ✓ The intrinsic α -particle structure is maintained, although the particle separation changes.



$$\left. \begin{array}{l} z = Z_0/2 \\ z = -Z_0/2 \end{array} \right\} \begin{array}{l} \mathcal{I}_{rig} = \frac{1}{2} m_\alpha Z_0^2 + \frac{4}{5} m_\alpha R_\alpha^2 \\ R_\alpha^2 = \frac{5}{3} \langle r_\alpha^2 \rangle \quad \langle r_\alpha^2 \rangle^{1/2} \approx 1.7 \text{ fm} \end{array} \left\{ \begin{array}{l} Z_0 = 3 \text{ fm} \\ B_1 = \frac{\hbar^2}{2\mathcal{I}_{rig}} = 0.621 \text{ MeV} \end{array} \right.$$

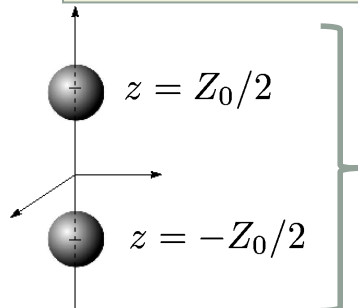
Transitions between rotational nuclear few-body states in the continuum

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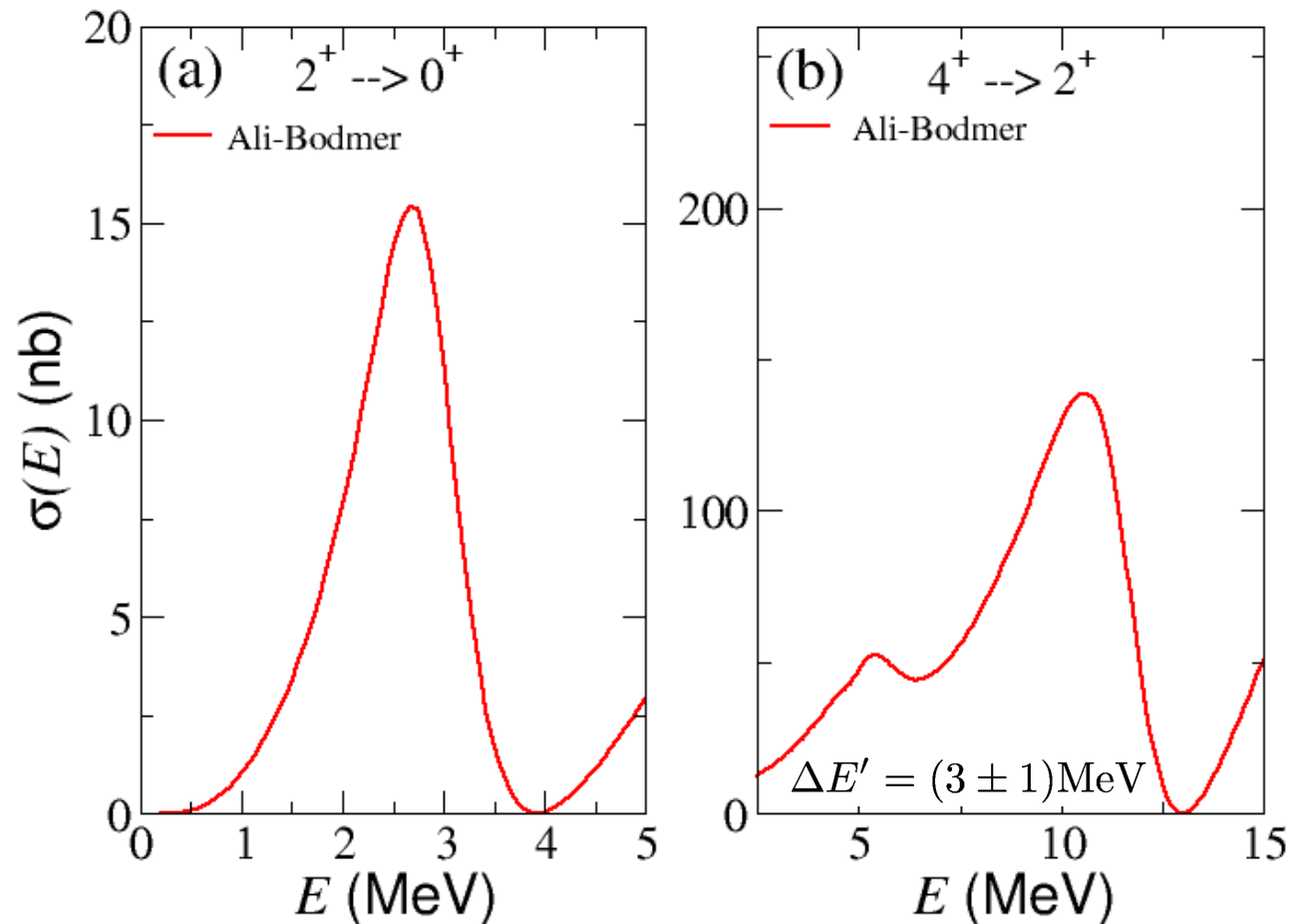
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$E_r^{(1)} = E_0 + B_1 J(J+1)$	$E_0 = 0.091$	3.8	12.5	26.2	44.8
$E_r^{(Z_0)} = E_0 + B_{Z_0} J(J+1)$	$E_0 = 0.091$	3.2	12.9	28.5	49.1

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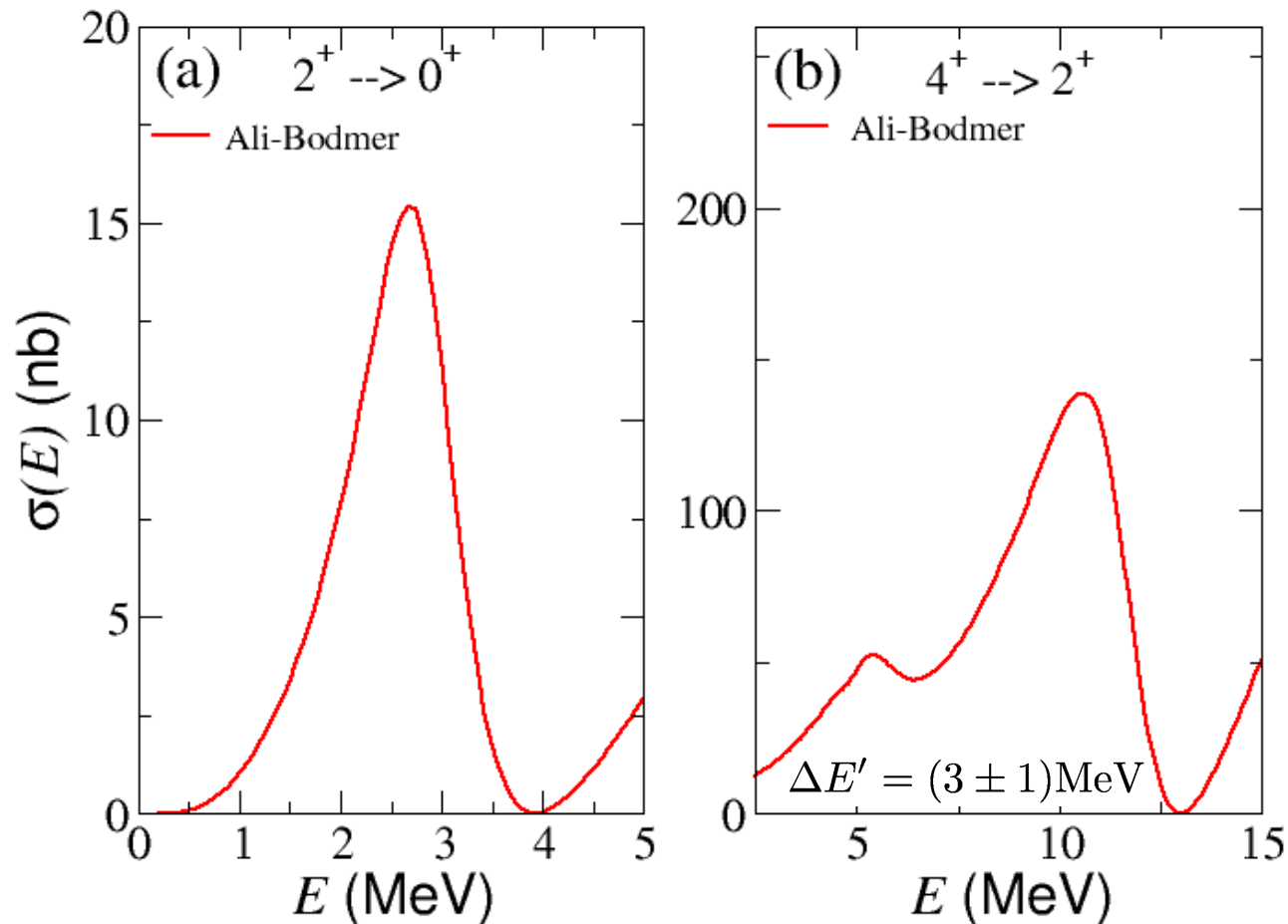
Transitions between rotational nuclear few-body states in the continuum



Ali-Bodmer potential

- ✓ Gaussian potential
- ✓ Partial wave dependent

Transitions between rotational nuclear few-body states in the continuum



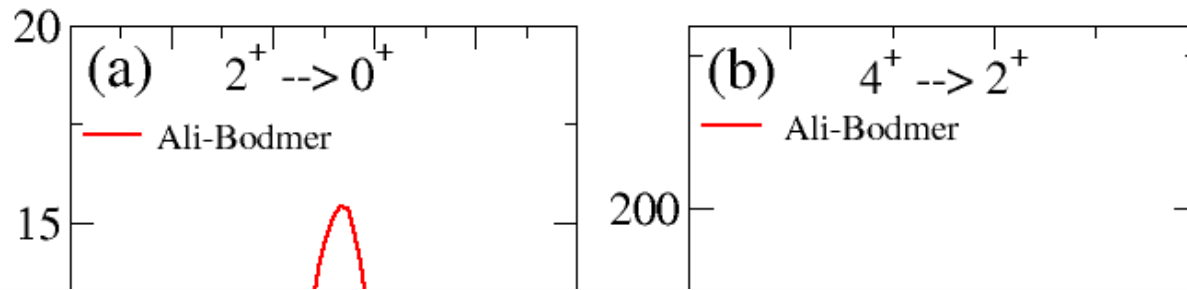
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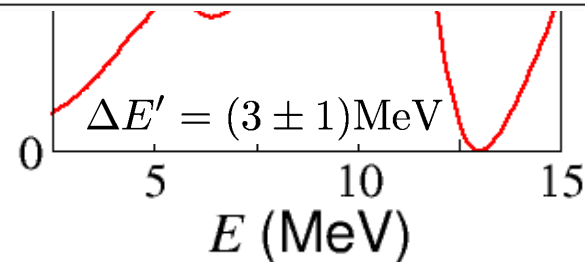
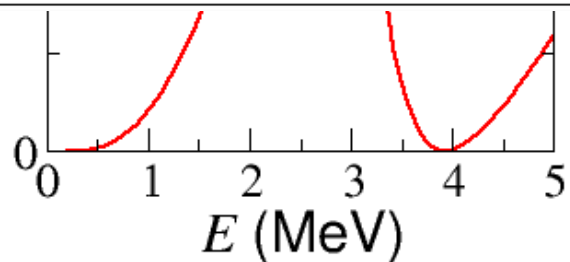
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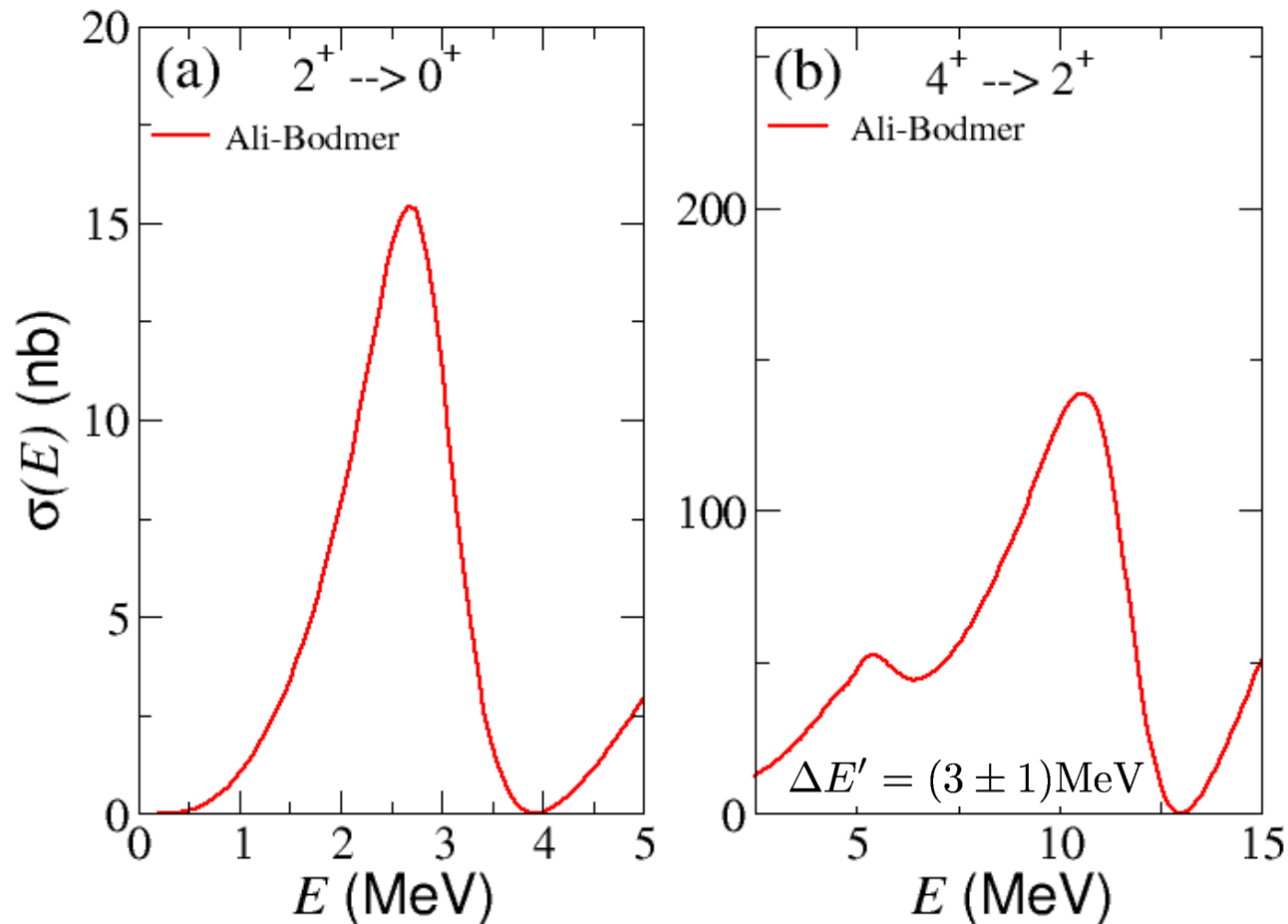
Ali-Bodmer potential

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- ✓ Partial wave dependent
- ✓ Reproduce the phase-shifts

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Transitions between rotational nuclear few-body states in the continuum



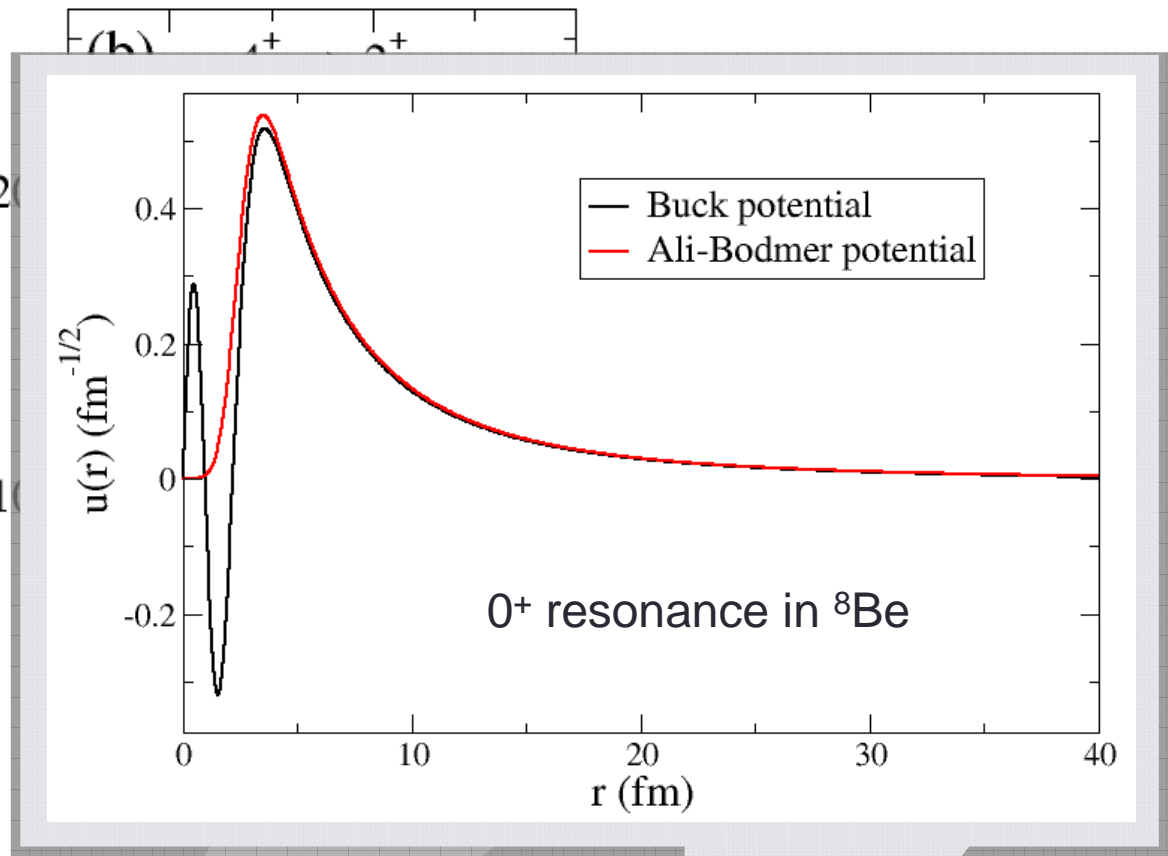
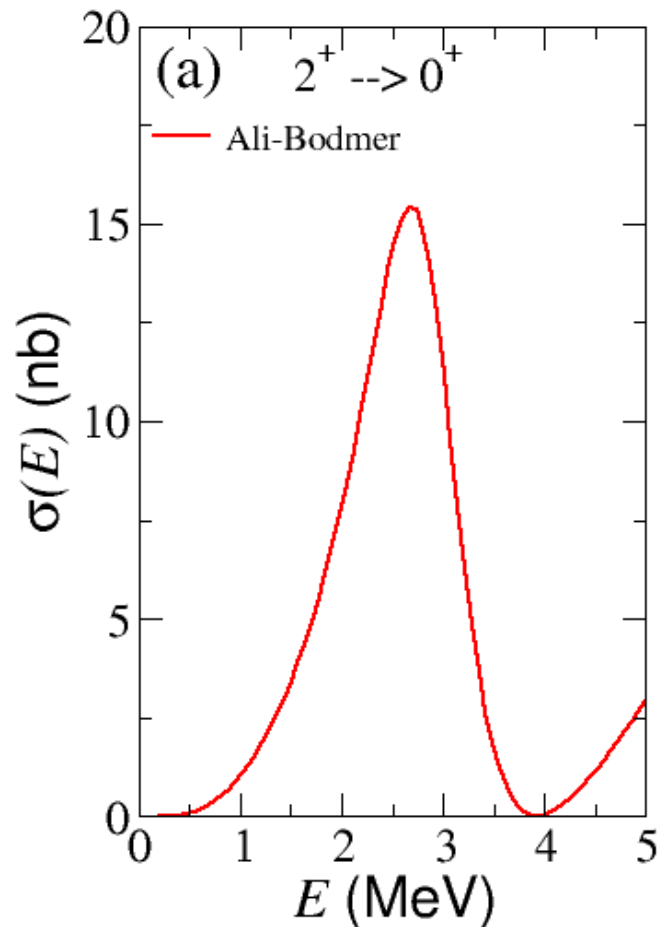
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Transitions between rotational nuclear few-body states in the continuum



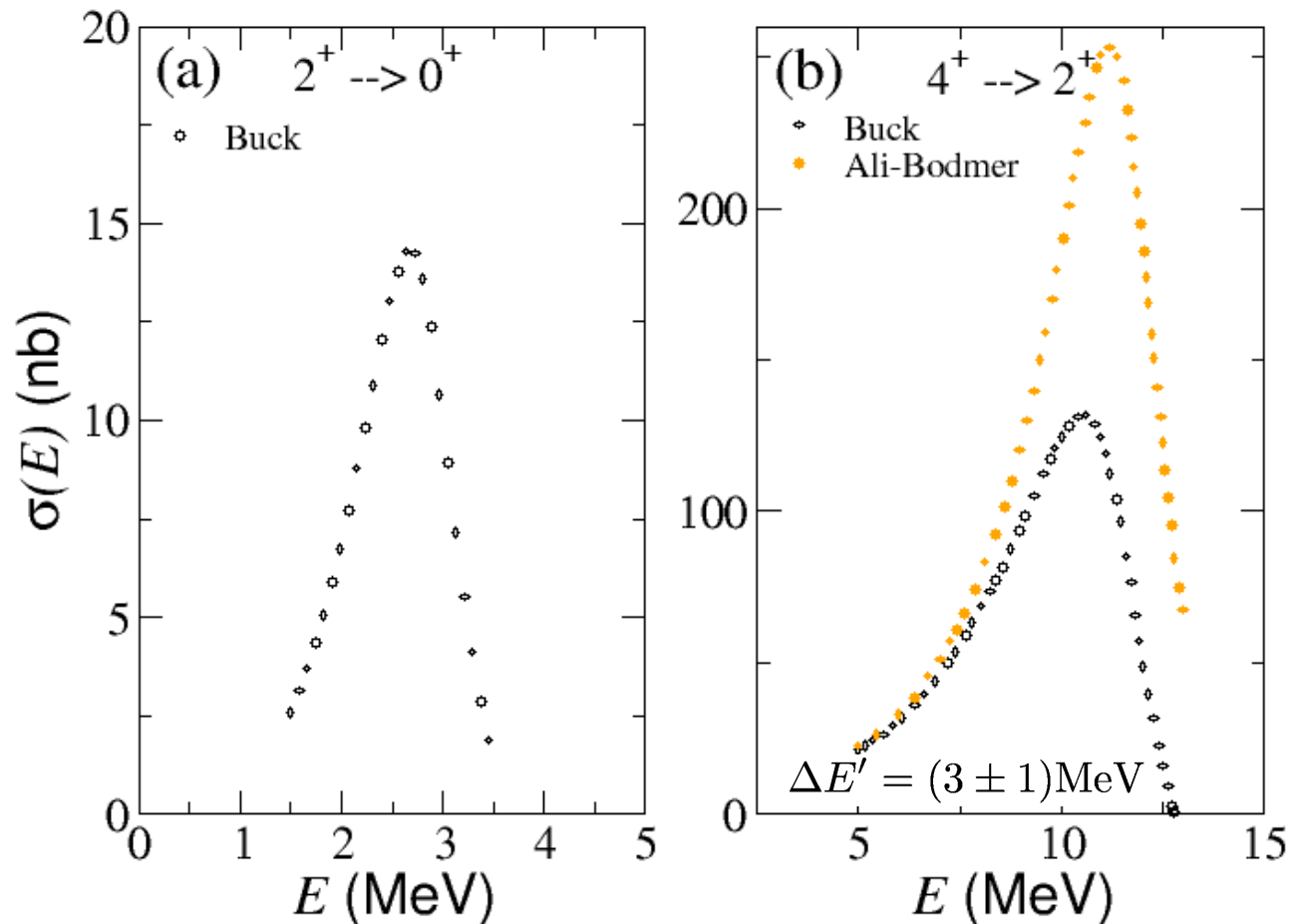
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Transitions between rotational nuclear few-body states in the continuum



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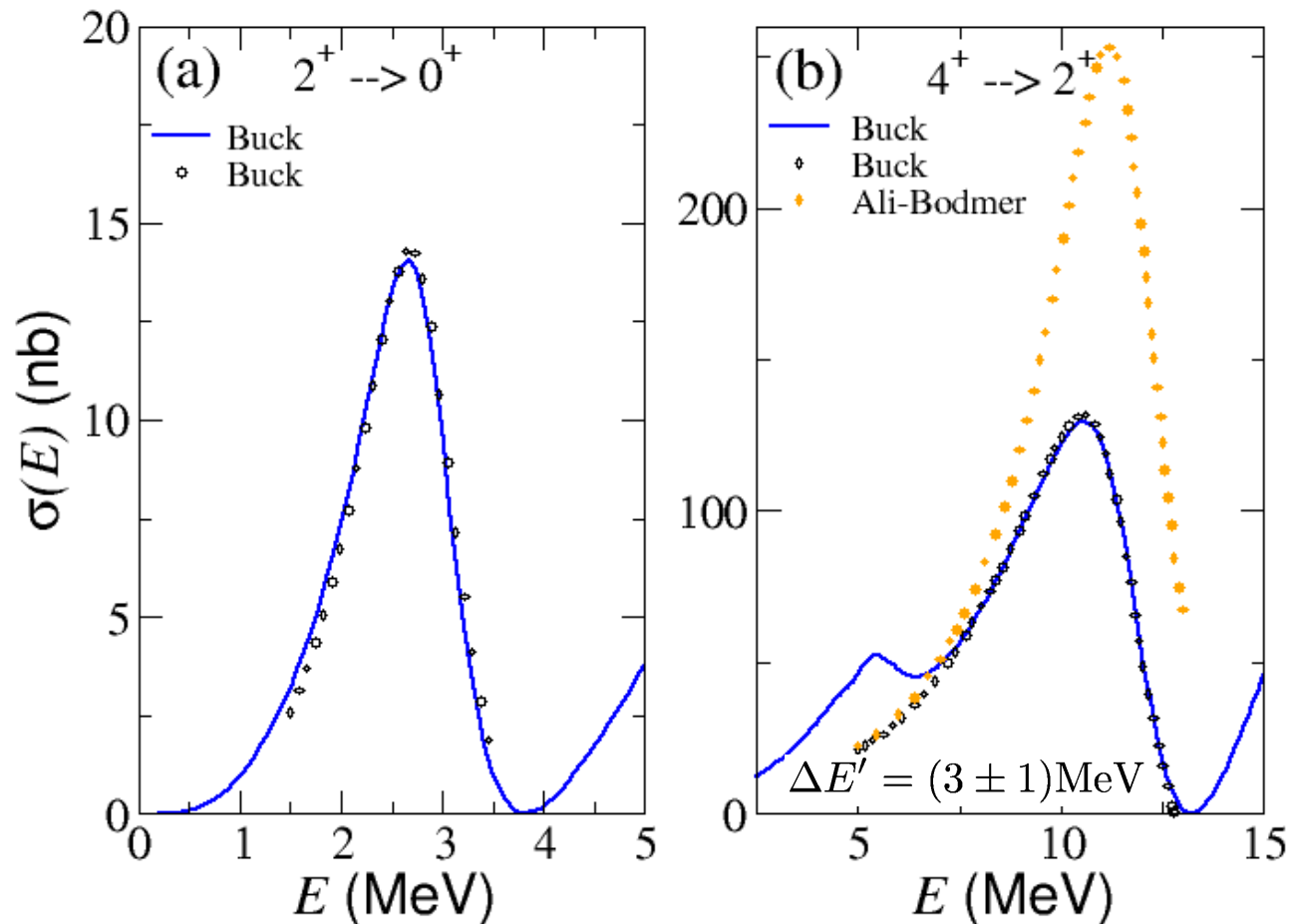
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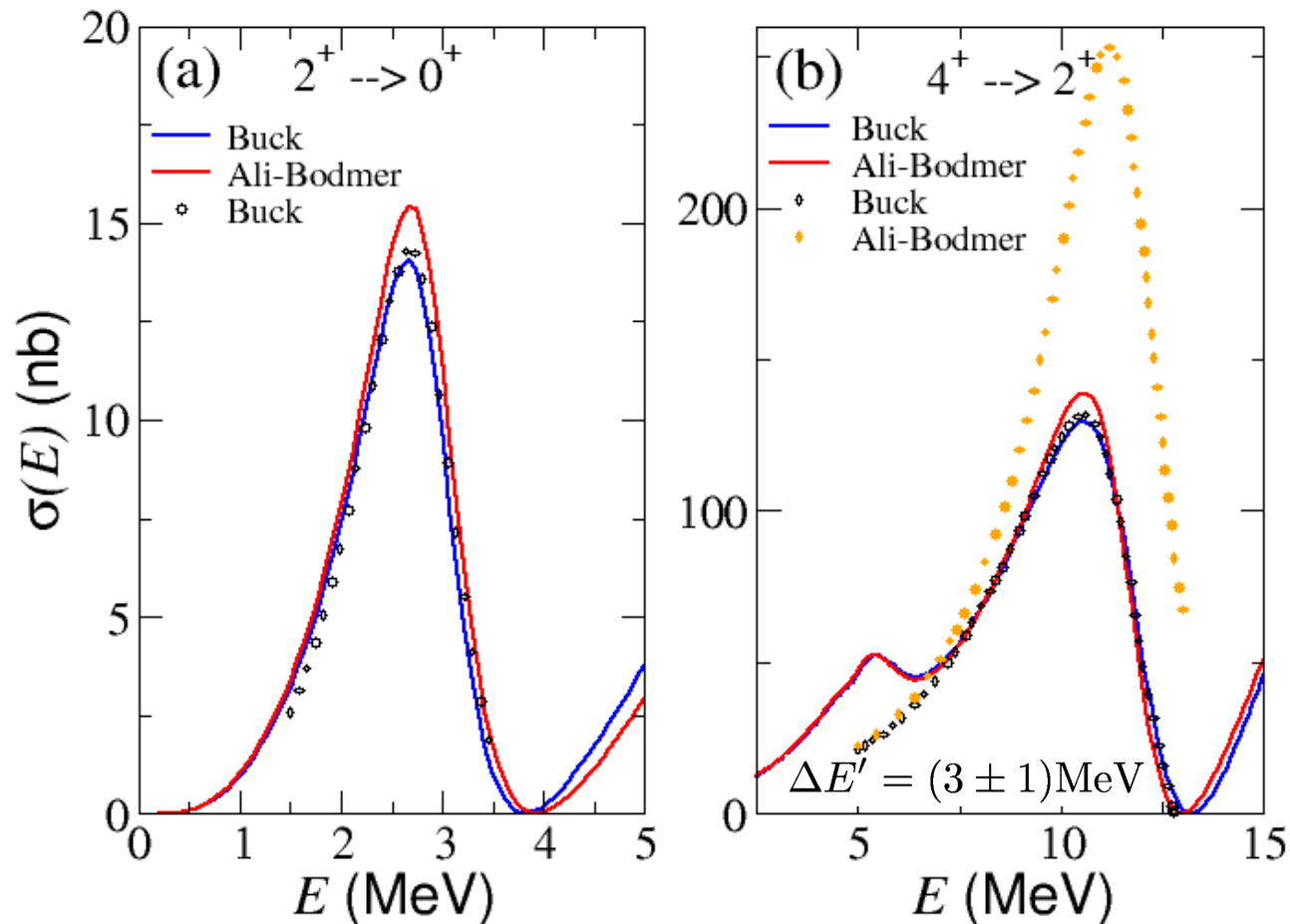
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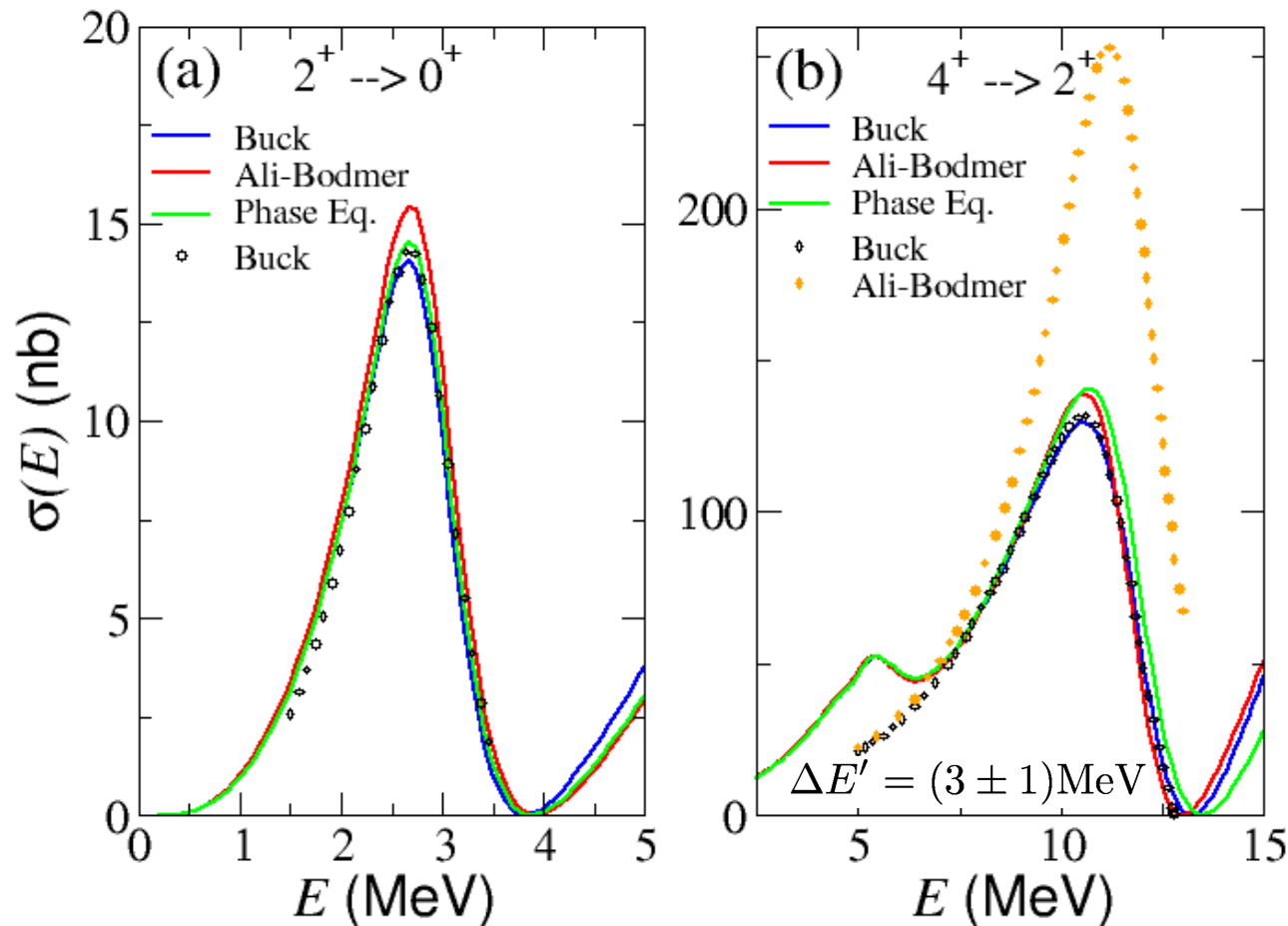
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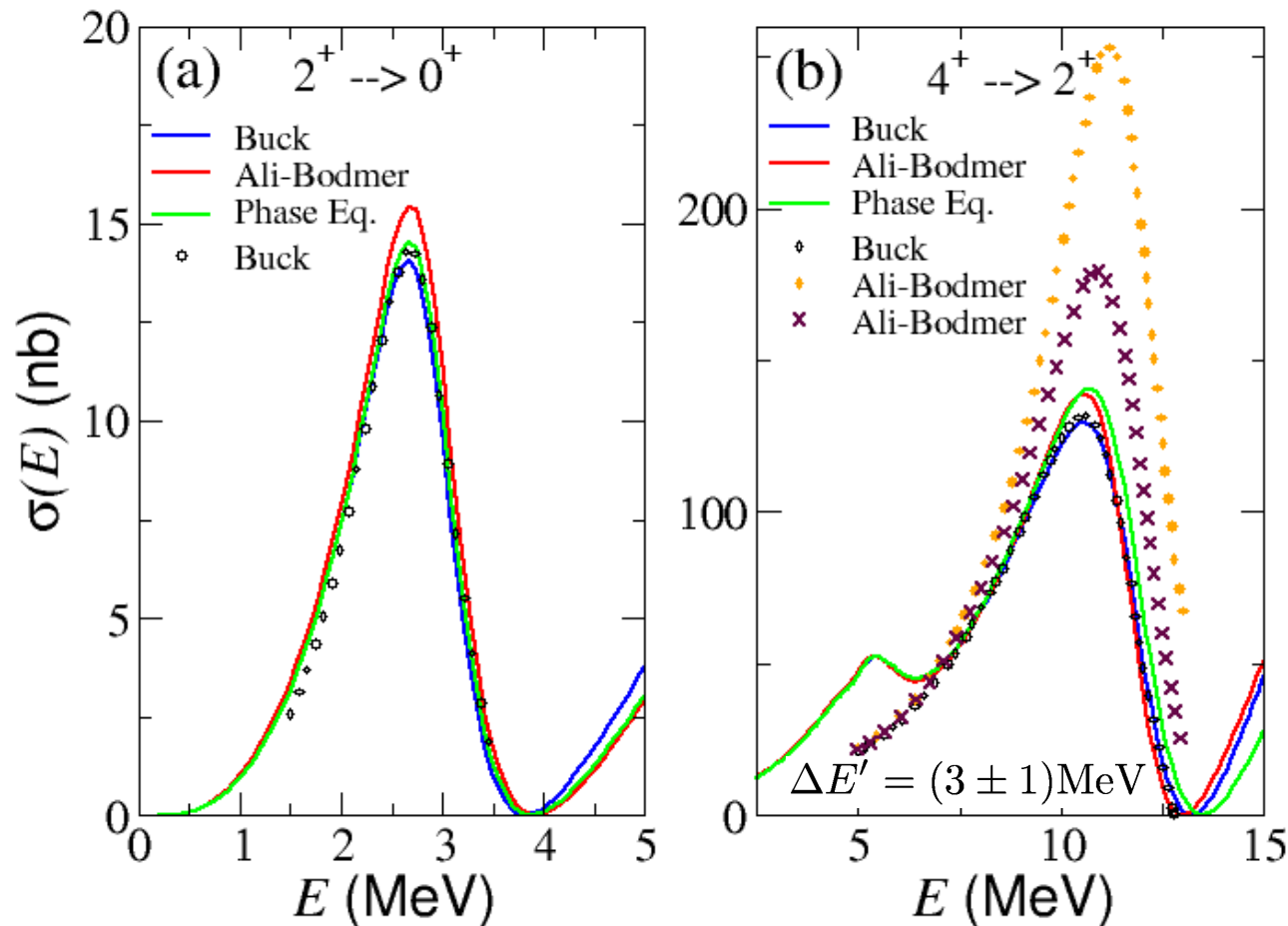
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D. Krolle et al.
 PRC 35 (1987) 1631

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Transitions between rotational nuclear few-body states in the continuum

THANK YOU !!!



A.S. Jensen, D.V. Fedorov, Aarhus University, *Denmark*



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Krakow, September 2013