Evgeny Epelbaum, RUB

EFB22, Krakow, Poland, 9-13 September 2013

Nuclear Physics with Chiral Effective Field Theory

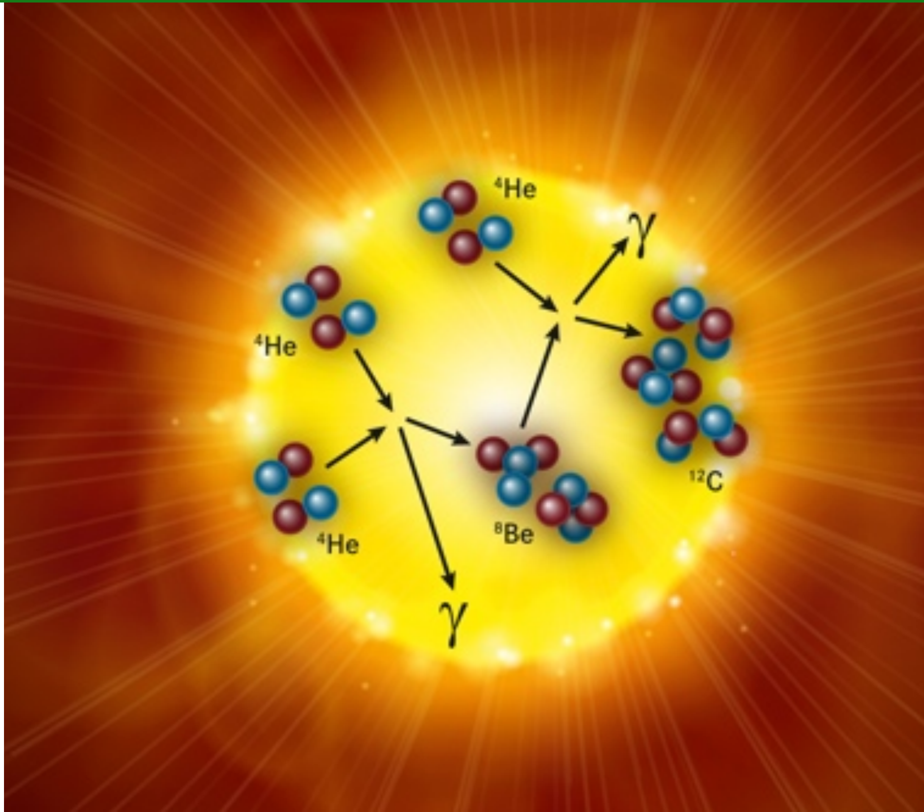
Chiral dynamics and the πN system

Chiral dynamics and nuclear forces

- The NN potential
- Quark mass dependence
- Three-nucleon forces

Nuclear chiral EFT on the lattice

Summary and outlook



Chiral dynamics
and the pion-nucleon system

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \, \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \, \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\Big(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\Big)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i c_i\, \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i d_i\, \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

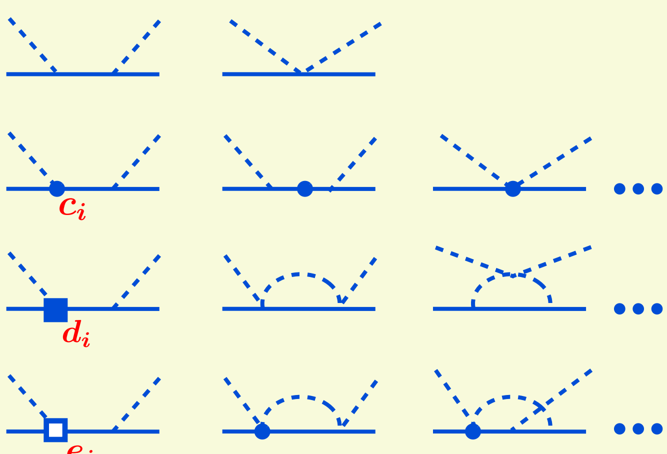
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

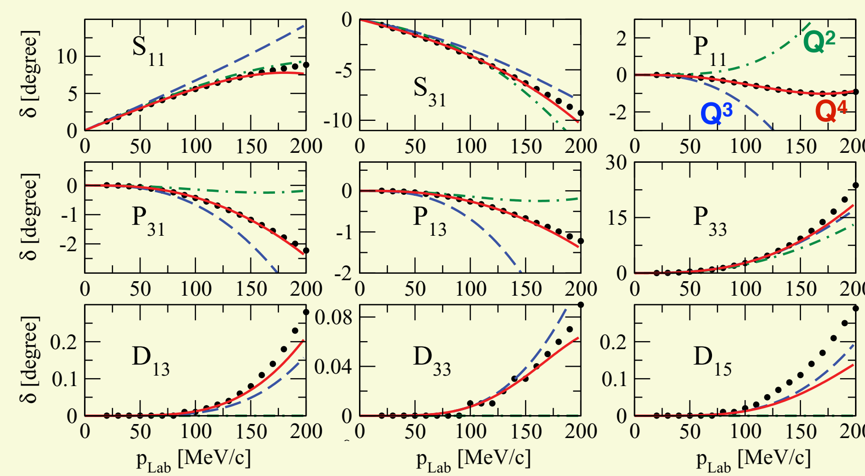
Order Q:

Order Q^2:

Order Q^3:

Order Q^4:





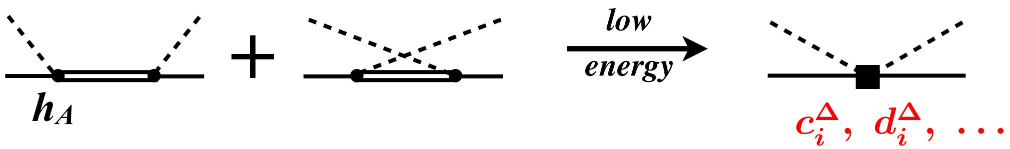
Chiral Perturbation Theory

Improving the convergence of ChPT:

- **Covariant formulations (no $1/m_N$-expansion)** Becher, Leutwyler, Fuchs, Gegelia, Japaridze, Scherer ...
- **Explicit treatment of the Δ(1232) isobar** Jenkens, Manohar, Hemmert, Holstein, Kambor, ...

For ChPT to be useful, (renormalized) LECs must be natural, i.e. $\sim \alpha_i/\Lambda_\chi^n, \quad \alpha_i = \mathcal{O}(1)$

LECs contain information about short-range physics such as the Δ:



$$c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9(m_\Delta - m_N)} \simeq 2.8\,\mathrm{GeV}^{-1}$$ Bernard, Kaiser, Meißner '97

$$\bar{d}_{14}^\Delta - \bar{d}_{15}^\Delta = -2(\bar{d}_1^\Delta + \bar{d}_2^\Delta) = 2\bar{d}_3^\Delta = \frac{-2h_A^2}{9(m_\Delta - m_N)^2} \simeq -4.8\,\mathrm{GeV}^{-2}$$ Krebs, Gasparyan, EE, to appear

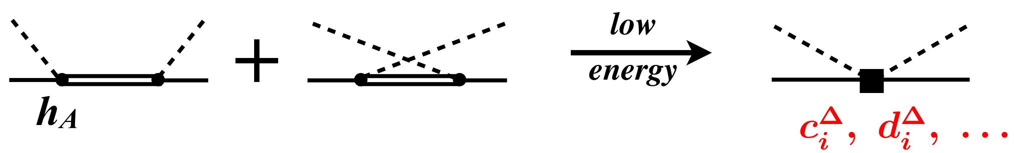
Chiral Perturbation Theory

Improving the convergence of ChPT:

- **Covariant formulations (no 1/m$_N$-expansion)** Becher, Leutwyler, Fuchs, Gegelia, Japaridze, Scherer ...
- **Explicit treatment of the Δ(1232) isobar** Jenkens, Manohar, Hemmert, Holstein, Kambor, ...

For ChPT to be useful, (renormalized) LECs must be natural, i.e. $\sim \alpha_i/\Lambda_\chi^n, \quad \alpha_i = \mathcal{O}(1)$

LECs contain information about short-range physics such as the Δ:



$$c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9(m_\Delta - m_N)} \simeq 2.8\,\text{GeV}^{-1}$$ Bernard, Kaiser, Meißner ’97

$$\bar{d}_{14}^\Delta - \bar{d}_{15}^\Delta = -2(\bar{d}_1^\Delta + \bar{d}_2^\Delta) = 2\bar{d}_3^\Delta = \frac{-2h_A^2}{9(m_\Delta - m_N)^2} \simeq -4.8\,\text{GeV}^{-2}$$ Krebs, Gasparyan, EE, to appear

LECs from pion-nucleon scattering (HB ChPT) in units of GeV^{-n}

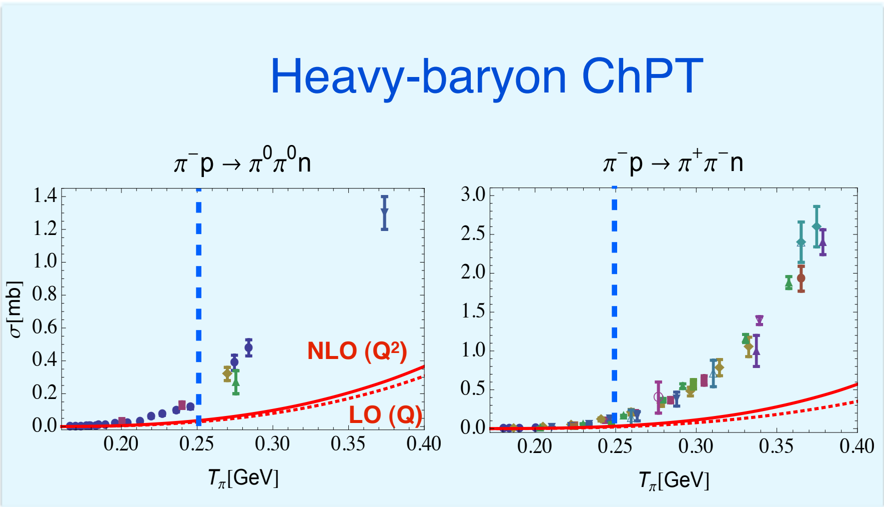
Krebs, Gasparyan, EE, to appear; similar results found by Fettes, Meißner; Büttiker, Meißner, ...

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Δ-less approach	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26
Δ-full approach	-0.95	1.90	-1.78	1.50	2.40	-3.87	1.21	-5.25	-0.24	-6.35	2.34	-0.39	2.81
Δ-contribution	0	2.81	-2.81	1.40	2.39	-2.39	0	-4.77	1.87	-4.15	4.15	-0.17	1.32

The hope: LECs of a more natural size $\longrightarrow$ better convergence of the EFT expansion...

The reaction $\pi N \to \pi\pi N$ (preliminary)

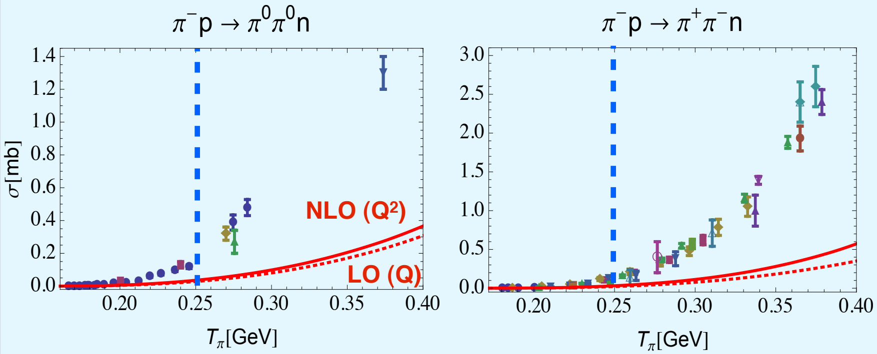
Siemens, Bernard, EE, Krebs, Meißner, to appear



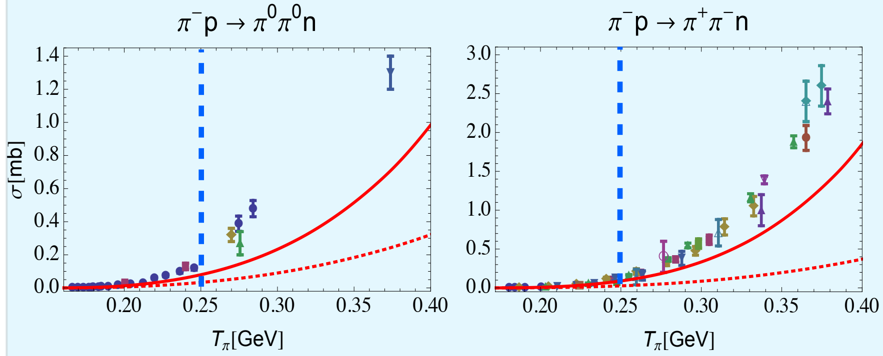
The reaction $\pi N \to \pi\pi N$ (preliminary)

Siemens, Bernard, EE, Krebs, Meißner, to appear

Heavy-baryon ChPT



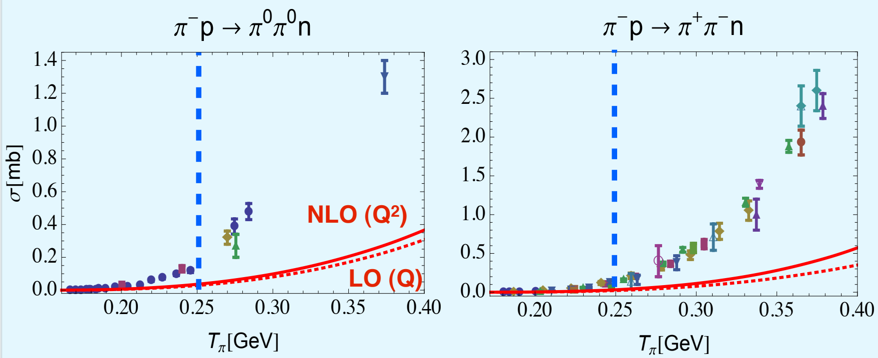
Covariant ChPT



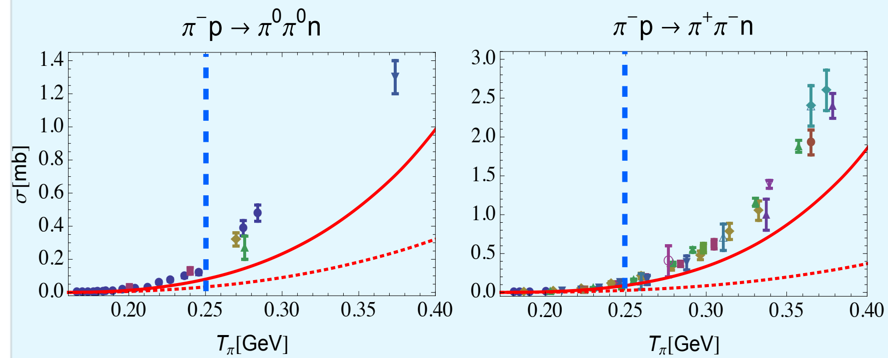
The reaction $\pi N \to \pi\pi N$ (preliminary)

Siemens, Bernard, EE, Krebs, Meißner, to appear

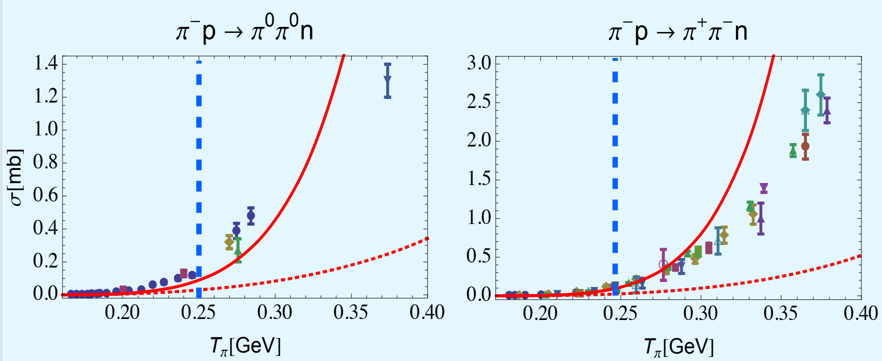
Heavy-baryon ChPT



Covariant ChPT



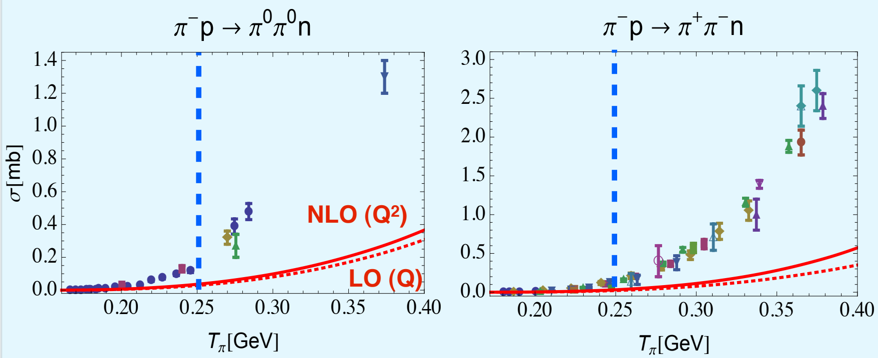
Heavy-baryon ChPT, explicit Δ



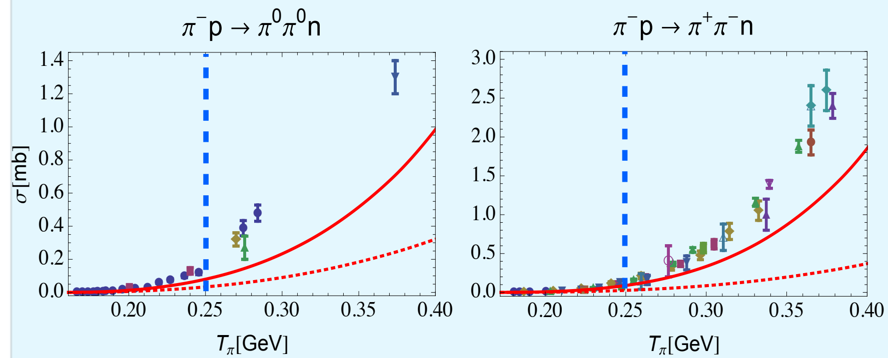
The reaction $\pi N \to \pi\pi N$ (preliminary)

Siemens, Bernard, EE, Krebs, Meißner, to appear

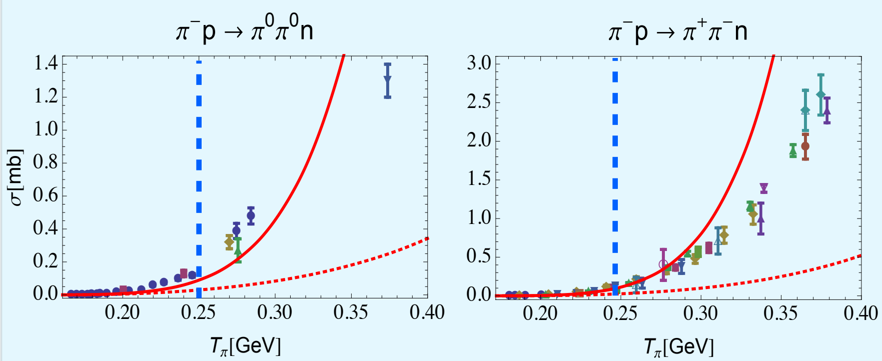
Heavy-baryon ChPT



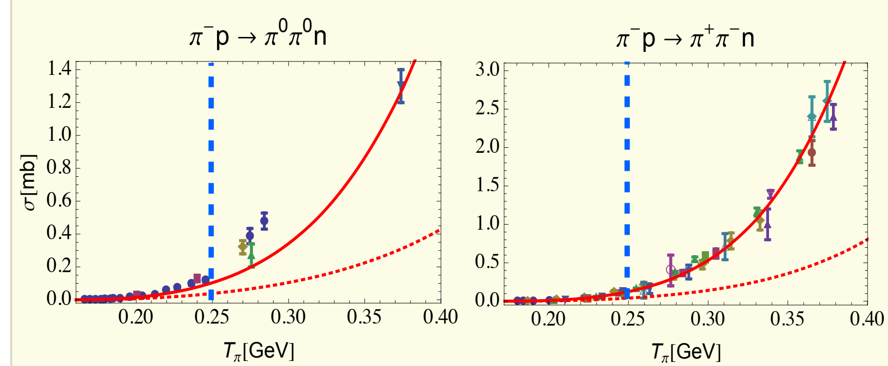
Covariant ChPT



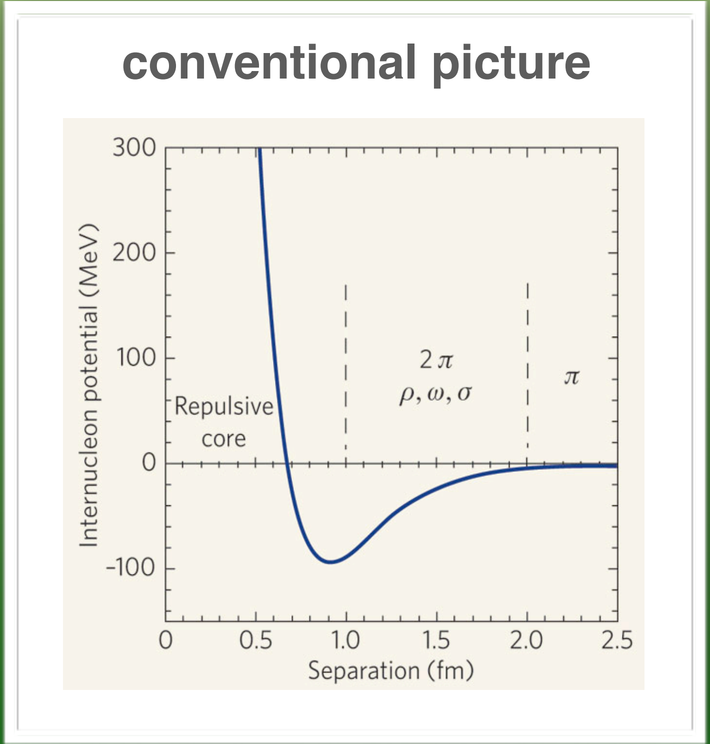
Heavy-baryon ChPT, explicit Δ

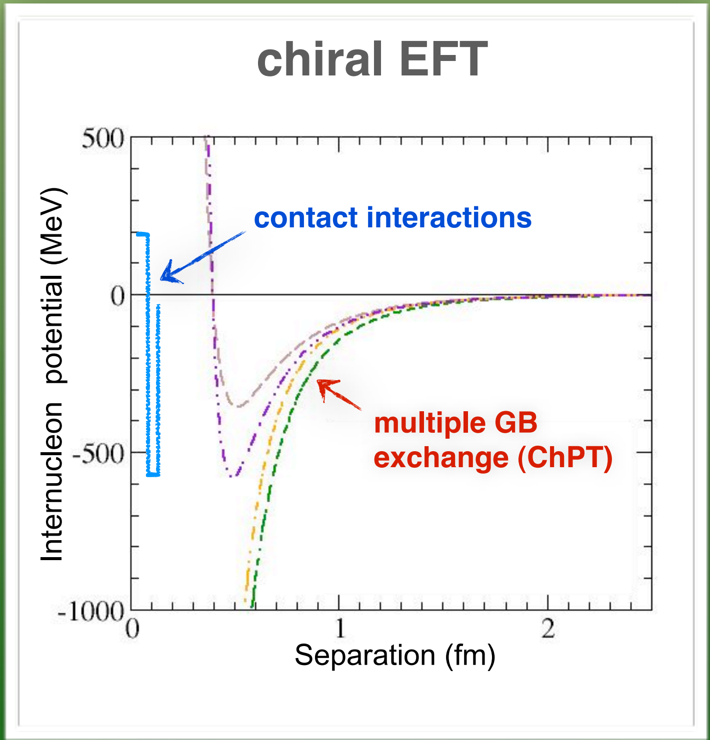


Covariant ChPT, explicit Δ

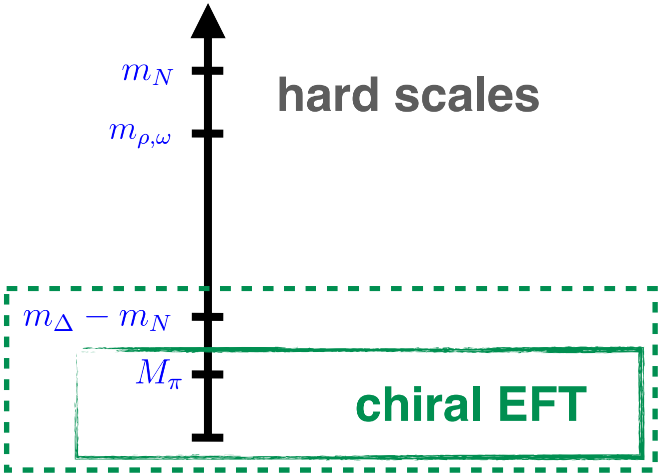


Chiral dynamics and nuclear forces

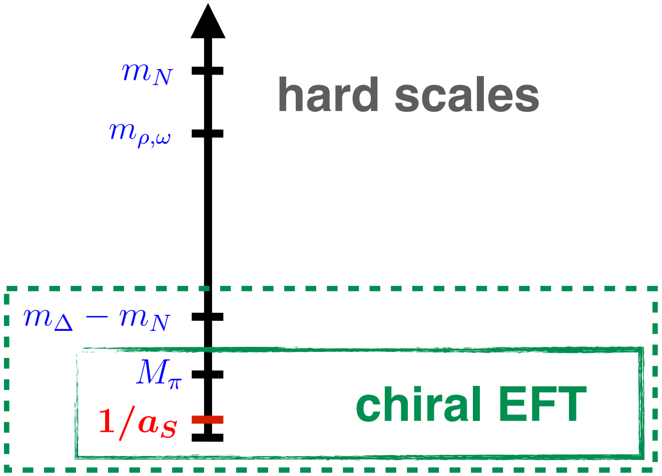




Chiral EFT for nuclei



Chiral EFT for nuclei

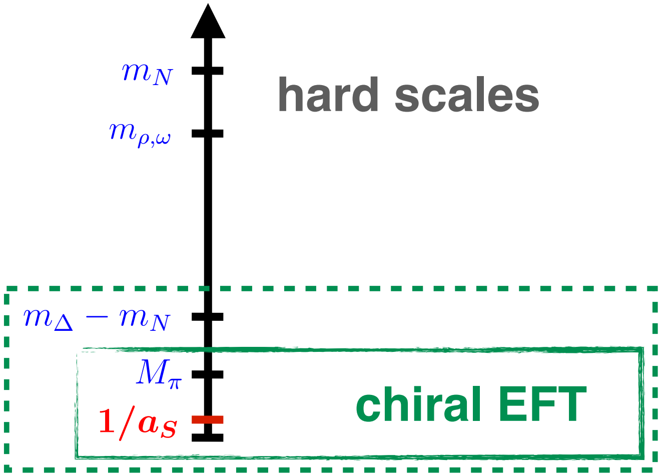


A new, soft scale associated with nuclear binding

$Q \sim 1/a_S \simeq 8.5$ MeV(36 MeV) in $^1S_0$ ($^3S_1$)

to be generated dynamically (need resummations...)

Chiral EFT for nuclei



A new, soft scale associated with nuclear binding

$Q \sim 1/a_S \simeq 8.5$ MeV(36 MeV) in $^1S_0$ ($^3S_1$)

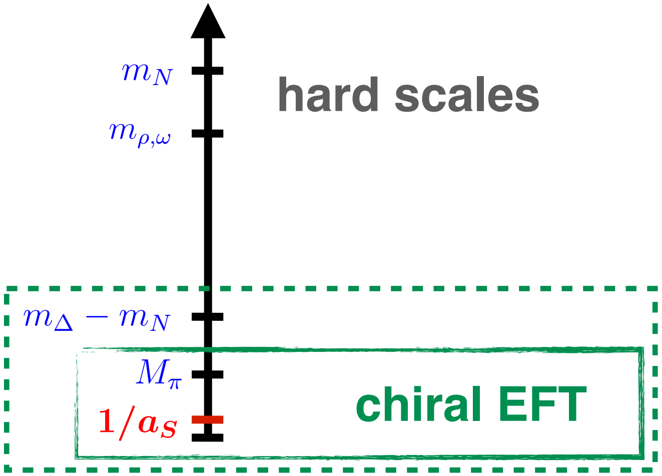
to be generated dynamically (need resummations...)

Pionless EFT (valid for $\sqrt{m_N E_B} \ll Q \ll M_\pi$)

- zero-range forces between nucleons
- for 2N equivalent to Effective Range Theory
- universality, Efimov physics, cold gases, halos,...

talk by Hans-Werner Hammer

Chiral EFT for nuclei



A new, soft scale associated with nuclear binding

$Q \sim 1/a_S \simeq 8.5\ \mathrm{MeV}(36\ \mathrm{MeV})$ in $^1S_0$ ($^3S_1$)

to be generated dynamically (need resummations...)

Pionless EFT (valid for $\sqrt{m_N E_B} \ll Q \ll M_\pi$)

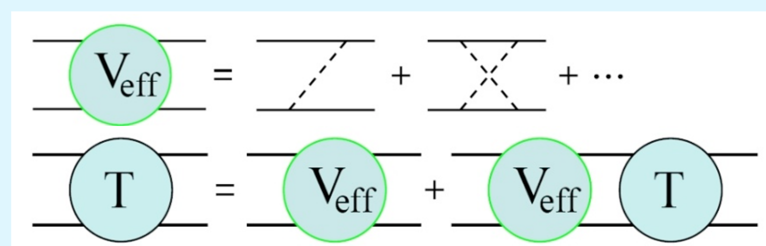
- zero-range forces between nucleons
- for 2N equivalent to Effective Range Theory
- universality, Efimov physics, cold gases, halos,...

talk by Hans-Werner Hammer

Chiral EFT (valid for $Q \sim M_\pi$)

Weinberg, van Kolck, EE, Glöckle, Meißner, Machleidt, Entem...

- Schrödinger equation for nucleons interacting via contact forces and long-range potentials (pion exchanges)

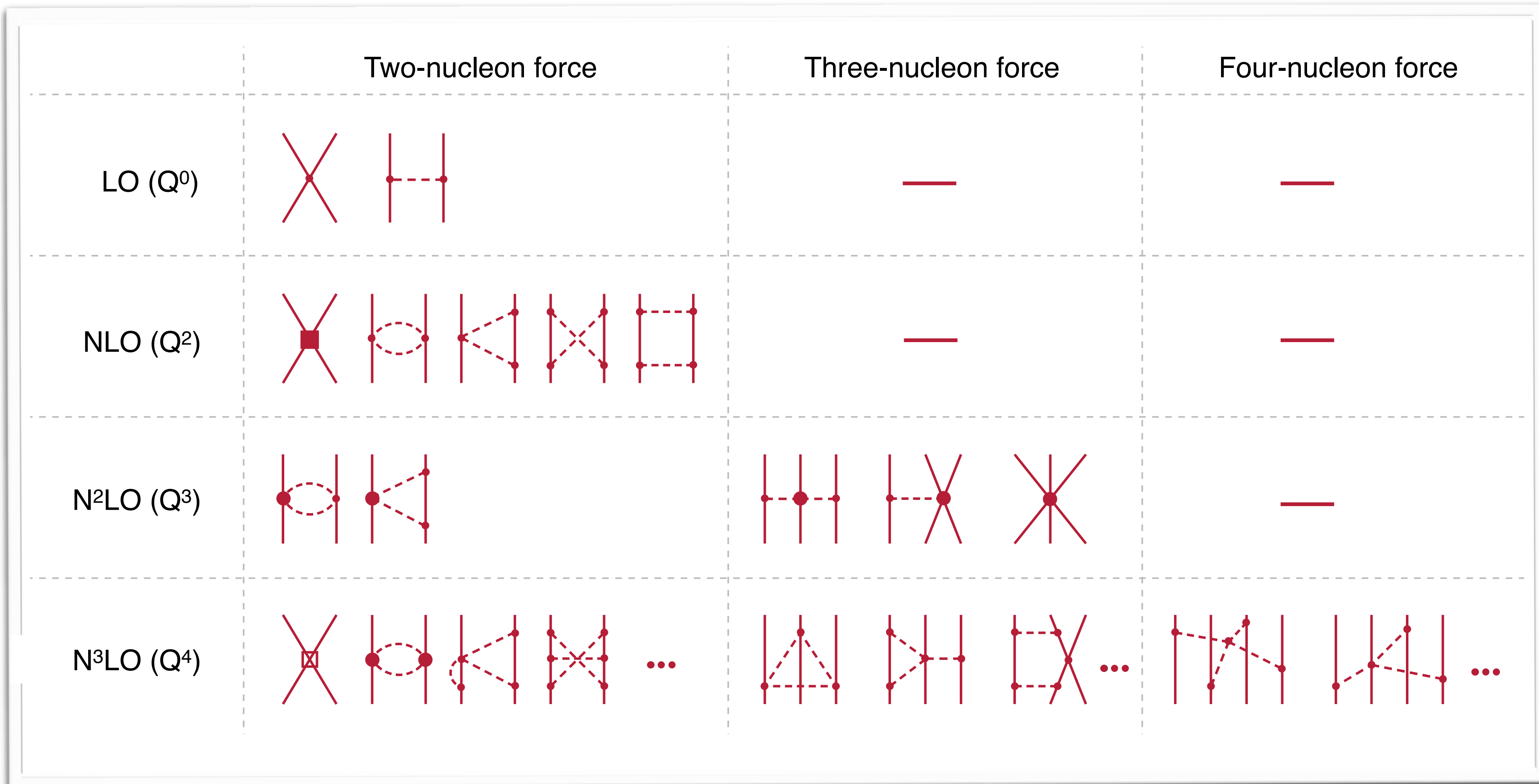


$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

- access to heavier nuclei (ab initio few-/many-body methods)

talks by Petr Navratil and Robert Roth

Chiral expansion of nuclear forces



$\langle V_{2N} \rangle \sim 20\ \mathrm{MeV/pair}$ $\langle V_{3N} \rangle \sim 1\ \mathrm{MeV/triplet}$ $\langle V_{4N} \rangle \sim 0.1\ \mathrm{MeV/quartet}$

(numbers from Pudliner et al. PRL 74 (95) 4396)

Regularization, renormalization and all that...

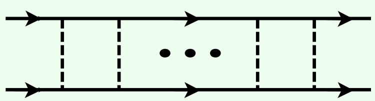
Kaplan, Savage, Wise, Fleming, Mehen, Stewart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Birse, Beane, Bedaque, van Kolck, Pavon Valderrama, Ruiz Arriola, Nogga, Timmermanns, EE, Meißner, Entem, Machleidt, Yang, Elster, Long, Gegelia, ...

→ see also talks by Jambul Gegelia and Varese Timoteo

Problem in essence: static OPEP has a singular short-distance tensor part ~ $1/r^3$

$$V_{1\pi}(\vec{r}) = \left(\frac{g_A}{2F_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[M_\pi^2 \frac{e^{-M_\pi r}}{12\pi r} \left(S_{12}(\hat{r}) \left(1 + \frac{3}{M_\pi r} + \frac{3}{(M_\pi r)^2} \right) + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) - \frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \delta^3(r) \right]$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



Regularization, renormalization and all that...

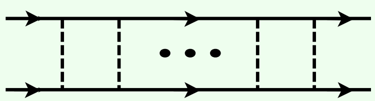
Kaplan, Savage, Wise, Fleming, Mehen, Stewart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Birse, Beane, Bedaque, van Kolck, Pavon Valderrama, Ruiz Arriola, Nogga, Timmermanns, EE, Meißner, Entem, Machleidt, Yang, Elster, Long, Gegelia, ...

→ see also talks by Jambul Gegelia and Varese Timoteo

Problem in essence: static OPEP has a singular short-distance tensor part ~ $1/r^3$

$$V_{1\pi}(\vec{r}) = \left(\frac{g_A}{2F_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \left[M_\pi^2 \frac{e^{-M_\pi r}}{12\pi r}\left(S_{12}(\hat{r})\left(1+\frac{3}{M_\pi r}+\frac{3}{(M_\pi r)^2}\right)+\vec{\sigma}_1\cdot\vec{\sigma}_2\right)-\frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta^3(r)\right]$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



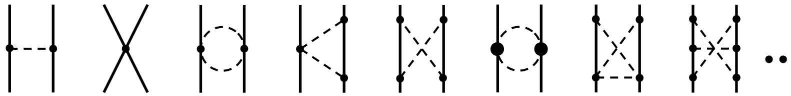
Attempted solutions:

- Solve the LS equation nonperturbatively and take $\Lambda \to \infty$: doesn‘t correspond to an EFT, fails to describe the data... EE, Gegelia ’09; Zeoli, Machleidt, Entem ‘12
- Don‘t iterate [KSW]: seems not to converge in low S=1 waves... Fleming, Mehen, Stewart ’00
- Don‘t do $1/m_N$ expansion: 3d equations fulfilling relativistic elastic unitarity (e.g. Kadyshevsky eq.), renormalizable at LO (only Log-divergent), higher orders to be treated perturbatively... EE, Gegelia’12; related work by Pavon Valderrama, Long, Yang, ...
- Use a finite UV cutoff a la Lepage (standard approach): simple, good description of data at N^3LO [EGM, EM], well suited for few-/many-body calculations but Λ-sensitivity...

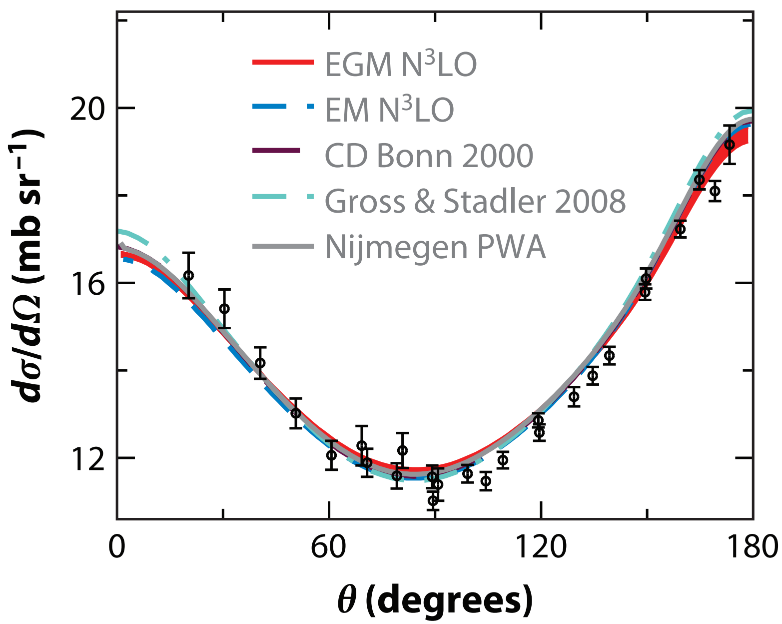
Nucleon-nucleon potential at N^3LO

van Kolck et al.'94; Friar & Coon '94; Kaiser et al. '97; E.E. et al. '98,'03; Kaiser '99-'01; Higa, Robilotta '03; ...

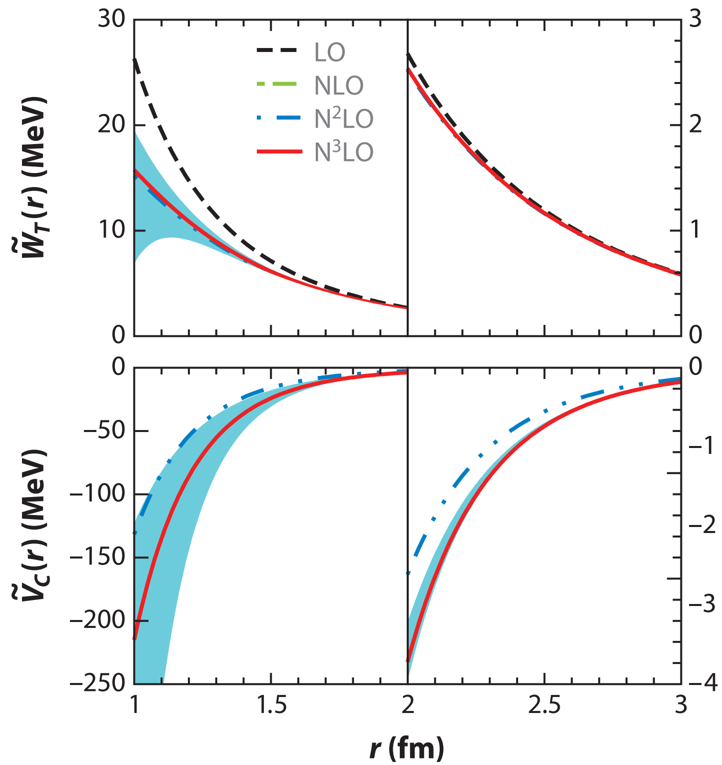
- Long-range: parameter-free (all LECs from πN)
- Short-range part: 24 LECs tuned to NN data
- Accurate description of NN data up to ~ 200 MeV Entem-Machleidt, EE-Glöckle-Meißner



np cross section @ 50 MeV



χ expansion of the long-range force

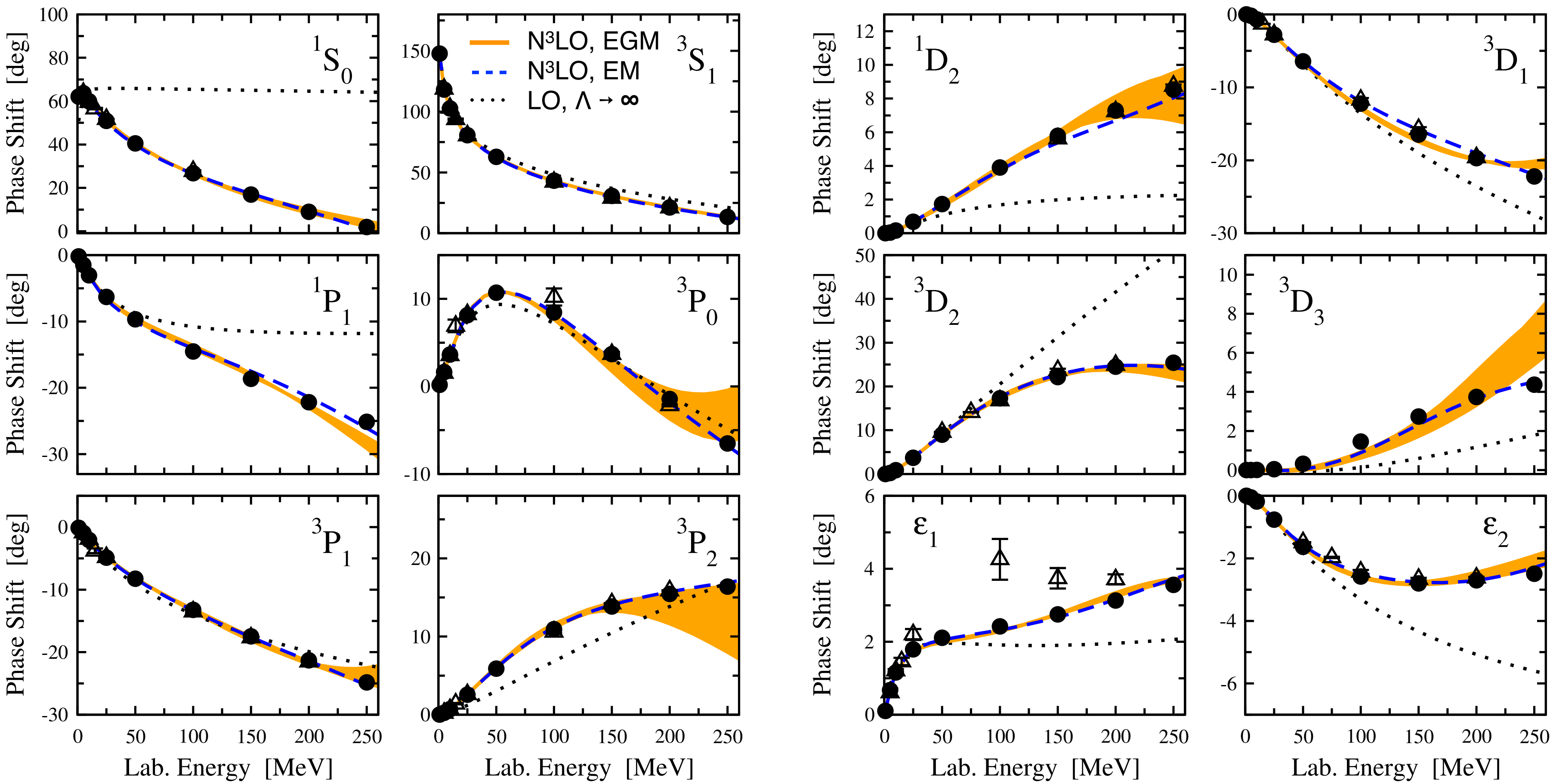


Recent reviews:

EE, Prog. Part Nucl. Phys. 57 (06) 654;
EE, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773;
Entem, Machleidt, Phys. Rept. 503 (11) 1;
EE, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (12) 159.

Neutron-proton phase shifts

Entem, Machleidt ’04; EE, Glöckle, Meißner ’05; EE, Gegelia ’12

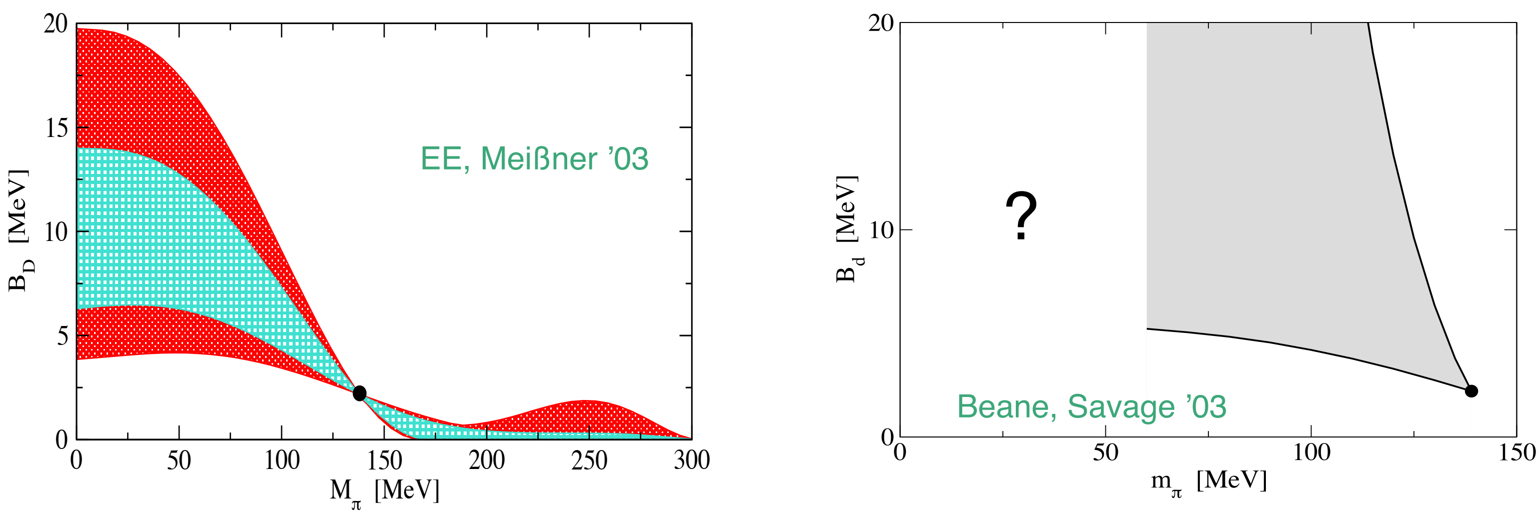


Quark mass dependence

Beane, Savage '02,'03; EE, Glöckle, Meißner '03; J. Donoghue '06; Chen, Lee, Liu, Liu '10; Soto, Tarrus '12; Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez '13, ...

In principle straightforward, but large uncertainties due to poorely known M_π -dependence of short-range contact interactions

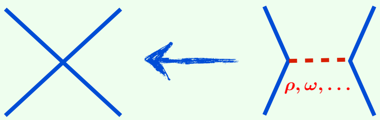
Chiral extrapolations of the deuteron BE at NLO

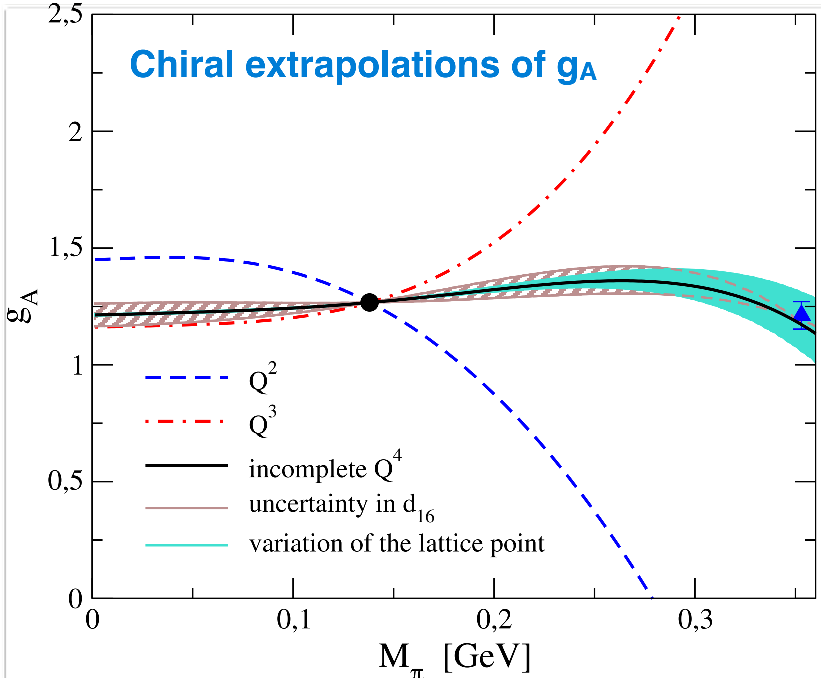


Quark mass dependence of the NN force

Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez ’13

- Use ChPT combined with lattice-QCD data to constrain the M_π-dependence of the nucleon mass and long-range part of the force
- M_π-dependence of contact interactions from resonance saturation [EE, Meißner, Glöckle, Elster ’02] + unitarized ChPT + lattice-QCD



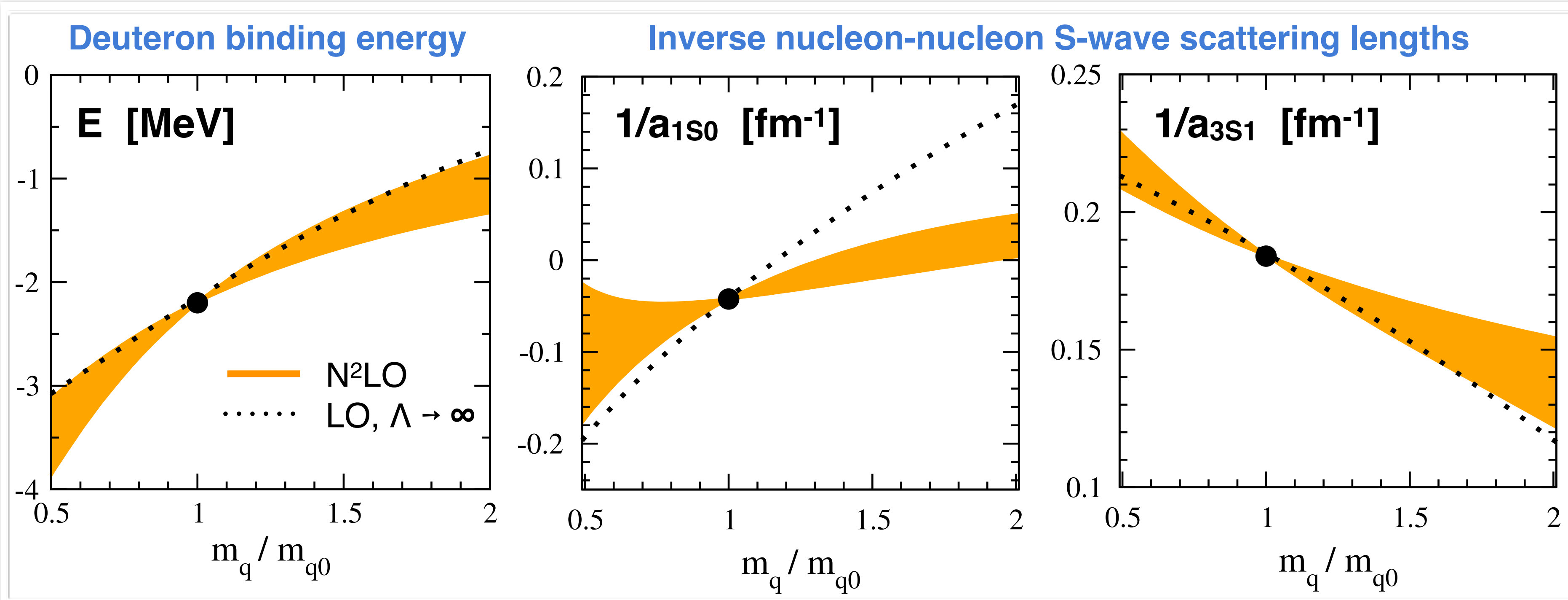


Resonance saturation of the various LECs based on the Bonn B potential

LEC	N^2LO fits	$\sigma + \rho + \omega$
$\tilde{C}^{\rm res}_{1S0}$	$-(0.12 \ldots 0.16)$	-0.12
$C^{\rm res}_{1S0}$	$(1.16 \ldots 1.37)$	1.28
$\tilde{C}^{\rm res}_{3S1}$	$-(0.13 \ldots 0.16)$	-0.10
$C^{\rm res}_{3S1}$	$(0.42 \ldots 0.72)$	0.66
$C^{\rm res}_{\epsilon 1}$	$-(0.36 \ldots 0.47)$	-0.41

Quark mass dependence of the NN force

Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez '13



In terms of K-factors $K_X^q \equiv \frac{m_q}{X}\frac{\partial X}{\partial m_q}\Big|_{m_q^{\rm phys}}$ we find: $K_{a_s}^q = 2.3^{+1.9}_{-1.8}, \quad K_{a_t}^q = 0.32^{+0.17}_{-0.18}$

to be compared with earlier calculations:

$K_{a_s}^q = 5 \pm 5, \quad K_{a_t}^q = 1.1 \pm 0.9$ (W, NLO) EE et al. '03

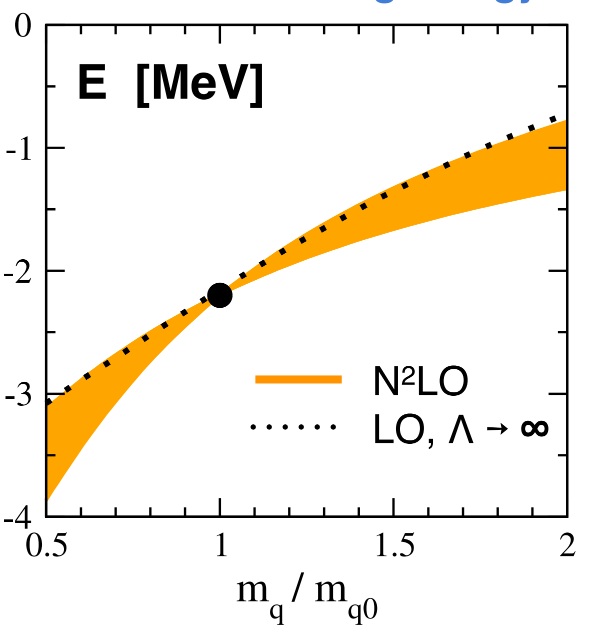
$K_{a_s}^q = 2.4 \pm 3.0, \quad K_{a_t}^q = 3.0 \pm 3.5$ (KSW, NLO) Beane, Savage '03

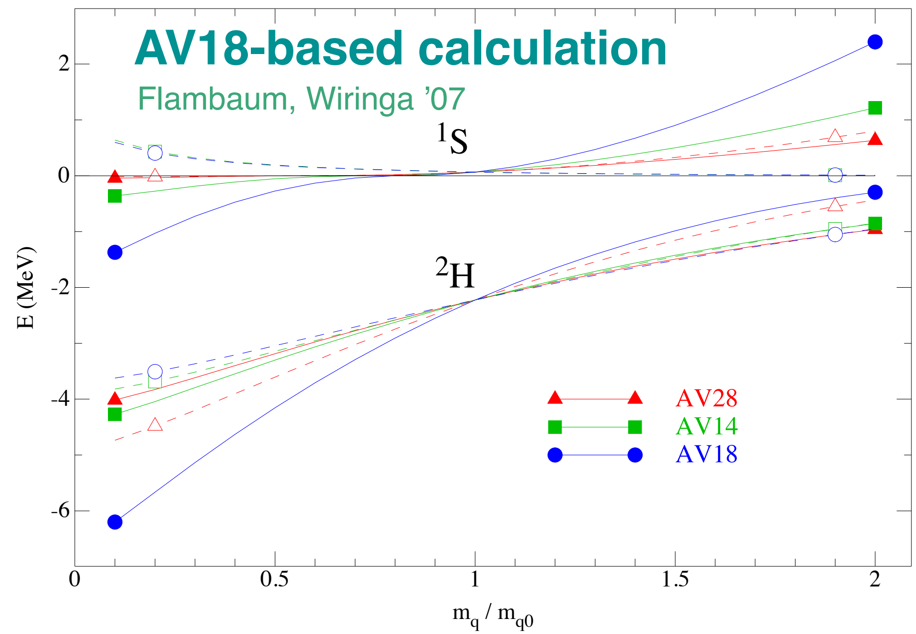
Impact on BBN: limits on m_q variation at the time of BBN: $\delta m_q/m_q = 0.02 \pm 0.04$

Quark mass dependence of the NN force

Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez ’13

Deuteron binding energy





In terms of K-factors $K_X^q \equiv \frac{m_q}{X}\frac{\partial X}{\partial m_q}\Big|_{m_q^{\rm phys}}$ we find: $K_{a_s}^q = 2.3^{+1.9}_{-1.8}, \quad K_{a_t}^q = 0.32^{+0.17}_{-0.18}$

to be compared with earlier calculations: $K_{a_s}^q = 5 \pm 5, \quad K_{a_t}^q = 1.1 \pm 0.9$ (W, NLO) EE et al. ’03

$K_{a_s}^q = 2.4 \pm 3.0, \quad K_{a_t}^q = 3.0 \pm 3.5$ (KSW, NLO) Beane, Savage ’03

Impact on BBN: limits on m_q variation at the time of BBN: $\delta m_q / m_q = 0.02 \pm 0.04$

Some ongoing developments

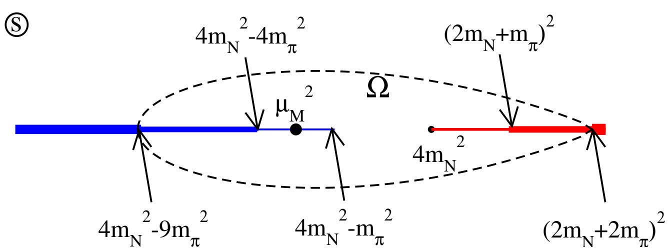
Generalization to the SU(3) sector
Talks by Johann Haidenbauer and Andreas Nogga

Nuclear parity violation
Talk by Matthias Schindler

Local nuclear forces
Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk ’13

- Up to N²LO, the long-range NN force is local → use local regulators + local contact operator basis to construct local chiral potentials (useful e.g. for QMC)

Merging chiral EFT with dispersion relations
Albaladejo, Oller ’11,’12; Gasparyan, EE, Lutz ’12; Guo, Oller, Rios ’13



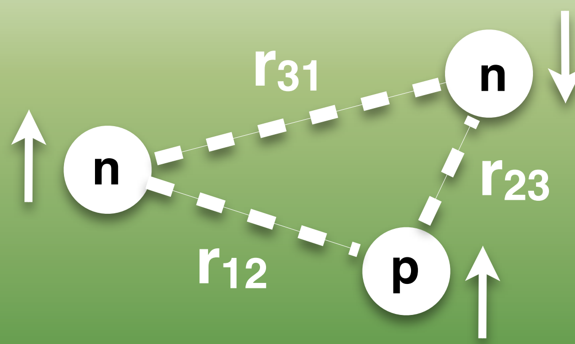
- Calculate the discontinuity of the amplitude along the left-hand cut using ChPT
- Reconstruct the amplitude in the physical region using dispersion relations + analytic cont. (conformal mapping)

Partial wave analysis
Talk by Navarro Perez

...and the role of the chiral two-pion exchange potential

N²LO: tuning the interaction to improve the χ^2
Ekström, Baardsen, et al. ’13

- It is possible to tune parameters (Λ, c_i, functional form of the regulator) to improve the description of the data (especially at higher E_{lab}). Justified from an EFT point of view?

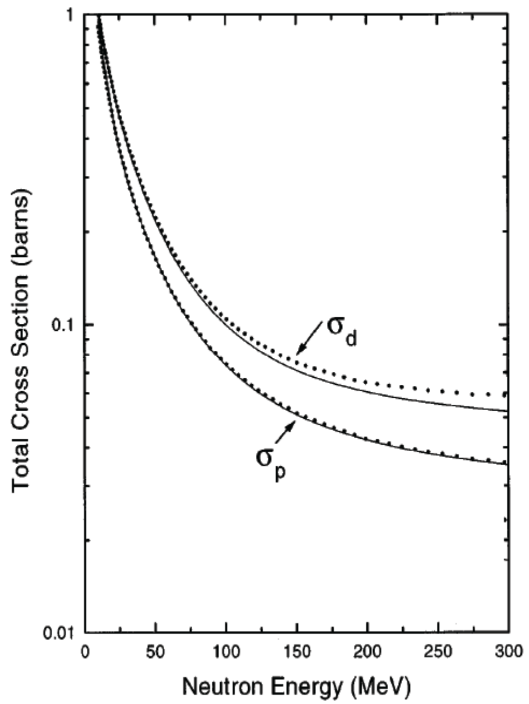


The three-nucleon force

Today‘s few- & many-body calculations have reached the level of accuracy at which it is necessary to include 3NFs

Inspite of decades of efforts, the structure of the 3NF is still poorely understood

Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rev. Mod. Phys. 75 (2012) 016301
Kistryn, Stephan, J. Phys. G: Nucl. Part. Phys. 40 (2013) 063101

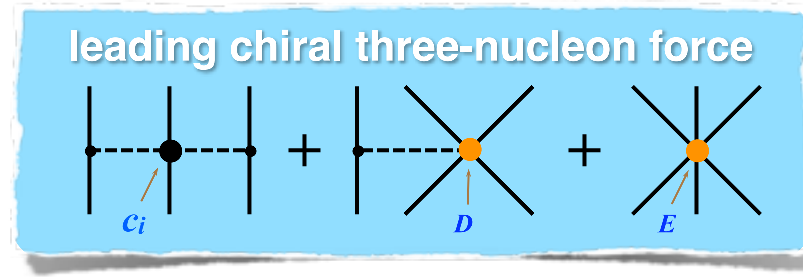


Chiral 3NF & nd elastic scattering

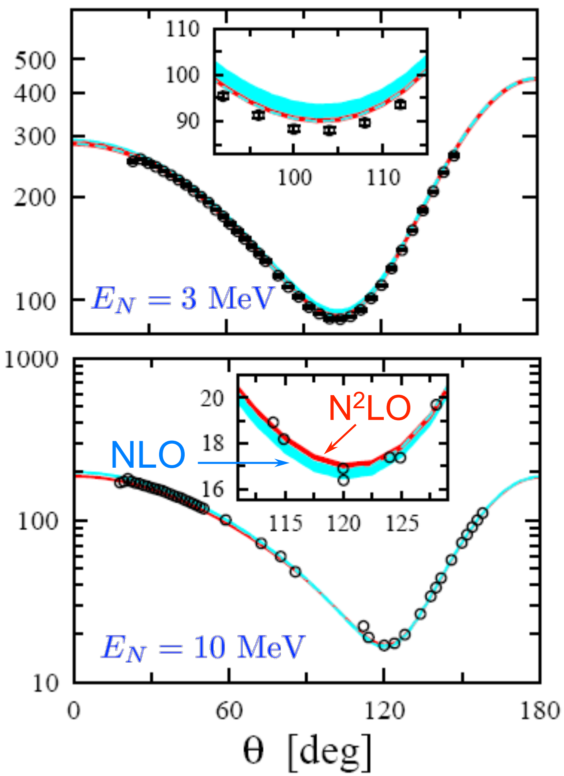
EE, Glöckle, Golak, Kamada, Nogga, Skibinski, Witala

The 3NF starts to contribute at N²LO

The LECs D,E can be fixed e.g. from ³H BE and nd doublet scattering length

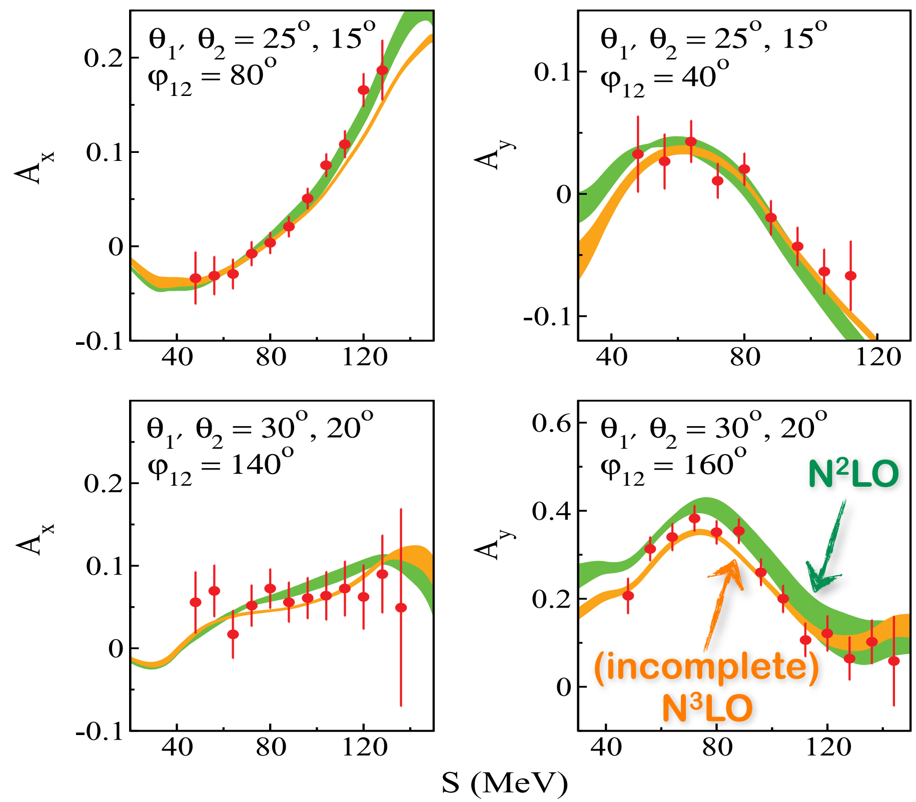


Nd elastic cross sections at low energies



Nd breakup at E_d=130 MeV

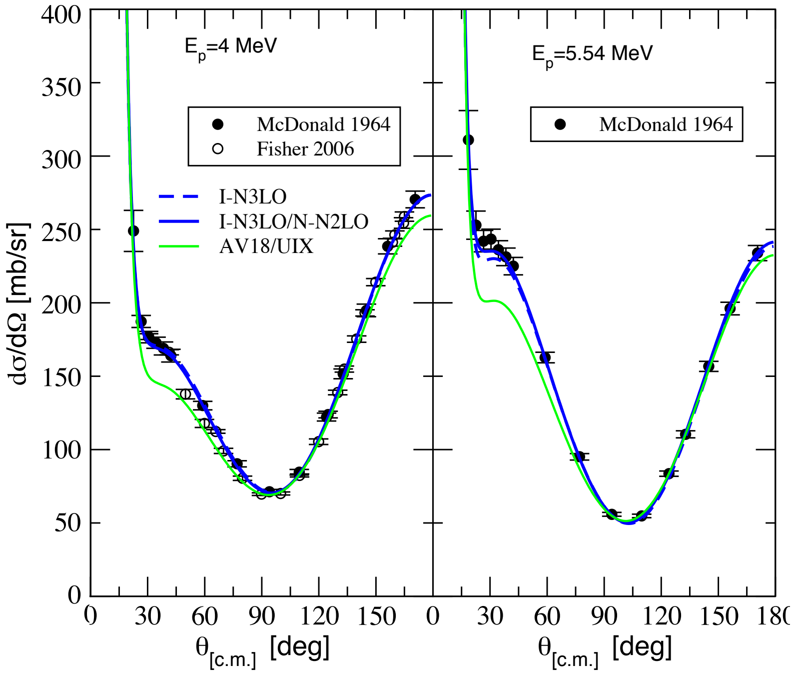
Stephan et al., PRC 82 (2010) 014003



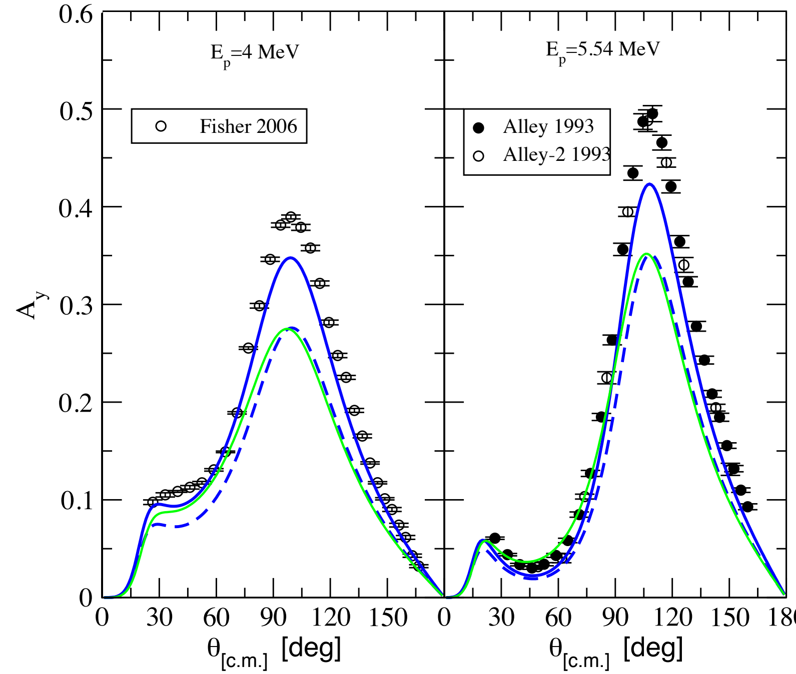
Chiral 3NF and 4N scattering

more results in the talk by Arnas Deltuva

p-^{3}He differential cross section



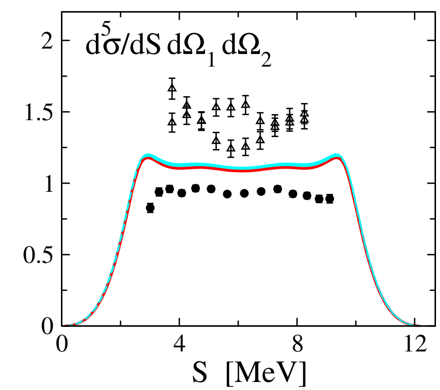
A_y-puzzle in p-^{3}He elastic scattering



LECs D,E tuned to the ^{3}H and ^{4}He binding energies, figure from Viviani et al., arXiv:1004.1306

To summarize:

- Nd scattering: accurate description at low energy except for A_y (fine tuned) and Space Star breakup configuration
- Uncertainty increases with energy (higher-order 3NF?)
- 4N continuum: an emerging field...



Most general structure of a local 3NF

Krebs, Gasparyan, EE '13

22 independent operators (coord. space)

$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1\,, \\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\,, \\
\tilde{\mathcal{G}}_3 &= \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)\,, \\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\,, \\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\,, \\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_2 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{13}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,,
\end{aligned}
$$

Building blocks:

$$\boldsymbol{\tau}_1,\ \boldsymbol{\tau}_2,\ \boldsymbol{\tau}_3,\ \vec{\sigma}_1,\ \vec{\sigma}_2,\ \vec{\sigma}_3,\ \vec{r}_{12},\ \vec{r}_{23}$$

Constraints:

- locality,
- isospin symmetry,
- parity and time-reversal invariance

$$\longrightarrow \quad V_{3\mathrm{N}} = \sum_{i=1}^{22} \mathcal{G}_i\, F_i(r_{12}, r_{23}, r_{31}) \ + \ 5 \text{ perm.}$$

Most general structure of a local 3NF

Krebs, Gasparyan, EE '13

22 independent operators (coord. space)

$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1\,, \\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\,, \\
\tilde{\mathcal{G}}_3 &= \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)\,, \\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\,, \\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\,, \\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_2 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{13}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,,
\end{aligned}
$$

Building blocks:

$$\boldsymbol{\tau}_1,\ \boldsymbol{\tau}_2,\ \boldsymbol{\tau}_3,\ \vec{\sigma}_1,\ \vec{\sigma}_2,\ \vec{\sigma}_3,\ \vec{r}_{12},\ \vec{r}_{23}$$

Constraints:

- locality,
- isospin symmetry,
- parity and time-reversal invariance

$$\longrightarrow \quad V_{3\mathrm{N}} = \sum_{i=1}^{22} \mathcal{G}_i\, F_i(r_{12}, r_{23}, r_{31}) + 5 \text{ perm.}$$

derivable in ChPT; long-range terms parameter-free predictions

Most general structure of a local 3NF

Krebs, Gasparyan, EE '13

22 independent operators (coord. space)

$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1\,,\\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,,\\
\tilde{\mathcal{G}}_3 &= \vec\sigma_1\cdot\vec\sigma_3\,,\\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\vec\sigma_1\cdot\vec\sigma_3\,,\\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\vec\sigma_1\cdot\vec\sigma_2\,,\\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec\sigma_1\cdot(\vec\sigma_2\times\vec\sigma_3)\,,\\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec\sigma_2\cdot(\hat r_{12}\times\hat r_{23})\,,\\
\tilde{\mathcal{G}}_8 &= \hat r_{23}\cdot\vec\sigma_1\,\hat r_{23}\cdot\vec\sigma_3\,,\\
\tilde{\mathcal{G}}_9 &= \hat r_{23}\cdot\vec\sigma_3\,\hat r_{12}\cdot\vec\sigma_1\,,\\
\tilde{\mathcal{G}}_{10} &= \hat r_{23}\cdot\vec\sigma_1\,\hat r_{12}\cdot\vec\sigma_3\,,\\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat r_{23}\cdot\vec\sigma_1\,\hat r_{23}\cdot\vec\sigma_2\,,\\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat r_{23}\cdot\vec\sigma_1\,\hat r_{12}\cdot\vec\sigma_2\,,\\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat r_{12}\cdot\vec\sigma_1\,\hat r_{23}\cdot\vec\sigma_2\,,\\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat r_{12}\cdot\vec\sigma_1\,\hat r_{12}\cdot\vec\sigma_2\,,\\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\hat r_{13}\cdot\vec\sigma_1\,\hat r_{13}\cdot\vec\sigma_3\,,\\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,\hat r_{12}\cdot\vec\sigma_2\,\hat r_{12}\cdot\vec\sigma_3\,,\\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\hat r_{23}\cdot\vec\sigma_1\,\hat r_{12}\cdot\vec\sigma_3\,,\\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec\sigma_1\cdot\vec\sigma_3\,\vec\sigma_2\cdot(\hat r_{12}\times\hat r_{23})\,,\\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec\sigma_3\cdot\hat r_{23}\,\hat r_{23}\cdot(\vec\sigma_1\times\vec\sigma_2)\,,\\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec\sigma_1\cdot\hat r_{23}\,\vec\sigma_2\cdot\hat r_{23}\,\vec\sigma_3\cdot(\hat r_{12}\times\hat r_{23})\,,\\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec\sigma_1\cdot\hat r_{13}\,\vec\sigma_3\cdot\hat r_{13}\,\vec\sigma_2\cdot(\hat r_{12}\times\hat r_{23})\,,\\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3)\,\vec\sigma_1\cdot\hat r_{23}\,\vec\sigma_3\cdot\hat r_{12}\,\vec\sigma_2\cdot(\hat r_{12}\times\hat r_{23})\,,
\end{aligned}
$$

Building blocks:

$$\boldsymbol{\tau}_1,\ \boldsymbol{\tau}_2,\ \boldsymbol{\tau}_3,\ \vec\sigma_1,\ \vec\sigma_2,\ \vec\sigma_3,\ \vec r_{12},\ \vec r_{23}$$

Constraints:

- locality,
- isospin symmetry,
- parity and time-reversal invariance

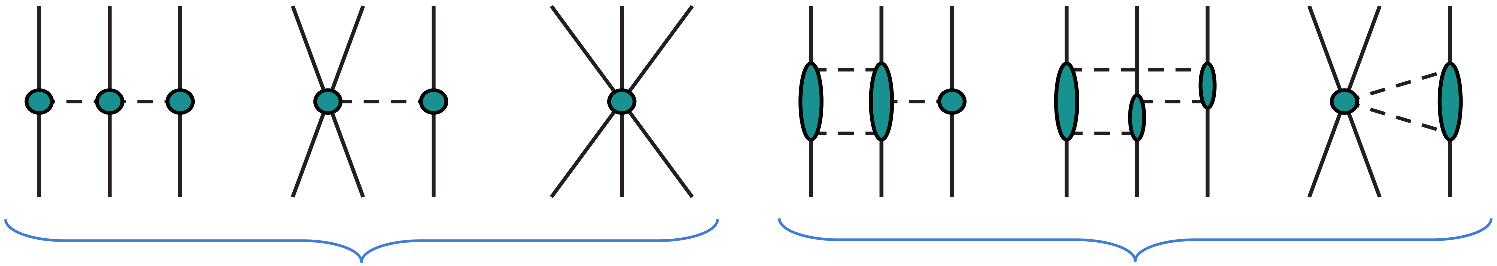
$$\longrightarrow\quad V_{3\mathrm{N}} = \sum_{i=1}^{22} \mathcal{G}_i\, F_i(r_{12}, r_{23}, r_{31}) + 5 \text{ perm.}$$

derivable in ChPT; long-range terms parameter-free predictions

Most general structure of a nonlocal 3NF and constraints on F_i from large-N_c also worked out. Schat, Phillips '13

3N force beyond N^2LO

3NF topologies (up to N^4LO)



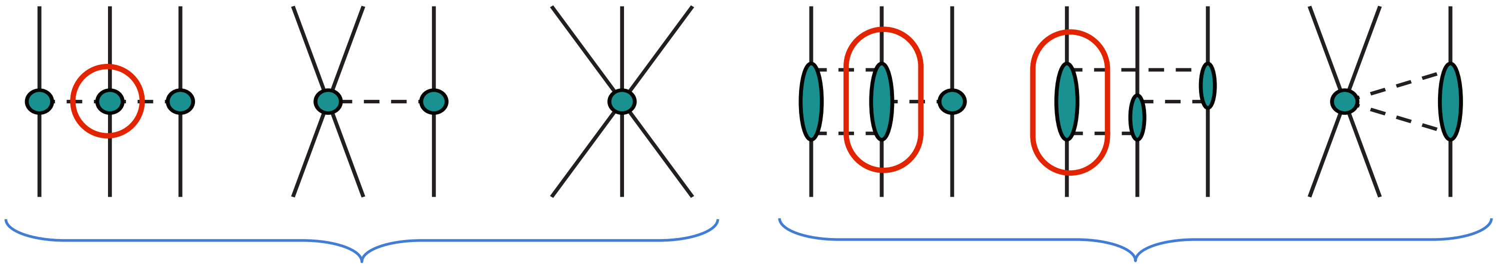
Q^3 (N^2LO) + Q^4 (N^3LO) + Q^5 (N^4LO) + ...

Q^4 (N^3LO) + Q^5 (N^4LO) + ...

3N force beyond N²LO

3NF topologies (up to N⁴LO)

3NF structure functions at large distance are model-independent, parameter-free predictions based on chiral symmetry of QCD + exp. information on πN system

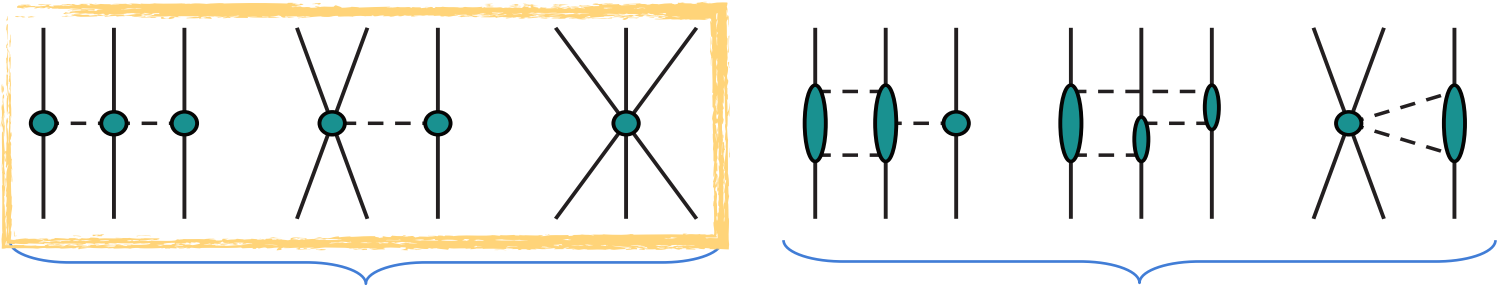


Q^3 (N²LO) + Q^4 (N³LO) + Q^5 (N⁴LO) + ...

Q^4 (N³LO) + Q^5 (N⁴LO) + ...

3N force beyond N^2LO

3NF topologies (up to N^4LO)



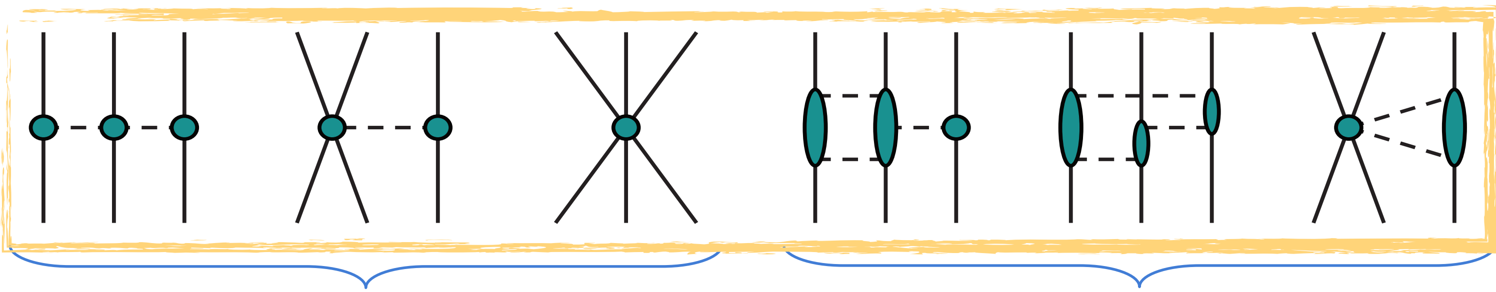
Q^3 (N^2LO) + Q^4 (N^3LO) + Q^5 (N^4LO) + ...

Q^4 (N^3LO) + Q^5 (N^4LO) + ...

- N^2LO contributions (leading 3NF) nowadays included in most few-/many-body calculations

3N force beyond N^2LO

3NF topologies (up to N^4LO)



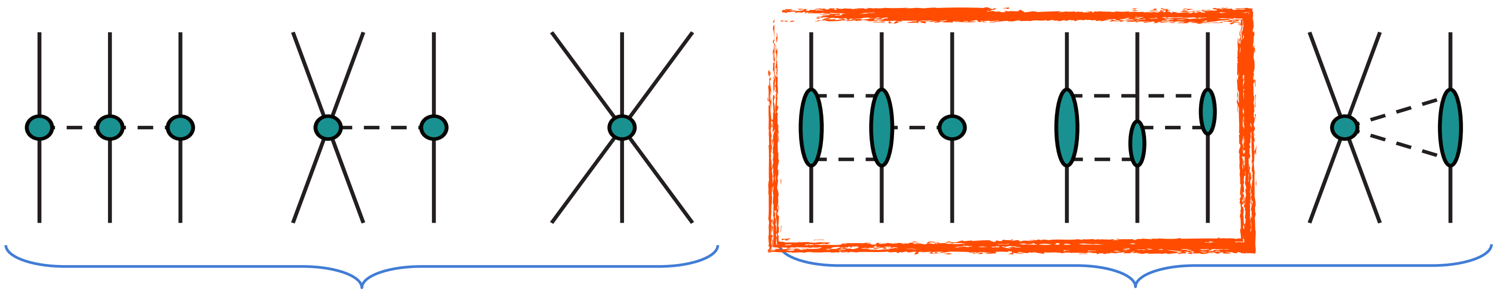
Q^3 (N^2LO) + Q^4 (N^3LO) + Q^5 (N^4LO) + ...

Q^4 (N^3LO) + Q^5 (N^4LO) + ...

- N^2LO contributions (leading 3NF) nowadays included in most few-/many-body calculations
- First corrections (N^3LO)
  Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); PRC84 (11)
  - parameter-free, rich spin-momentum structure (especially from ring diagrams)

3N force beyond N^2LO

3NF topologies (up to N^4LO)



Q^3 (N^2LO) + Q^4 (N^3LO) + Q^5 (N^4LO) + ...

Q^4 (N^3LO) + Q^5 (N^4LO) + ...

- N^2LO contributions (leading 3NF) nowadays included in most few-/many-body calculations
- First corrections (N^3LO)

  Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); PRC84 (11)

  - parameter-free, rich spin-momentum structure (especially from ring diagrams)
  - intermediate-range contributions not converged (effects of **Δ**(1232) are missing...)

subleading loop corrections (N^4LO)
in the **Δ**-less theory

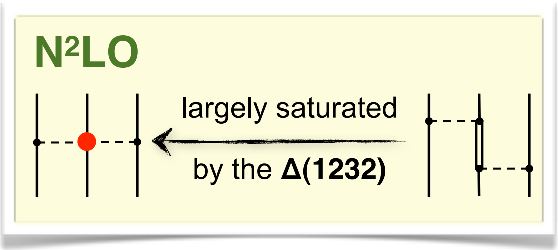
Krebs, Gasparyan, EE '12,'13

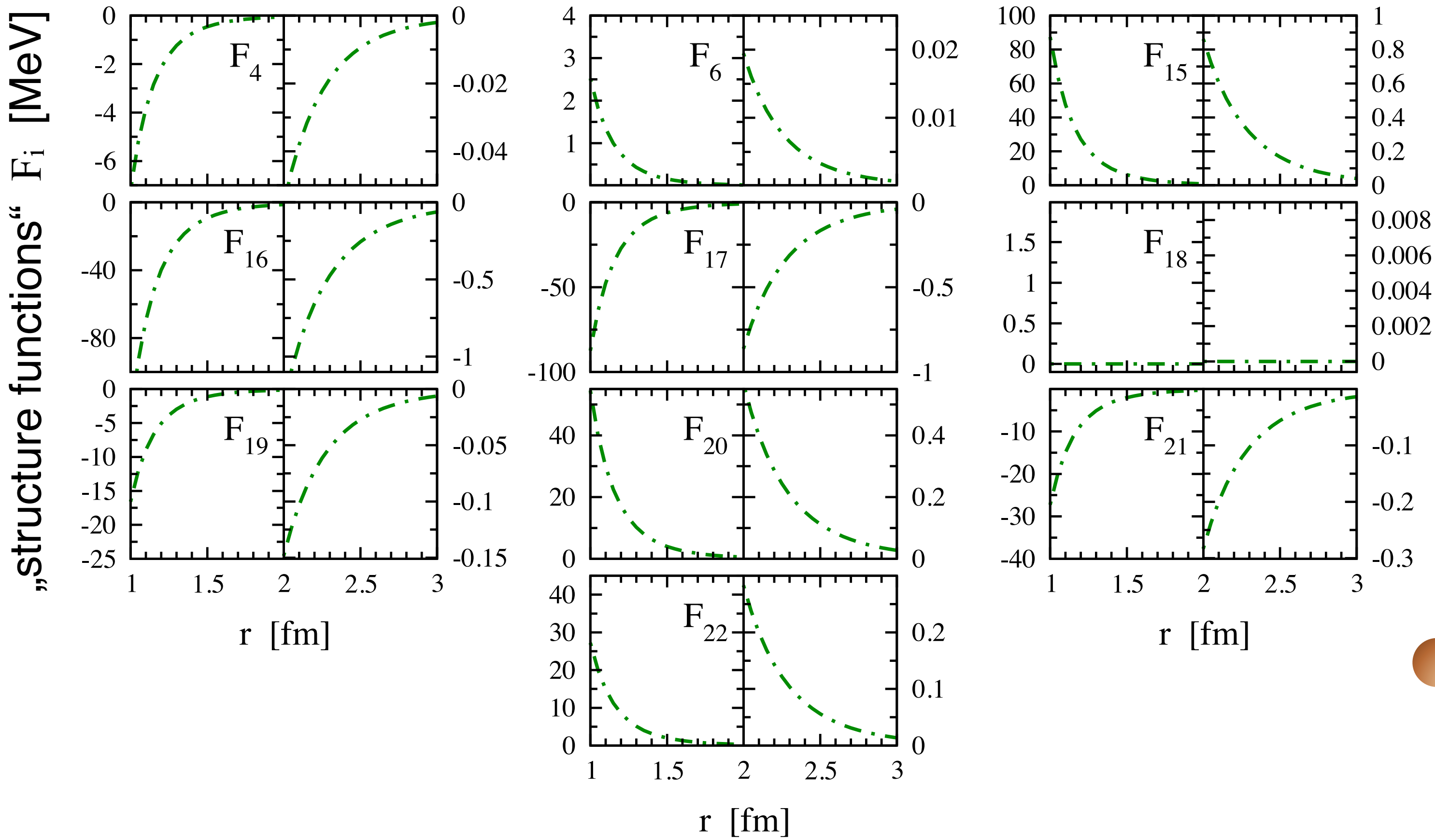
leading loop corrections (N^3LO)
in EFT with explicit **Δ**

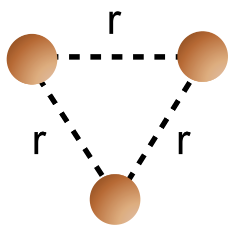
Krebs, Gasparyan, EE, to appear

Two-pion exchange 3NF up to N^{4}LO

Krebs, Gasparyan, EE ’12

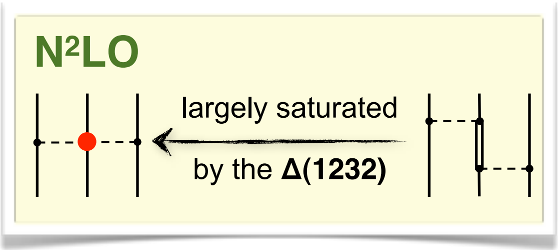


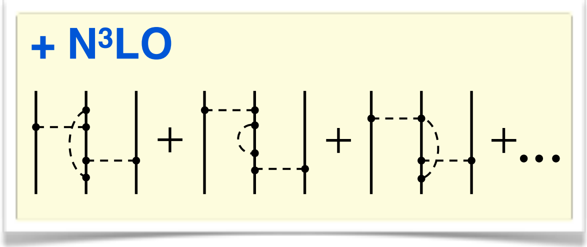


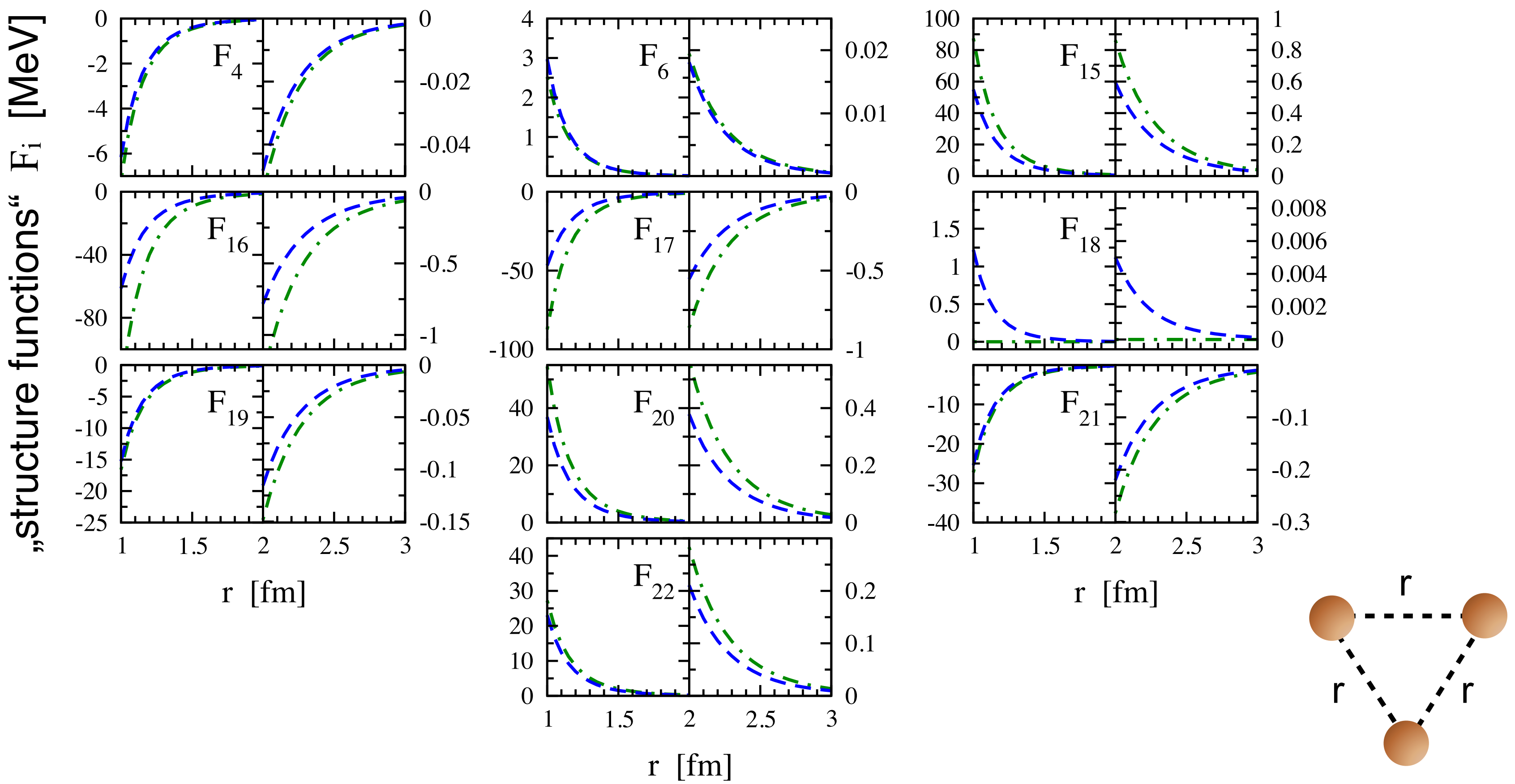


Two-pion exchange 3NF up to N⁴LO

Krebs, Gasparyan, EE '12

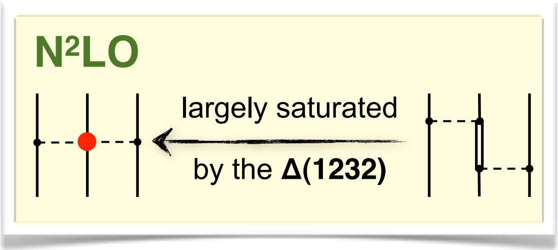


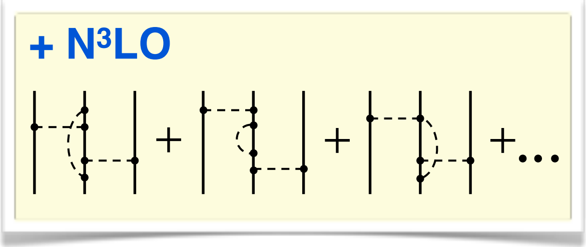


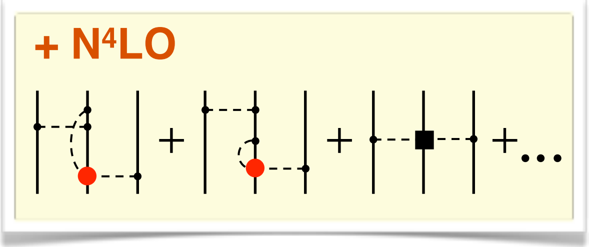


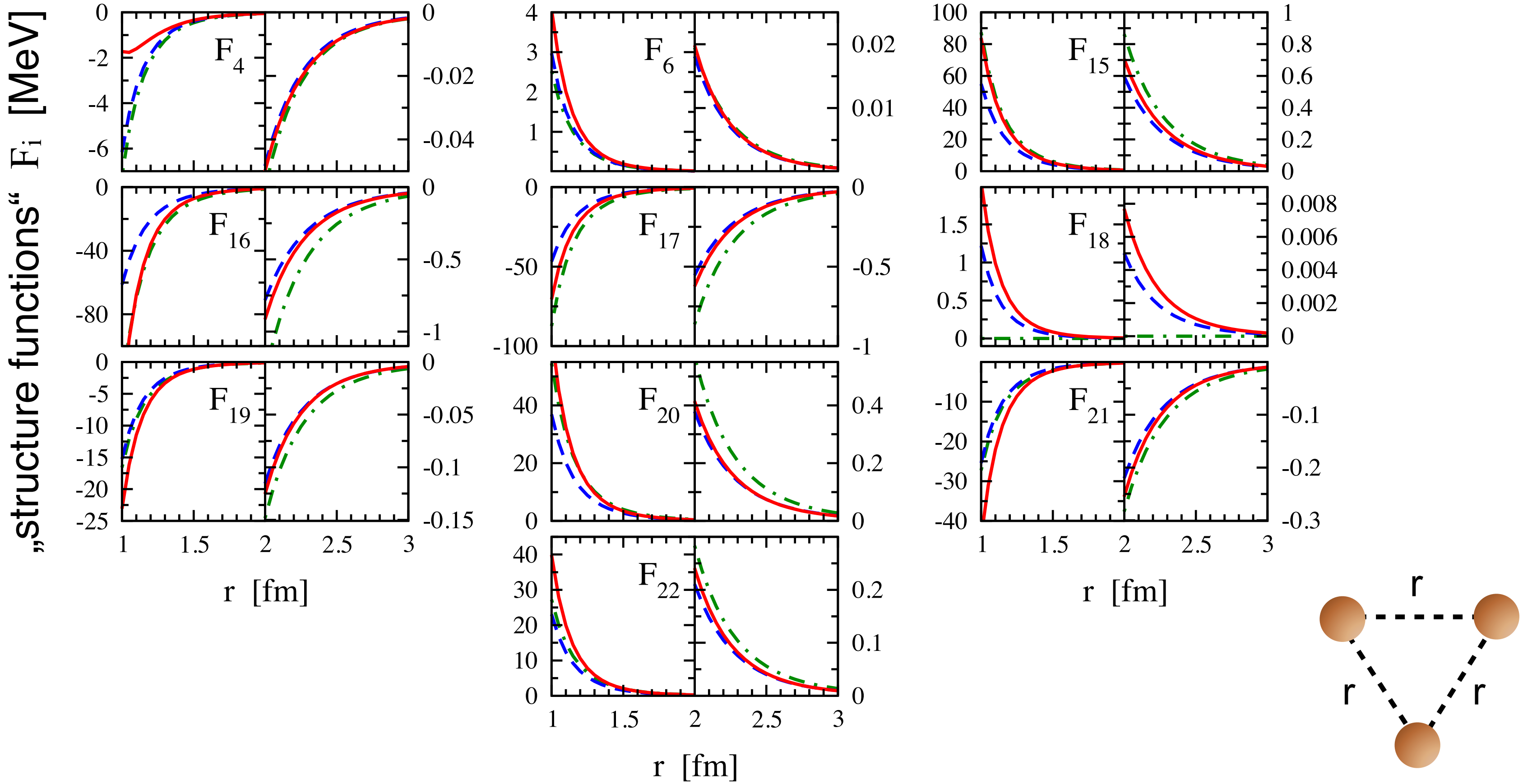
Two-pion exchange 3NF up to N⁴LO

Krebs, Gasparyan, EE '12





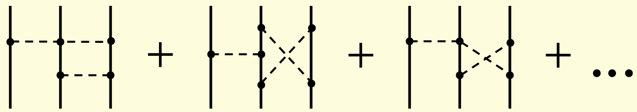


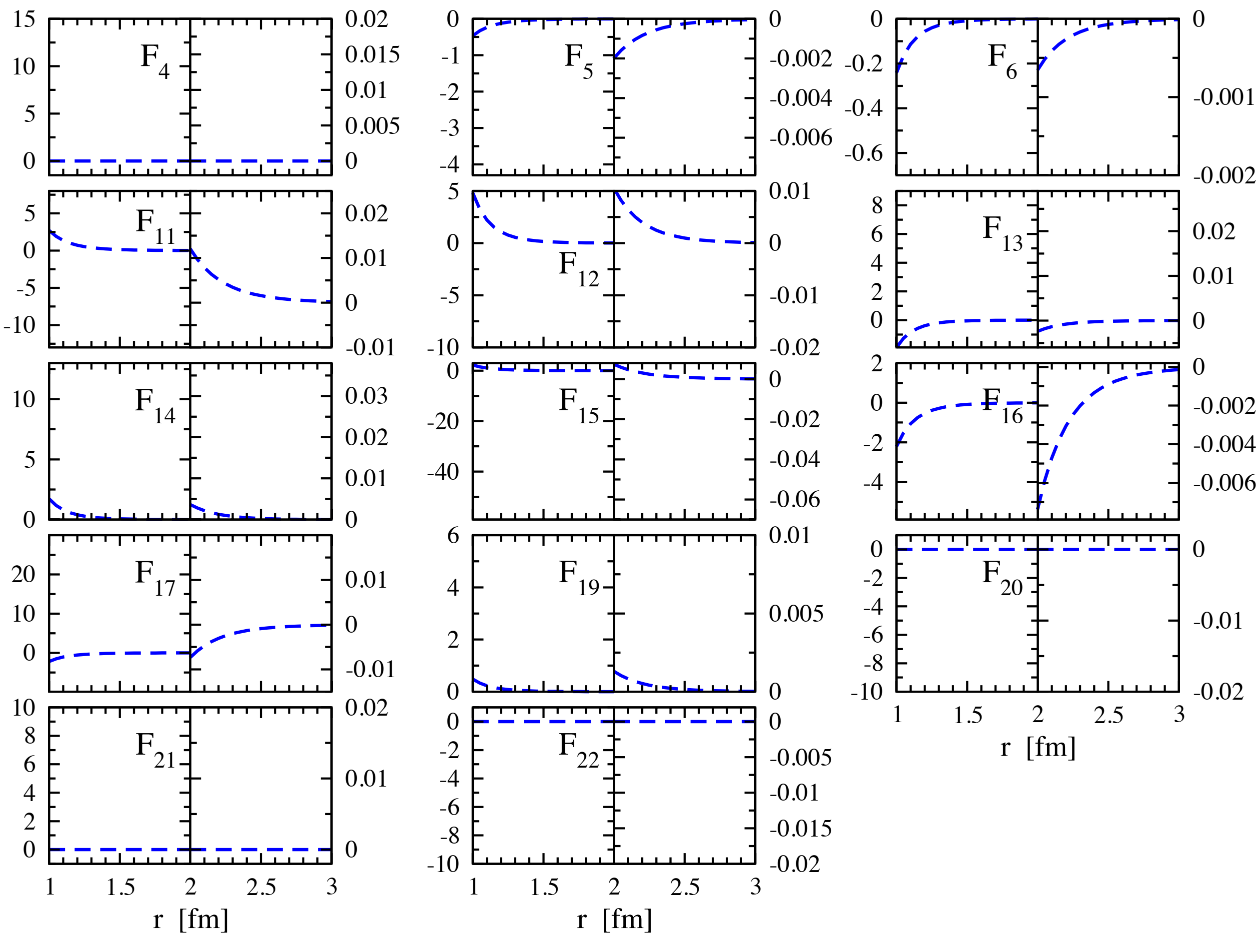


Intermediate-range 3NF up to N^{4}LO

Krebs, Gasparyan, EE ’13

N^{3}LO Bernard, EE, Krebs, Meißner ’08

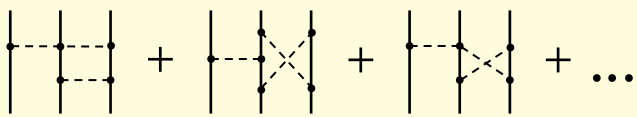




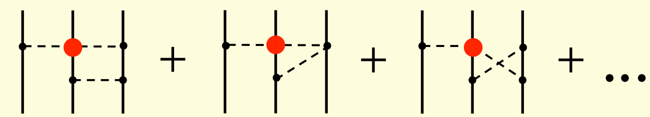
Intermediate-range 3NF up to N⁴LO

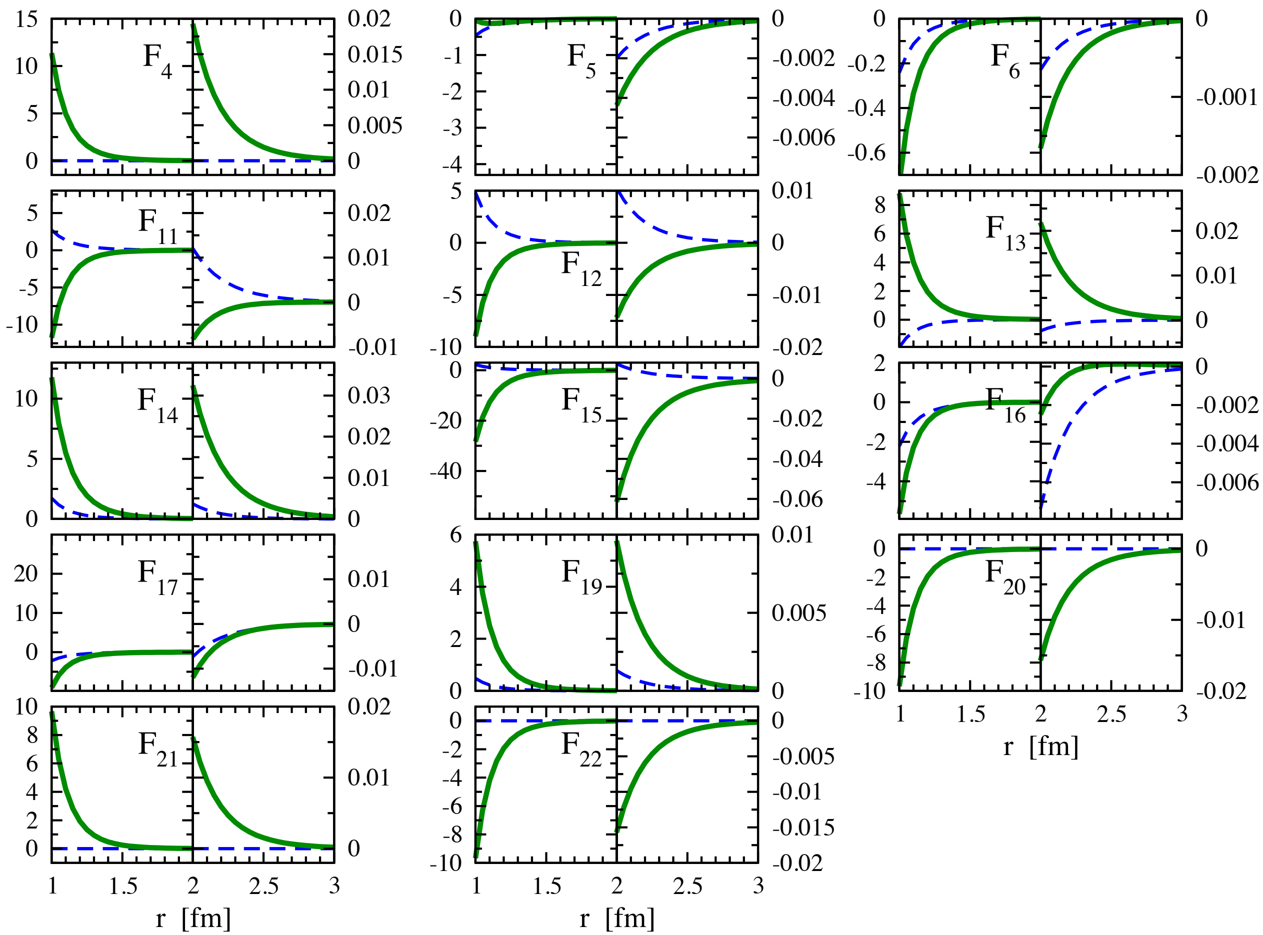
Krebs, Gasparyan, EE ’13

N³LO Bernard, EE, Krebs, Meißner ’08



+ N⁴LO Krebs, Gasparyan, EE ’13

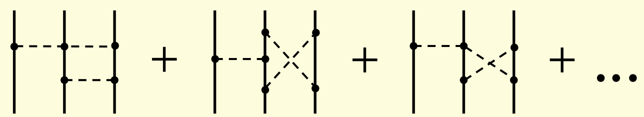




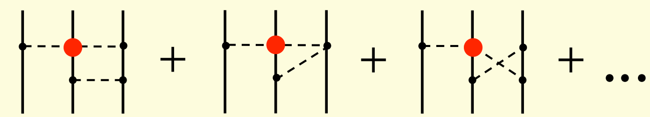
Intermediate-range 3NF up to N^4LO

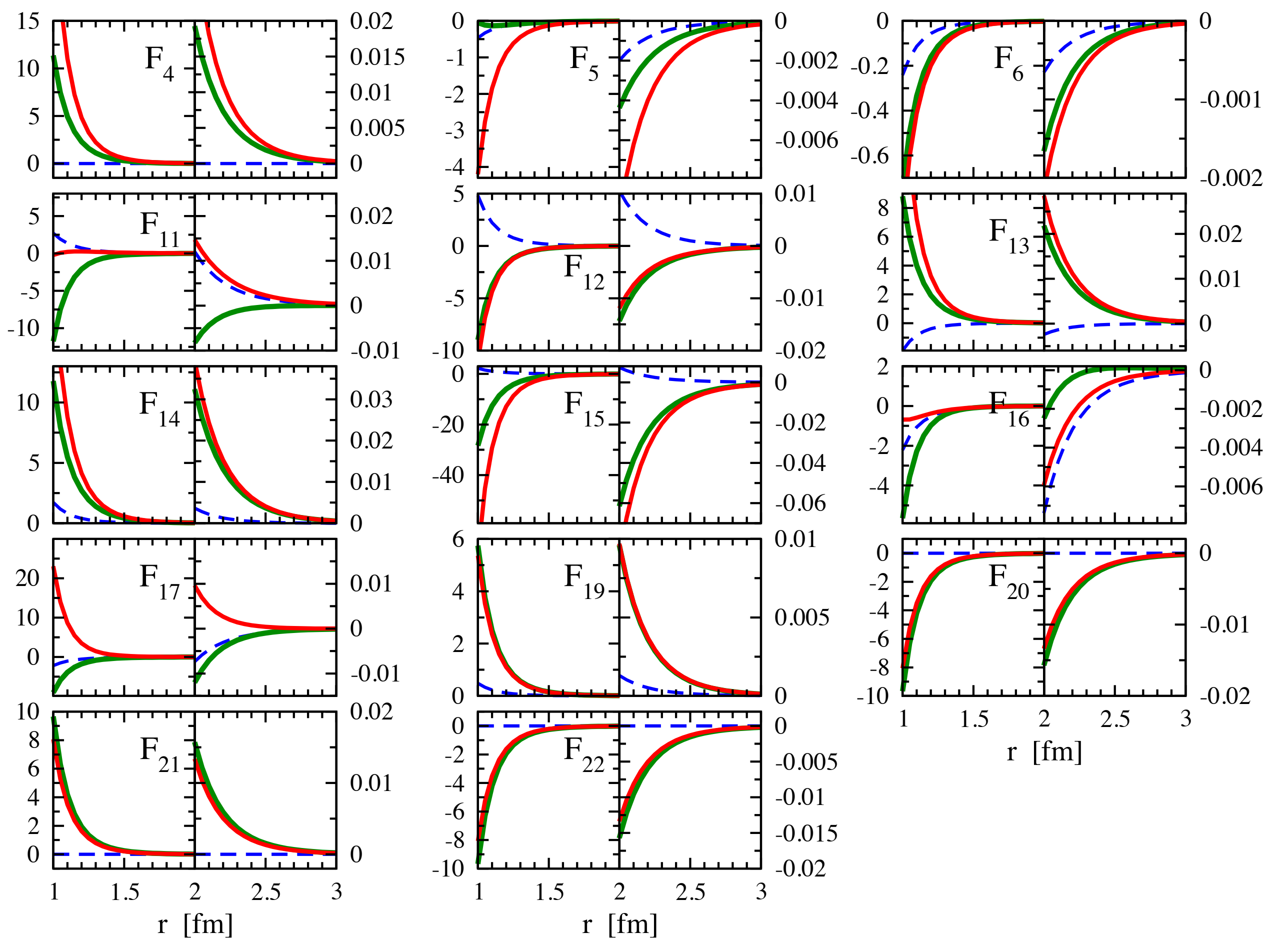
Krebs, Gasparyan, EE ’13

N^3LO Bernard, EE, Krebs, Meißner ’08



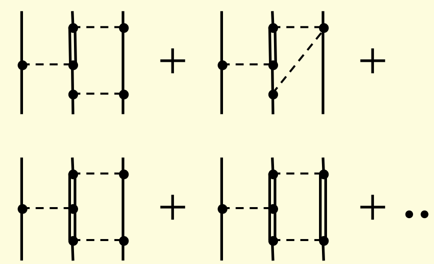
+ N^4LO Krebs, Gasparyan, EE ’13





N^3LO-Δ + N^4LO (nucl.)

Krebs, Gasparyan, EE, to appear

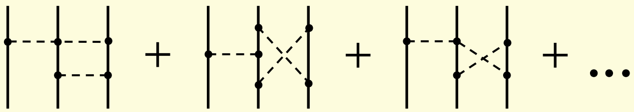


- Double-Δ excitations even more significant for the ring topology talk by Hermann Krebs
- Shorter-range topologies in progress; contact terms at N^4LO already worked out Girlanda et al.’12

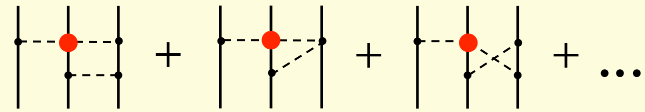
Intermediate-range 3NF up to N^4LO

Krebs, Gasparyan, EE ’13

N^3LO Bernard, EE, Krebs, Meißner ’08



+ N^4LO Krebs, Gasparyan, EE ’13



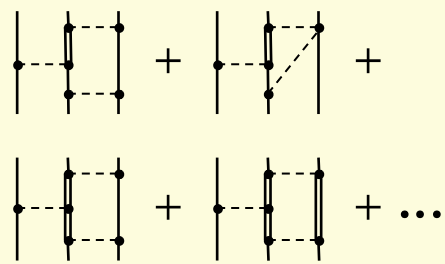
Numerical partial wave decomposition in progress (currently on JUQUEEN@Jülich, INTREPID@Argonne)

Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration **(LENPIC)**

J. Golak, R. Skibinski, K. Topolnicki, H. Witala (Cracow)
EE, H.Krebs (Bochum)
S. Binder, A. Calci, K. Hebeler, J. Langhammer, R. Roth (Darmstadt)
P. Maris, H. Potter, J. Vary (Iowa State)
R. J. Furnstahl (Ohio State)
A. Nogga (Jülich)
H. Kamada (Kyushu)
U.-G. Meißner (Bonn)
V. Bernard (Orsay)

N^3LO-Δ + N^4LO (nucl.)

Krebs, Gasparyan, EE, to appear

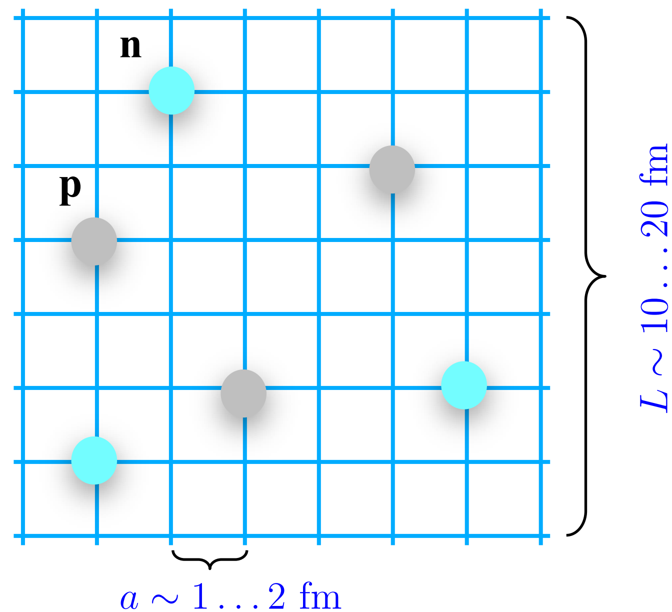


- Double-Δ excitations even more significant for the ring topology talk by Hermann Krebs
- Shorter-range topologies in progress; contact terms at N^4LO already worked out Girlanda et al.’12

Nuclear Lattice Effective Field Theory

The Collaboration: E.E., Hermann Krebs (Bochum), Timo Lähde (Jülich), Dean Lee (NC State),
Ulf-G. Meißner (Bonn/Jülich), Gautam Rupak (Mississippi State)

Borasoy, E.E., Krebs, Lee, Meißner, Eur. Phys. J. A31 (07) 105,
Eur. Phys. J. A34 (07) 185,
Eur. Phys. J. A35 (08) 343,
Eur. Phys. J. A35 (08) 357,
E.E., Krebs, Lee, Meißner, Eur. Phys. J A40 (09) 199,
Eur. Phys. J A41 (09) 125,
Phys. Rev. Lett 104 (10) 142501,
Eur. Phys. J. 45 (10) 335,
Phys. Rev. Lett. 106 (11) 192501,
E.E., Krebs, Lähde, Lee, Meißner Phys. Rev. Lett. 109 (12) 252501,
Phys. Rev. Lett. 110 (13) 112502,
Eur. Phys. J. A49 (13) 82



European Research Council

Calculation strategy

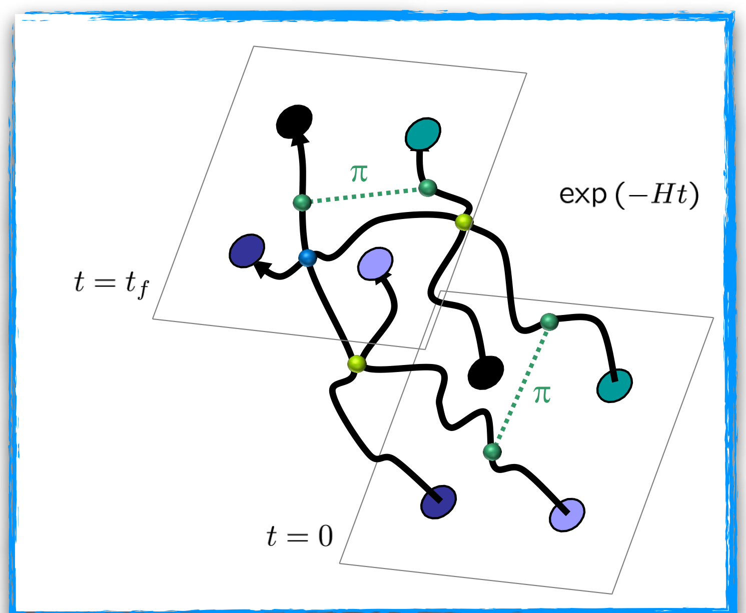
- Eucl.-time propagation of A nucleons → transition amplitude $Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

  → ground-state energies $E_A = -\lim_{t\to\infty} d(\ln Z_A)/dt$

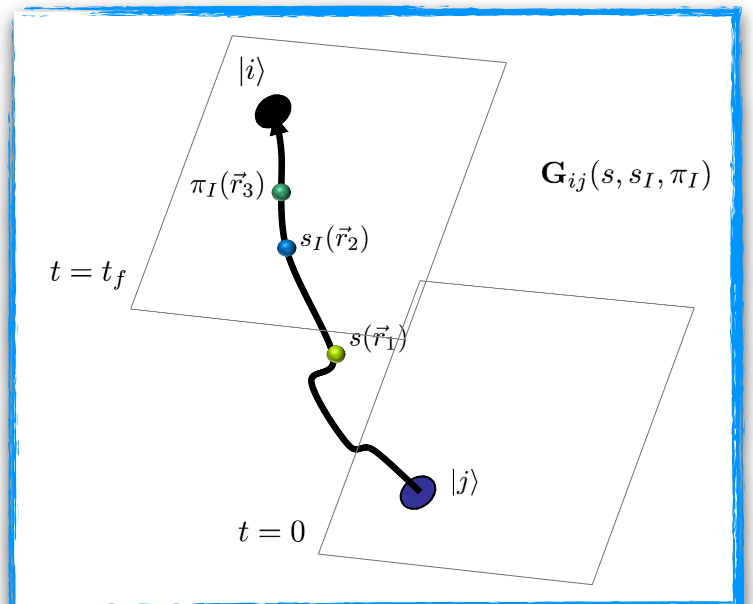
- Excited state energies can be obtained from a large-t limit of a correlation matrix $Z_A^{ij}(t) = \langle \Psi_A^i | \exp(-tH) | \Psi_A^j \rangle$ between A-nucleon states Ψ_A^j with the proper quantum numb.

- Use $H_{\rm LO}$ to run the simulation, higher-order terms (incl. Coulomb, 3NF, ...) taken into account perturvatively via $Z_A^O(t) = \langle \Psi_A | \exp(-tH/2)\, O\, \exp(-tH/2) | \Psi_A \rangle$

- We use Auxiliary-Field QMC method



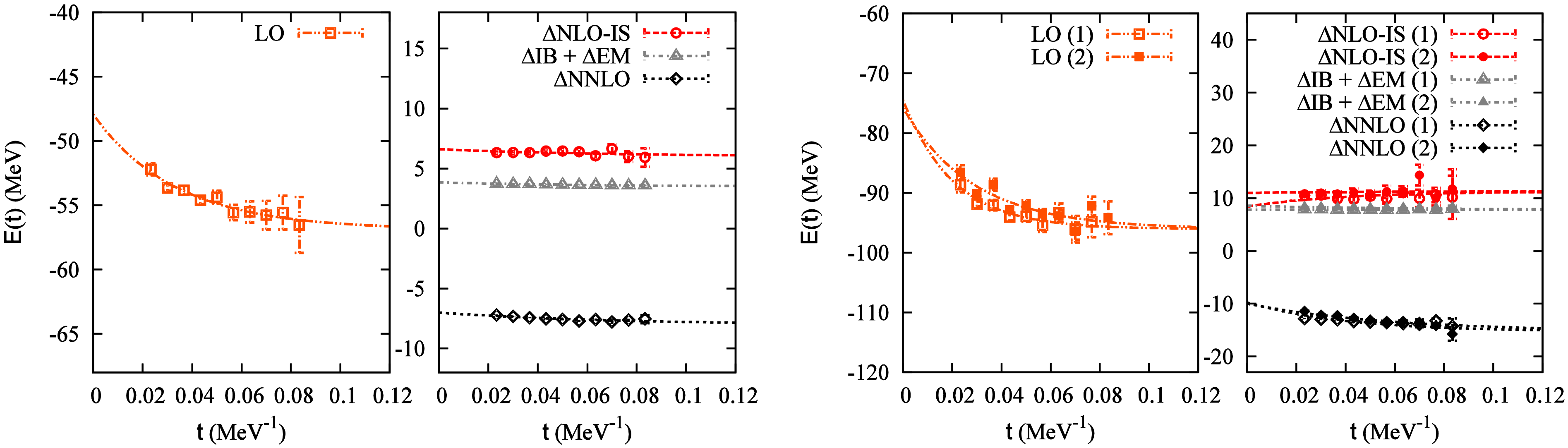
→



Ground states of ^{8}Be and ^{12}C

E.E., Krebs, Lee, Meißner, PRL 106 (11) 192501

Simulations for ^{8}Be and ^{12}C, L=11.8 fm



Ground state energies (L=11.8 fm) of ^{4}He, ^{8}Be, ^{12}C & ^{16}O

	^{4}He	^{8}Be	^{12}C	^{16}O
LO $[Q^0]$, in MeV	$-28.0(3)$	$-57(2)$	$-96(2)$	$-144(4)$
NLO $[Q^2]$, in MeV	$-24.9(5)$	$-47(2)$	$-77(3)$	$-116(6)$
NNLO $[Q^3]$, in MeV	$-28.3(6)$	$-55(2)$	$-92(3)$	$-135(6)$
Experiment, in MeV	-28.30	-56.5	-92.2	-127.6

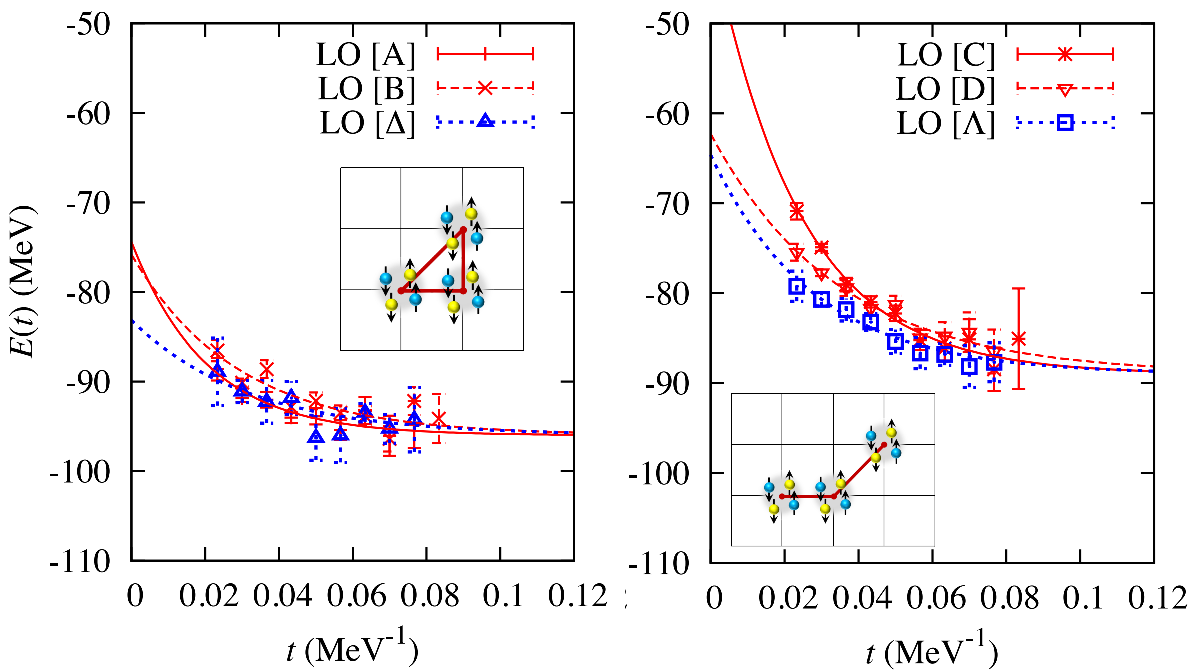
The Hoyle state

EE, Krebs, Lähde, Lee, Meißner, PRL 106 (2011) 192501; PRL 109 (2012) 252501

Lattice results for low-lying even-parity states of ^{12}C

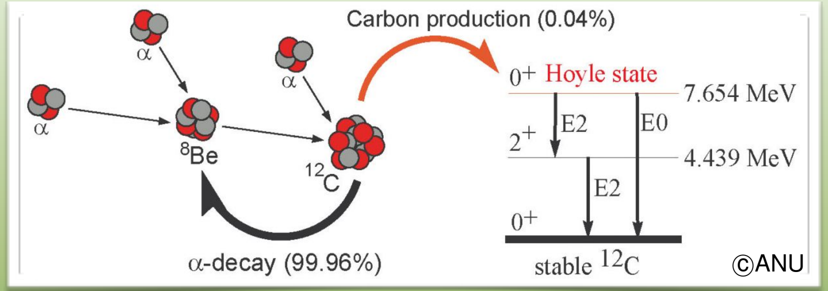
	0_1^+	$2_1^+(E^+)$	0_2^+	$2_2^+(E^+)$
LO	$-96(2)$	$-94(2)$	$-89(2)$	$-88(2)$
NLO	$-77(3)$	$-74(3)$	$-72(3)$	$-70(3)$
NNLO	$-92(3)$	$-89(3)$	$-85(3)$	$-83(3)$
Exp	-92.16	-87.72	-84.51	$-82(1)$

Probing (α-cluster) structure of the 0_1^+, 0_2^+ states



RMS radii and quadrupole moments

	LO	Experiment
$r(0_1^+)$ [fm]	2.2(2)	2.47(2) [26]
$r(2_1^+)$ [fm]	2.2(2)	–
$Q(2_1^+)$ [$e\,\mathrm{fm}^2$]	6(2)	6(3) [27]
$r(0_2^+)$ [fm]	2.4(2)	–
$r(2_2^+)$ [fm]	2.4(2)	–
$Q(2_2^+)$ [$e\,\mathrm{fm}^2$]	$-7(2)$	–



The triple alpha reaction rate as a function of the quark mass

Production of ^{12}C in stars depends sensitively on the energy differences:

$$\Delta E_b \equiv E_8 - 2E_4\,,$$
$$\Delta E_h \equiv E_{12}^\star - E_8 - E_4$$

Reaction rate for the triple alpha process:

$$r_{3\alpha} \simeq 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_\mathrm{B} T}\right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_\mathrm{B}T}\right)$$

Oberhummer, Csoto, Schlattl, Science 289 (2000) 88

where $\varepsilon \equiv \Delta E_b + \Delta E_h = E_{12}^\star - 3E_4 = 379.47(18)\ \mathrm{keV}$ - crucial control parameter

Changing ε by ~100 keV destroys production of either ^{12}C or ^{16}O Livio et al.'89; Oberhummer, et al.'00

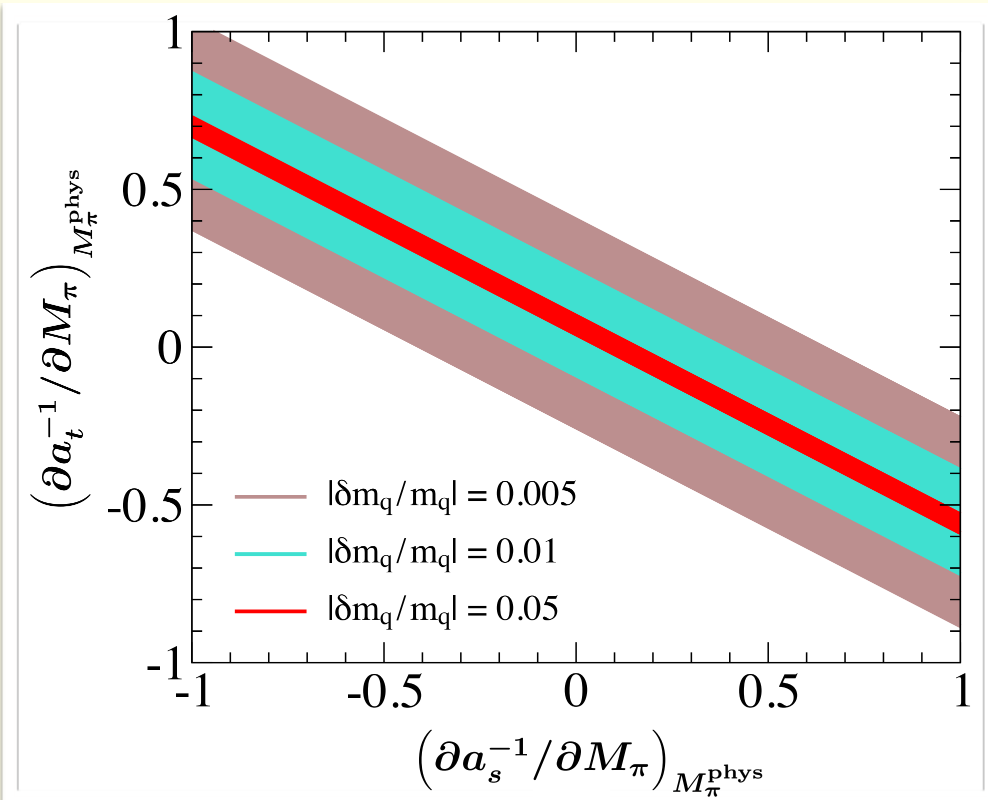
How robust is ε with respect to variations of the light quark mass?

Quark mass dependence of the triple-α reaction rate

EE, Krebs, Lähde, Lee, Meißner, PRL 110 (2013) 112502; arXiv:1303.4856 (to appear in EPJA)

$$|\delta\varepsilon| < 100 \text{ keV} \longrightarrow \left| \left(0.771(14) \left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.934(11) \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$

„Survivability bands“ for carbon-oxygen based life due to 0.5%, 1%, 5% variation of m_q

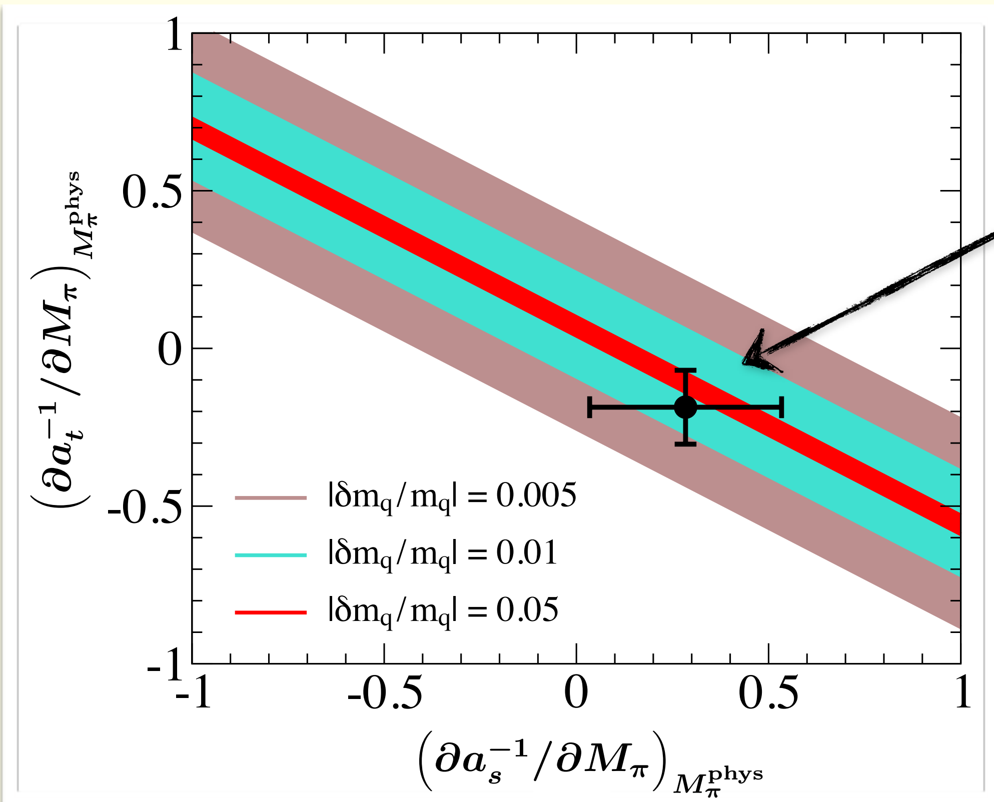


Quark mass dependence of the triple-α reaction rate

EE, Krebs, Lähde, Lee, Meißner, PRL 110 (2013) 112502; arXiv:1303.4856 (to appear in EPJA)

$$|\delta\varepsilon| < 100\ \text{keV} \longrightarrow \left| \left(0.771(14) \left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\text{phys}}} + 0.934(11) \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\text{phys}}} - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$

„Survivability bands“ for carbon-oxygen based life due to 0.5%, 1%, 5% variation of m_q



up-to-date chiral EFT calculation (N^2LO):

$$K_{a_s}^q = 2.3^{+1.9}_{-1.8}, \quad K_{a_t}^q = 0.32^{+0.17}_{-0.18}$$

Berengut et al., PRD 87 (2013)

Summary and outlook

Nuclear chiral dynamics enters precision era:

- low-energy NN scattering is accurately described at N^3LO
- many high-precision few-N studies: pion-N scatt. lengths, Compton scattering, pion photoproduction, FFs, radiative/muon capture...
- impressive progress in ab initio many-body methods, precise nuclear structure calculations for light nuclei become reality!
- The main source of uncertainty is presently due to the 3NF... (higher-order corrections in progress)

Nuclear lattice simulations:

- combining EFT and lattice simulations → access to (light) nuclei
- exciting results for the ^{12}C spectrum, first ab initio calculation of the Hoyle state

Work in progress: heavier systems, volume dependence, reactions ...