Three-body calculation of the rate of reaction $p+p+e^{-}\rightarrow d+\nu_e$ in the Sun

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The sun is yellow
The sun is orange
The sun is bright
The sun is our heater
The sun is our fire
The sun is our light
Let me ask you a question,
What do you think of the sun?

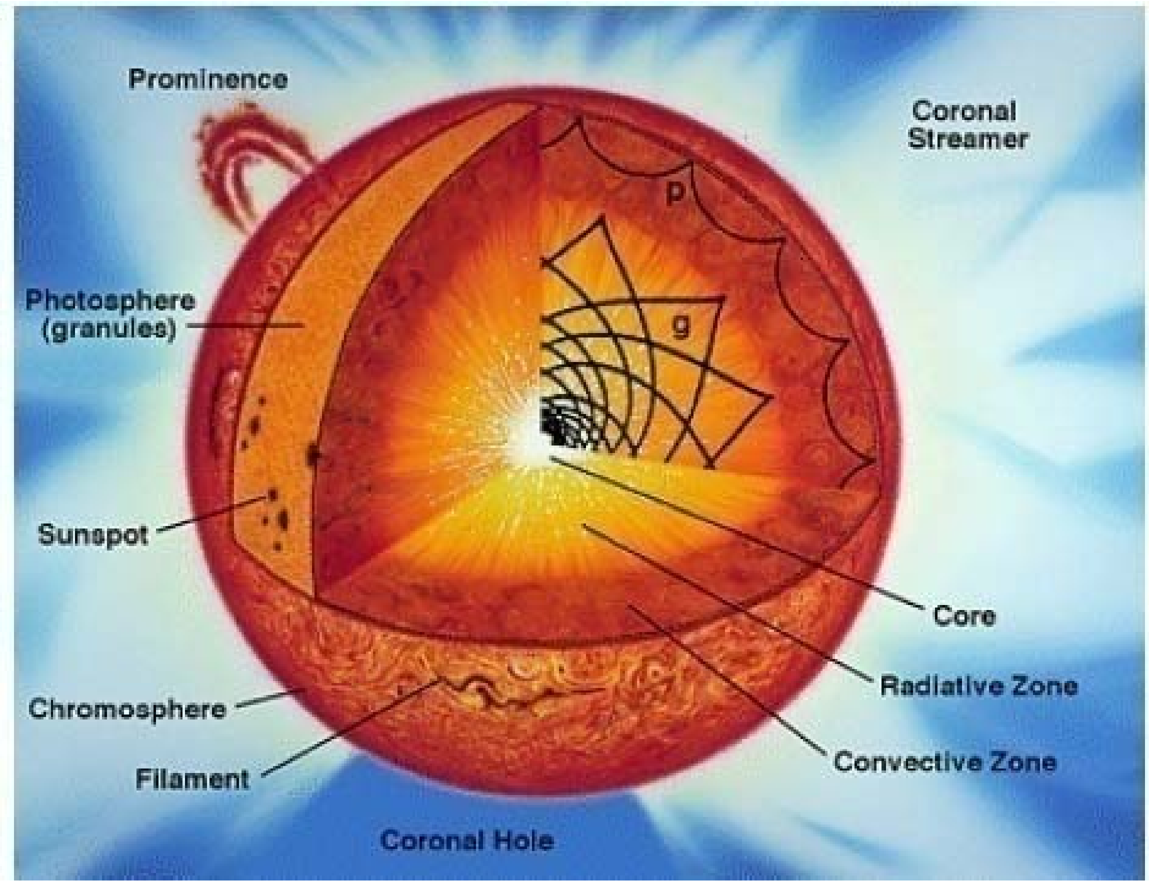
Krystal Galvis

Outline

- 1. Introduction
- 2. Motivation
- 3. Inputs
- 4. Wave function of the initial state
- 5. Transition amplitude
- 6. Rate and neutrino flux
- 7. Survival probability of neutrino
- 8. Comparison with Borexino experimental data and Bahcall's results
- 9.Conclusion

Introduction

SOLAR STRUCTURE .



Calculating a solar model means the determination of pressure, temperature and chemical composition as a function of mass or radius through the Sun, (Chandrasekhar 1967; Kourganoff 1973).

Table 1. Internal structure of the Sun (R = 6.96 10^5 km)

Internal region	extension in terms of solar radius	chemical composition
core	0.20 $R_\odot$	center only: He: 0.63, H: 0.35, metals: 0.02 (almost actively ionized matter)
radiative zone	0.50 $R_\odot$	He: 0.23, H: 0.75, metals: 0.02 (highly ionized)
convective zone	0.30 $R_\odot$	same (less ionized)
photosphere	0.002 $R_\odot$	same (less ionized)
solar surface	1.000 $R_\odot$	
chromosphere	0.02	same (less ionized)
corona	$\approx$ 5	same (highly ionized)

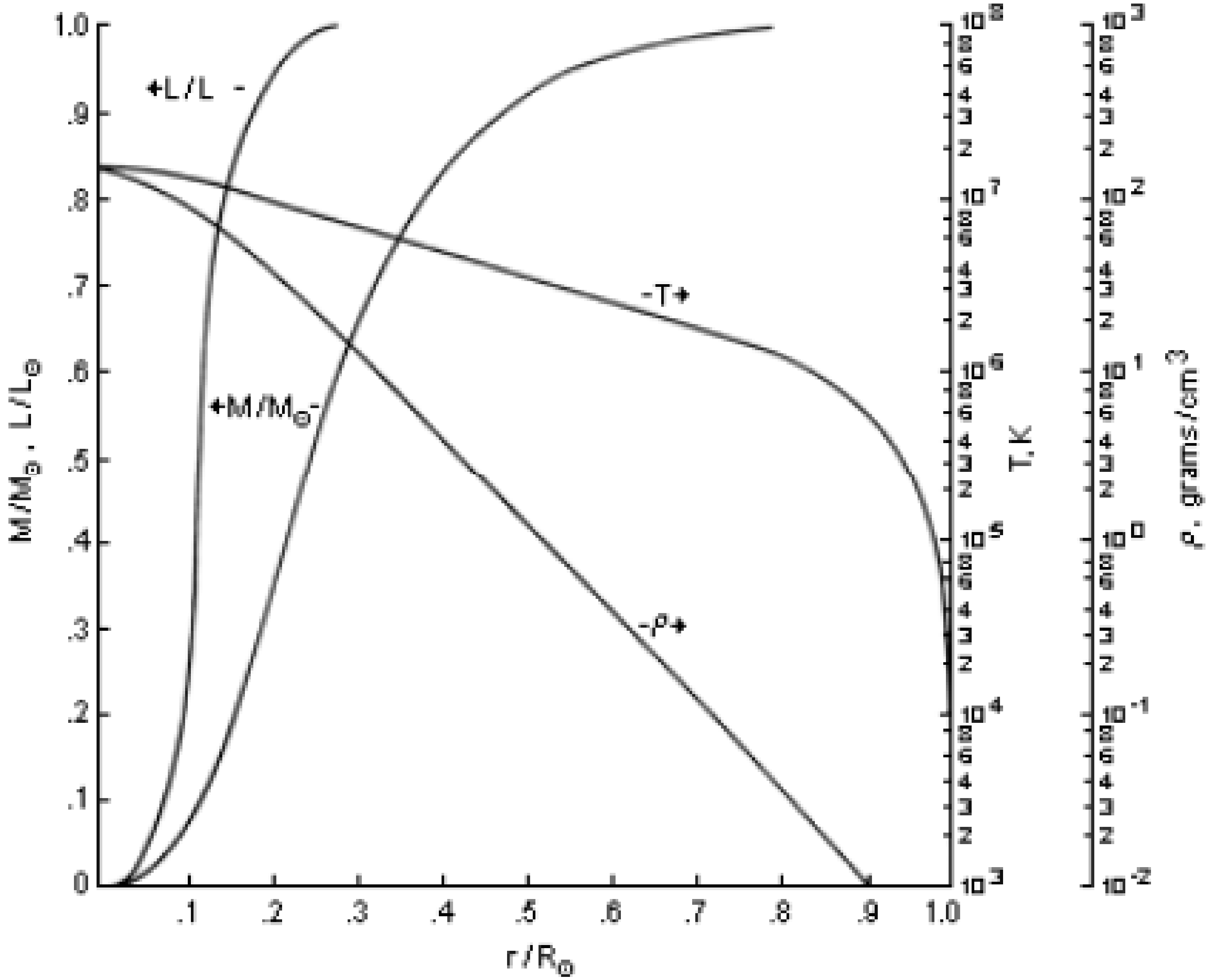
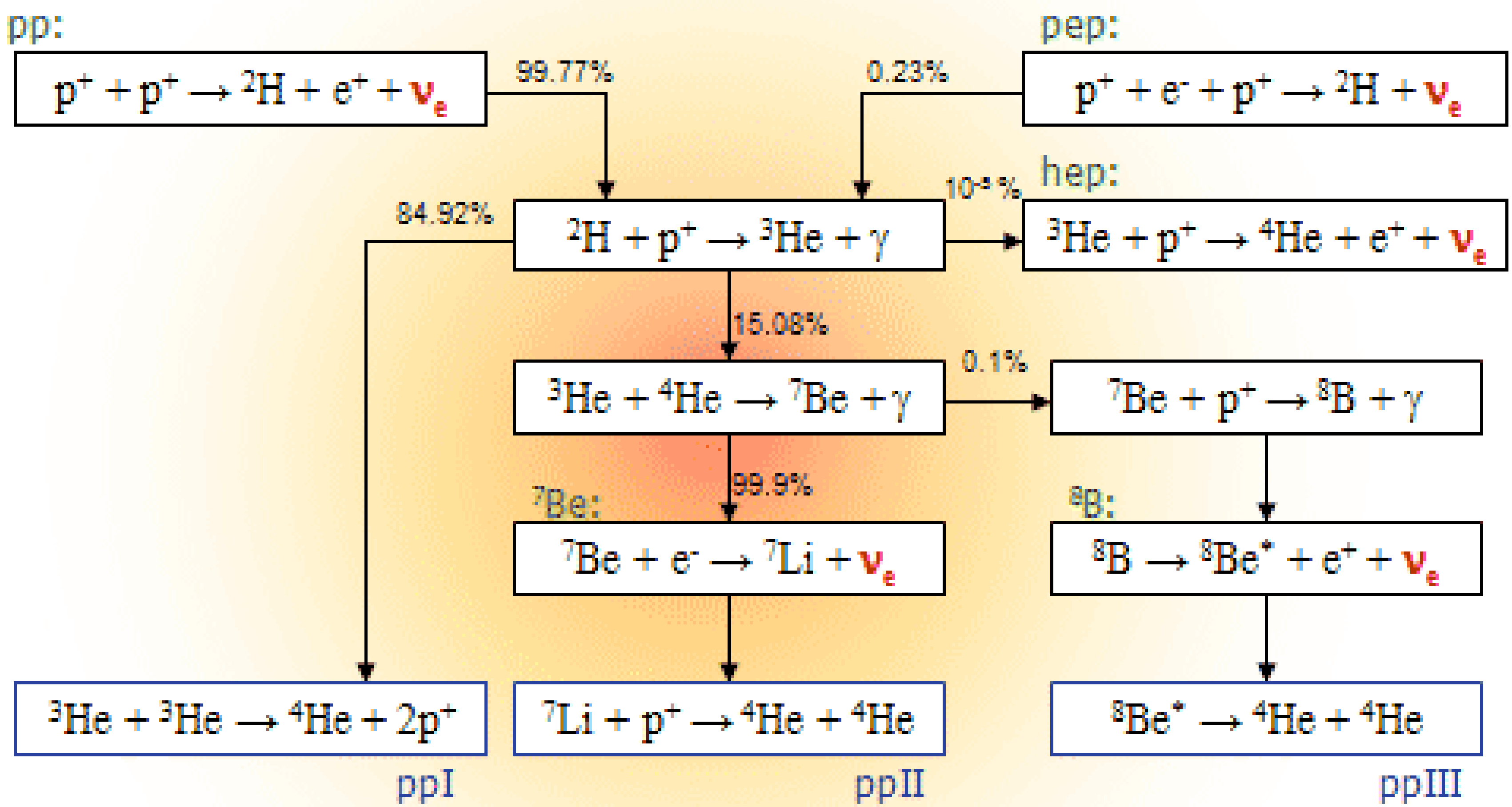


Figure 1. A standard solar model of the present solar interior:
$X = 0.708$, $Y = 0.272$, $Z = 0.0020$, $\rho_c = 158 gcm^3$, $T_c = 1.57\ 10^7 K$.

The proton–proton chain reactions in the Sun



The neutrinos in the ppI, ppII and ppIII chains carry away 2.0%, 4.0%, and 28.3% of the energy in those reactions, respectively.

Motivation

a) The neutrinos released by the pep reaction are far more energetic: while neutrinos produced in the first step of the pp reaction range in energy up to 0.42 MeV, the pep reaction produces sharp-energy-line neutrinos of 1.442 MeV.

The first detection of solar neutrinos from the pep reaction were reported by the Borexino collaboration in 2012.

b) Absence of three-body treatment for the pep reaction. Bahcall's calculation was based on two-body approximation.

Neutrino flux estimation from luminosity of the Sun

- Distance from the Sun to the Earth R= 1.496×10^{13} cm;
- Luminosity $L_\odot$ =2.4×10^{39} MeV/s; $\Phi = n_\nu L_\odot \cdot P/(\Delta E \cdot 4\pi R^2)$

The pp chain releases energy $\Delta E=(4m_p+2m_e-m_{4He})c^2=26.73$ MeV. Number of emitted neutrinos in processes of ppI branch $n_\nu=2$; From the energy ΔE the energy of the emitted 2 neutrinos should be subtracted. The mean energy of neutrino is $E_\nu=0.263$ MeV for pp reaction and $E_\nu=1.442$ MeV for pep one (from Cauldrons). Probability of ppI branch P_{ppI} = 84.72 ÷ 91.0 %, while probability of **pep is** P_{pep}=0.23 ÷0.33 %. If the luminosity $L_\odot$ is multiplied by the probability of the branch and divided by ΔE we get number of the circles $4p \rightarrow {}^4He$ in reactions, then multiplying by n_ν and dividing on the area of sphere of 1 AU we get the flux.

The lowest limit of the flux at 1 AU from pp → d+e⁺+v

$$\Phi_{pp} = 5.52\times10^{10} \div 5.91\times10^{10} \text{ neutrinos/cm}^2\text{/s}$$

The lowest limit of the flux at 1 AU from pep→d+v

$$\Phi_{pep} = 1.40\times10^{8} \div 2.00\times10^{8} \text{ neutrinos/cm}^2\text{/s}$$

Inputs

- Weak Hamiltonian (effective)

$$H_w = \frac{1}{\sqrt{2}} \tau^{(+)} \frac{1-\vec{\sigma}\cdot\hat{\nu}}{\sqrt{2}} \sum_{i=1}^{2} \tau_i^{(-)} \{ G_V 1 \cdot 1_i + G_A \vec{\sigma}\cdot\vec{\sigma}_i$$

$$- G_P \vec{\sigma}\cdot\hat{\nu}\vec{\sigma}_i \cdot\hat{\nu} \} \delta(\vec{r}-\vec{r}_i), \qquad (1)$$

G_V, G_A and G_P are vector, axial vector and "induced" pseudoscalar coupling constants, respectively; $\vec{r}$ and $\vec{r}_i$ are space coordinates of the lepton and the ith nucleon; $\vec{\tau}, \vec{\tau}_i$ are isobaric-spin operators which transform a lepton electron state into a lepton neutrino state and an ith nucleon proton state into an ith nucleon neutron state; 1, 1_i, and $\vec{\sigma}, \vec{\sigma}_i$ are 2×2 matrix unit operators and spin angular momentum operators for the lepton and the ith nucleon,

$$\hat{\nu} = \frac{\vec{\nu}}{|\vec{\nu}|}, \; G_p \text{ is proportional to } \frac{\nu}{2m_p c}.$$

NN experimental data:

$$^{s}a_{pp} = -7.882 \pm 0.004 \text{ fm}, \quad ^{s}r_{pp} = 2.830 \pm 0.017 \text{ fm}$$

$$^{t}a_{np} = 5.44 \pm 0.005 \text{ fm}, \quad ^{t}r_{np} = 1.750 \pm 0.005 \text{ fm}, \quad \varepsilon_d = 2.225 \text{ MeV}$$

- Nuclear potentials

a) Guass potential

$$V(r) = -V_0 \exp(-r^2 / R_N^2) \qquad (2)$$

$$V_0^s = 30.36 \text{ MeV}, \quad R_N^s = 1.816 \text{ fm}$$

$$^{s}a_{pp} = \text{-}7.884 \text{ fm}, \quad ^{s}r_{pp} = 2.673 \text{ fm}$$

$$V_0^t = 60.752 \text{ MeV}, \quad R_N^t = 1.65 \text{ fm}$$

$$^{t}a_{np} = 5.48 \text{ fm}, \quad ^{t}r_{np} = 1.85 \text{ fm},$$

$$\varepsilon_d = 2.225 \text{ MeV}$$

b) Yukawa potential.

$$V(r) = -\frac{V_0}{r/R_N}\exp(-r/R_N) \qquad (3)$$

$$V_0^s = 44.05 \text{ MeV}, \;\; R_N^s = 1.206 \text{ fm}$$

$${}^s a_{pp} = -7.782 \text{ fm}, \;\; {}^s r_{pp} = 2.868 \text{ fm}$$

$$V_0^t = 53.27 \text{ MeV}, \quad R_N^t = 2.43 \text{ fm}$$

$${}^t a_{np} = 5.626 \text{ fm}, \quad {}^t r_{np} = 1.895 \text{ fm},$$

$$\varepsilon_d = 2.225 \text{ MeV}$$

c) exponential potential.

$$V(r) = -V_0 \exp(-r / R_N) \qquad (4)$$

$$V_0^s = 98.10 \text{ MeV}, \;\; R_N^s = 0.744 \text{ fm}$$

$${}^s a_{pp} = \text{-}7.874 \text{ fm}, \;\; {}^s r_{pp} = 2.804 \text{ fm}$$

$$V_0^t = 184.08 \text{ MeV}, \quad R_N^t = 0.683 \text{ fm}$$

$${}^t a_{np} = 5.403 \text{ fm}, \qquad {}^t r_{np} = 1.716 \text{ fm},$$

$$\varepsilon_d = 2.225 \text{ MeV}$$

d) Malfliet-Tjon potential

$$V(r) = V_A \frac{\exp(-r/R_A)}{r/R_A} + V_R \frac{\exp(-r/R_R)}{r/R_R} \qquad (5)$$

$$V_A^s = \text{-898.75 MeV}, \;\; R_A^s = 0.617 \text{ fm};$$

$$V_R^s = 4319.85 \text{ MeV}, \;\; R_R^s = 0.325 \text{ fm}$$

$$^s a_{pp} = \text{-7.88 fm}, \;\; ^s r_{pp} = 2.69 \text{ fm}$$

$$V_A^t = \text{-945.50 MeV}, \quad R_A^t = 0.645 \text{ fm};$$

$$V_R^t = 4476.845 \text{ MeV}, \quad R_R^t = 0.322 \text{ fm};$$

$$^t a_{np} = 5.52 \text{ fm}, \qquad ^t r_{np} = 1.89 \text{ fm}, \; \varepsilon_d = 2.225 \text{ MeV}$$

Wave function of the initial state

- Hamiltonian for pep system

$$H = H_0 + V_{23}^N + V_{123}^C, \qquad (6)$$

$$V_{123}^C = V_{12}^C + V_{13}^C + V_{23}^C,$$

1- electron, 2 and 3 - protons

- Jacobi variables

$$\vec{x}_i = \sqrt{\frac{m_j m_k}{(m_j + m_k)\mu_{23}}}\left(\vec{r}_j - \vec{r}_k\right), \qquad (7)$$

$$\vec{y}_i = \sqrt{\frac{m_i(m_j + m_k)}{(m_i + m_j + m_k)\mu_{23}}}\left(-\vec{r}_i + \frac{m_j\vec{r}_j + m_k\vec{r}_k}{m_j + m_k}\right), \qquad (8)$$

$$\mu_{23} = m_2 m_3 /(m_2 + m_3) = m_p / 2.$$

- Hyperharmonics:

$$\rho^2 = x_i^2 + y_i^2 = x_j^2 + y_j^2 = x_k^2 + y_k^2, \quad x_l = \rho\cos\alpha_l, y_l = \rho\sin\alpha_l,$$

$$H_0 = -\frac{\hbar^2}{2\mu_{23}}(\Delta_{x_i} + \Delta_{y_i}) = -\frac{\hbar^2}{2\mu_{23}}\left(\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{1}{\rho^2}K^2\right), \qquad (9)$$

$$K^2 = -\frac{\partial^2}{\partial\alpha_i^2} - 4\cot 2\alpha_i \frac{\partial}{\partial\alpha_i} + \frac{l^2(\hat{x}_i)}{\cos^2\alpha_i} + \frac{l^2(\hat{y}_i)}{\sin^2\alpha_i};$$

$K^2\Phi(\Omega) = \Lambda\Phi(\Omega), \qquad \Omega = \{\hat{x}_i, \hat{y}_i, \alpha_i\}$ is a set of angles,

$\Lambda = K(K+4), \qquad K = 0, 2, 4\ldots$ is hypermoment

$$\Phi_{KM}^{Ll_xl_y}(\Omega) = N_K^{l_xl_y}\sum_{m_xm_y} C_{l_xm_xl_ym_y}^{LM}\left(\cos\alpha\right)^{l_x}\left(\sin\alpha\right)^{l_y}\times$$

$$P_n^{l_y+1/2,l_x+1/2}\left(\cos 2\alpha\right)Y_{l_xm_x}(\hat{x})Y_{l_ym_y}(\hat{y}), \quad l^2(\hat{x}_i) = -\Delta_{\hat{x}_i}, l^2(\hat{y}_i) = -\Delta_{\hat{y}_i},$$

$P_n^{l_1 l_2}$ is the Jacobi polinomial, and $n = (1/2)(K - l_x - l_y)$,

$$N_K^{l_x l_y} = \sqrt{\frac{2n!(K+2)(n+l_x+l_y)!}{\Gamma(n+l_x+3/2)\Gamma(n+l_y+3/2)}}\,. \qquad (10)$$

$$V_{123}(\rho,\Omega) = \frac{1}{\rho} f(\Omega), \quad f(\Omega) = \frac{a_1}{\cos\alpha_1} + \frac{a_2}{\cos\alpha_2} + \frac{a_3}{\cos\alpha_3},$$

$$a_1 = \sqrt{\frac{m_2 m_3}{(m_2+m_3)\mu_{23}}} e^2 \equiv e^2, \; a_2 = \sqrt{\frac{m_1 m_3}{(m_1+m_3)\mu_{23}}} e^2 \simeq \sqrt{\frac{2m_e}{m_p}} e^2,$$

$$a_3 = a_2.$$

- Schrödinger equation for the pep system

$$H\Psi(\rho,\Omega) = E\Psi(\rho,\Omega)$$

$$\Psi(\rho,\Omega) = (2\pi)^3 \sum_{K l_x l_y L M} i^K \frac{u_K^{L\, l_x l_y}(\rho)}{(\kappa\rho)^2} \Phi_{KM}^{L\, l_x l_y}(\Omega). \qquad (11)$$

$$\kappa^2 = p^2 + q^2 = \frac{2\mu_{23}E}{\hbar^2}, \quad p \text{ and } q \text{ are canonically conjugate}$$

to x and y, respectively.

We consider case for $l_x = l_y = L = 0$, $K=0$ because only these moments are important in the pep state. Therefore we have

$$\frac{d^2u(\rho)}{d\rho^2} + \frac{1}{\rho}\frac{du(\rho)}{d\rho} - \left[v(\rho) + \frac{4}{\rho^2} - \kappa^2 \right] u(\rho) = 0, \qquad (12)$$

$$v(\rho) = v^N(\rho) + v^C(\rho), \qquad v^C(\rho) = -\frac{2\eta_3}{\rho},$$

$$\eta_3 = \frac{8}{3\pi}\frac{m_p}{\hbar^2\kappa}(a_1 + a_2 + a_3) \text{ is the Coulomb factor}$$

for three charges similar to the Sommerfeld parameter.

- The Gauss's potential:

$$v^N(\rho) = -\frac{4m_p V_0}{\hbar^2} \exp\left[-\rho^2/2R_N^2\right] I_1\left(\rho^2/2R_N^2\right) / \left(\rho/\mathrm{R_N}\right)^2 , \quad (13)$$

where $I_n(x)$ is the modified Bessel function of the first kind;

- The exponential potential:

$$v^N(\rho) = -\frac{4\pi m_p V_0}{\hbar^2} \left[\frac{2\pi\, I_2\left(\frac{\rho}{R_N}\right) + \frac{\rho}{R_N} I_3\left(\frac{\rho}{R_N}\right)}{(\rho/R_N)^2} - \frac{8}{15}\frac{\rho}{R_N}\; {}_1F_2\left(2; \frac{3}{2}, \frac{7}{2}; \frac{\rho^2}{4R_N^2}\right) \right]$$

(14)

- The Yukawa potential:

$$v^N(\rho) = -\frac{8m_p V_0}{\hbar^2} \frac{\left\{ \frac{2\rho}{R_N} - 3\pi \left[I_2\left(\frac{\rho}{R_N}\right) - L_2\left(\frac{\rho}{R_N}\right) \right] \right\}}{3\ \pi\ (\rho / R_N)^2}, (15)$$

where $L_n(x)$ is the modified Struve function.

- The Malfliet-Tjon potential:

$$v^N(\rho) = \frac{8m_p V_A}{\hbar^2} \frac{\left\{ \frac{2\rho}{R_A} - 3\pi \left[I_2\left(\frac{\rho}{R_A}\right) - L_2\left(\frac{\rho}{R_A}\right) \right] \right\}}{3\ \pi\ (\rho / R_A)^2} +$$

$$\frac{8m_p V_R}{\hbar^2} \frac{\left\{ \frac{2\rho}{R_R} - 3\pi \left[I_2\left(\frac{\rho}{R_R}\right) - L_2\left(\frac{\rho}{R_R}\right) \right] \right\}}{3\ \pi\ (\rho / R_R)^2} \qquad (16)$$

- Boundary conditions:

at $\rho \to 0$ (ρ_0=10^{-4}-10^{-6} fm) $u(\rho_0)=J_2(\kappa\rho_0)$, $u'(\rho_0)=J_2'(\kappa\rho_0)$;

at $\rho \to \infty$ $u(\rho) \propto \cos\delta_{3N} \left[F_{3/2}(\kappa\rho)\text{-}\tan\delta_{3N} G_{3/2}(\kappa\rho)\right]$, $\delta_{3N} \ll 1$.

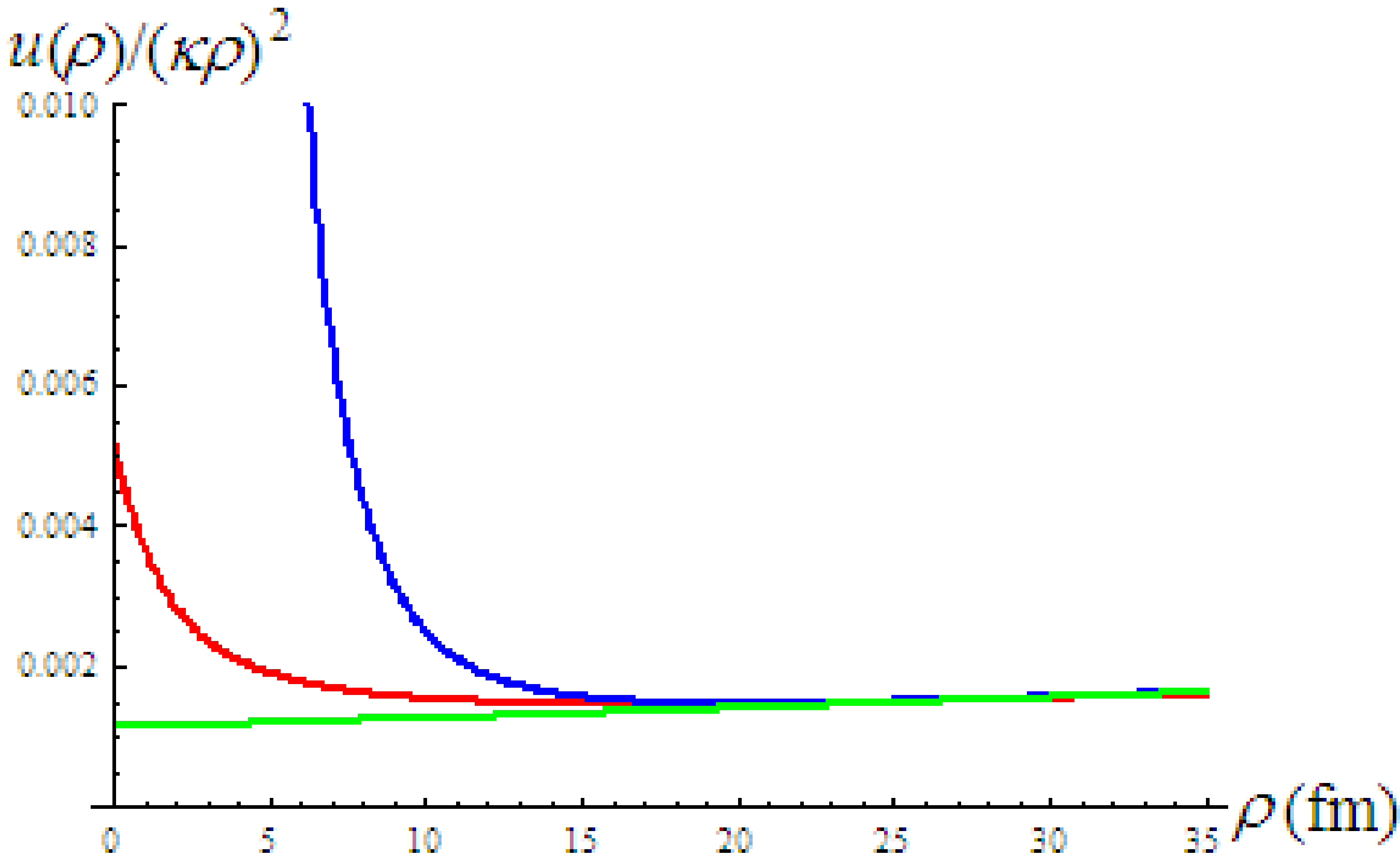


Fig.2 The pep radial wave function (red curve).
Its asymptotic at $\rho \to \infty$ (blue curve). The pure Coulomb wave is green curve. E_{pep}=6 keV.

Transition amplitude

- The effective weak Hamiltionian (Eq.1) is reduced to:

$$H_w = \tau^{(+)} \sum_{i=1}^{2} \tau_i^{(-)} G_A \vec{\sigma} \cdot \vec{\sigma}_i \delta(\vec{r} - \vec{r}_i), \qquad (17)$$

due to selection rule ($S_{pp} = 0,\ l_{pp} = 0,\ l_{np} = 0,\ S_{np} = 1,$

and $\dfrac{\nu}{2m_p c} \ll 1$).

- The amplitude:

$$M_{if} = G_A < \varphi_\nu \,|\, \vec{\sigma} \cdot \tau^{(+)} \,|\, \varphi_e > \sum_{i=1}^{2} < \Psi_d \,|\vec{\sigma}_i \cdot \tau_i^{(-)}|\, \Psi_{ppe} >, \qquad (18)$$

for $\nu r \ll 1$.

- The transition probability:

$$T_3 = \frac{2\pi}{\hbar} \overline{| M_{if} |^2} \rho(E_\nu), \qquad \rho(E_\nu) = \frac{E_\nu^2}{2\pi^2 \hbar^3 c^3} \quad (m_\nu = 0), \qquad (19)$$

$$E_\nu = E_{pp} + E_e + 2m_p c^2 + m_e c^2 - m_d c^2,$$

at $E_{pp} = E_e = 0, \quad E_\nu = 1.442$ MeV.

Taking into account that $\overline{|<\varphi_\nu | \vec{\sigma}\cdot\tau^{(+)} | \varphi_e>|^2} = 3$ we get

$$\mathrm{T}_3 = \frac{3E_\nu^2 G_A}{\pi\hbar^4 c^3} Q, \qquad Q = \left| \int \Psi_d^*(\vec{r})\Psi_{ppe}(\vec{r}, r_1)\Big|_{r_1 = r/2} d\vec{r} \right|^2. \qquad (19)$$

- The **rate law** for a elementary chemical reaction $a\mathrm{A}+b\mathrm{B} \to \mathrm{C}$ is defined by $r = k\,[\mathrm{A}]^a[\mathrm{B}]^b$, (20)

  where [A] and [B] express the concentration of the species A and B, respectively; *a* and *b* are called the stoichiometric coefficients of the balanced equation of the chemical reaction; *k* is the ***rate coefficient* or *rate constant*** of the reaction.
- According to Eq. (18) $\mathbf{T_3}$ is **a rate constant** for the reaction $ppe^- \to d+\nu_e$.

Rate and neutrino flux

$$r_3 = n_p^2 n_e < T_3 >; \qquad (21)$$

$$< T_3 >= \int \varphi(v_e)\varphi(v_{p_1})\varphi(v_{p_2})T_3(E)dv_e dv_{p_1} dv_{p_2}, \qquad (22)$$

where $n_p (n_e)$ is proton (electron) number density;

$\varphi(v_i)$ is the standard Maxwell-Boltzman distribution function.

$$\varphi(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right). \qquad (23)$$

After simplification of Eq. (16) we get:

$$< T_3 >= \frac{1}{2(kT)^3} \int_0^\infty \exp\left(-\frac{E}{kT}\right) T(E)E^2 dE. \qquad (24)$$

In analogy with two particle case {$v\sigma(E) \sim |F_0(r=0)|^2 S(E)$, $|F_0(r=0)|^2 = C_0^2(E) = 2\pi\eta/[\exp(2\pi\eta) - 1]$, where $S(E)$ is so-called astrophysical S-factor} we separate $C_{3/2}(E) = e^{-\pi\eta/2} G(5/2+i\eta)$ from the pep wave function in Eq. (17) and define the astrophysical S-factor for the $ppe^- \to d+\nu$ reaction as

$$T_3(E) = |C_{3/2}(E)|^2 \, S_3(E), \qquad (25)$$

where $S_3(E)$ is smooth function of E and we call it astrophysical S-factor for pep reaction, while

$$|C_{3/2}(E)|^2 = 2\pi \frac{\exp(-2\pi\eta_3)}{1+\exp(-2\pi\eta_3)} \left(\frac{1}{4}+\eta_3\right)\left(\frac{9}{4}+\eta_3\right), \qquad (26)$$

is exponentially decreasing factor (Gamow factor).

Finally for the average rate constant as a function of temperature T we obtain

$$< T_3 >= \frac{\pi}{(kT)^3} \int_0^\infty \exp\left(-\frac{E}{kT} - 2\pi\eta_3 \right) G_3(E) dE, \qquad (27)$$

$$G_3(E) = \frac{\left(\frac{1}{4} + \eta_3\right)\left(\frac{9}{4} + \eta_3\right)}{1 + \exp(-2\pi\eta_3)} S_3(E) E^2, \qquad (28)$$

The expression

$G_W(E) = \exp\left(-\frac{E}{kT} - 2\pi\eta_3 \right)$ defines the Gamow window. The maximum of the function at $E_{max} = (kT/2\sqrt{E_G})^{2/3}$, where $E_G = m_p (2\pi e^2 z_{eff}/\hbar)^2$, and

$$z_{eff} = \frac{16}{3\pi}(a_1 + a_2 + a_3)/e^2 = 1.588.$$

Note for pp→d+e$^+$ +ν_e reaction Gamow window is defined by the same expression but z_{eff} = 1. Therefore

$$E^{ppe}_{max} = z^{2/3}_{eff} E^{pp}_{max} = 1.344 E^{pp}_{max}.$$

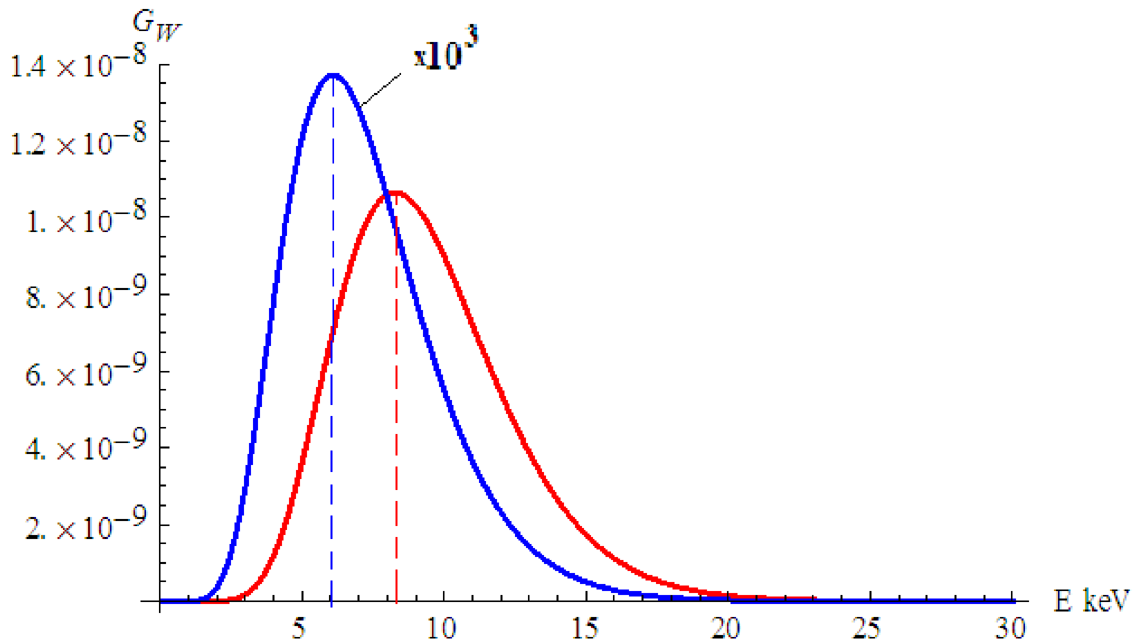


Fig.3. Gamow window vs. energy. Red curve for ppe$^-$→d+ ν_e, while blue curve for pp → d + e$^+$ + ν_e.

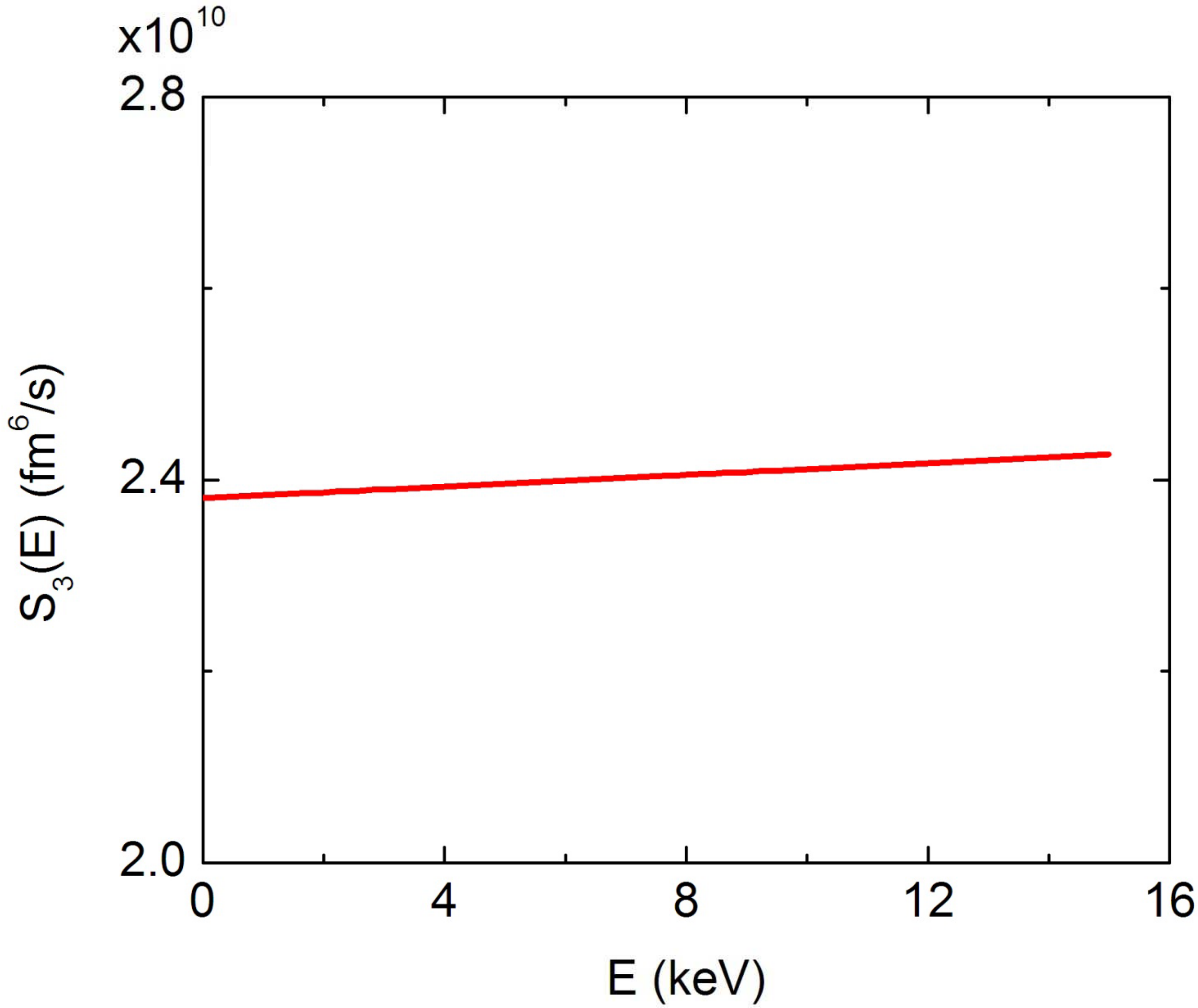


Fig.4. The pep astrophysical S_3-factor as a function of energy. Gauss potential case. The results with other NN potentials are the same

As we see from the last figure S-factor can be expanded in a Taylor series and we can restrict by three first terms:

$$S_3(E) = S_{(3)0} + S_{(3)1}E + S_{(3)2}E^2 \qquad (29)$$

The calculation gives the following values of the coefficients of expansion:

- Gauss potential:

$$S_{(3)0} = 2.38\times10^{10}\ \text{fm}^6/\text{s},$$

$$S_{(3)1} = 3.03\times10^{10}\ \text{fm}^6/(\text{Mev s}),$$

$$S_{(3)2} = 1.45\times10^{10}\ \text{fm}^6/(\text{Mev}^2\text{s}) \qquad (30)$$

- Yukawa potential:

$$S_{(3)0} = 2.33\times 10^{10}\ \text{fm}^6/\text{s},$$

$$S_{(3)1} = 3.01\times 10^{10}\ \text{fm}^6/(\text{Mev s}),$$

$$S_{(3)2} = 1.78\times 10^{10}\ \text{fm}^6/(\text{Mev}^2\ \text{s}) \qquad (31)$$

- exponential potential:

$$S_{(3)0} = 2.49\times 10^{10}\ \text{fm}^6/\text{s},$$

$$S_{(3)1} = 3.35\times 10^{10}\ \text{fm}^6/(\text{Mev s}),$$

$$S_{(3)2} = 2.21\times 10^{10}\ \text{fm}^6/(\text{Mev}^2\ \text{s}) \qquad (32)$$

- Malfliet-Tjon potential:

$$S_{(3)0} = 2.11\times 10^{10}\ \text{fm}^6/\text{s},$$

$$S_{(3)1} = 3.70\times 10^{10}\ \text{fm}^6/(\text{Mev s}),$$

$$S_{(3)2} = 1.76\times 10^{10}\ \text{fm}^6/(\text{Mev}^2\ \text{s}) \qquad (33)$$

Using these parameters we find the rate of reaction vs. radius of the Sun. We use the Standard Solar Model (SSM) versions BS2005-OP and BP2000

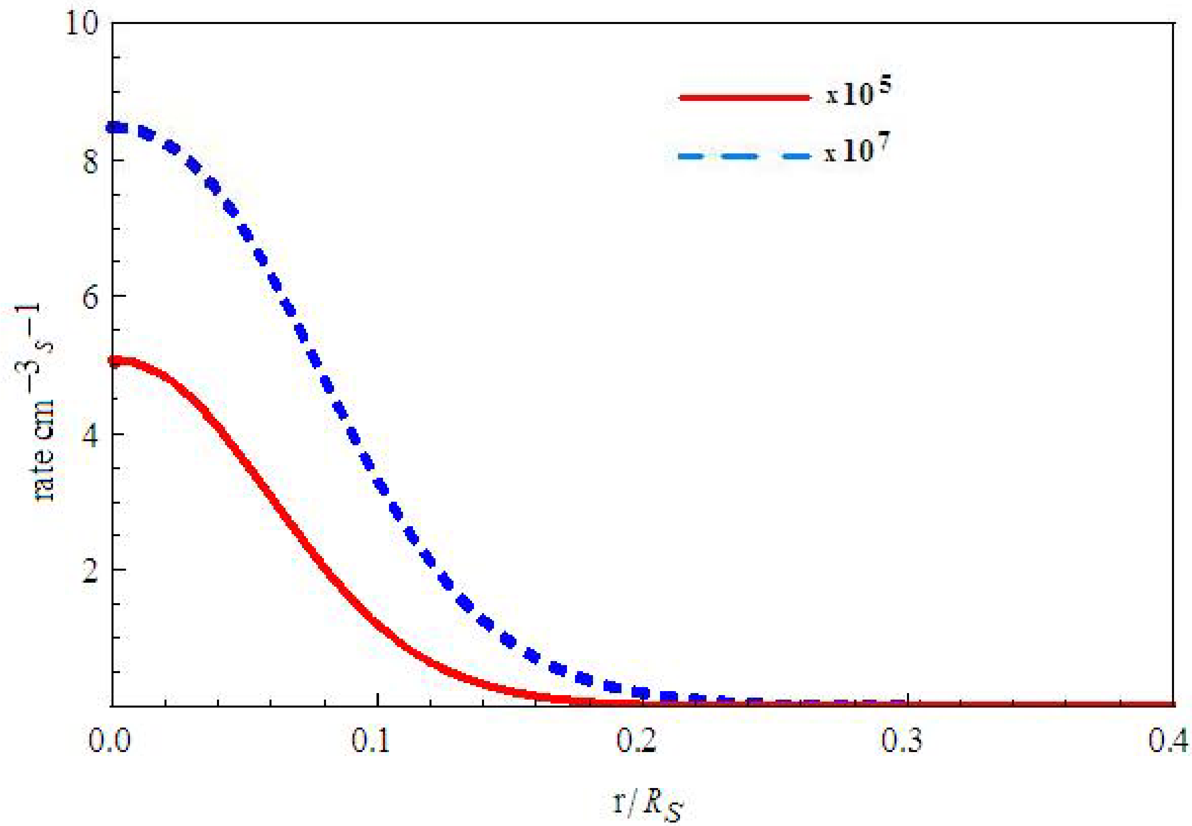


Fig. 5. Red curve for pep rate, blue curve for pp rate. SSM is BS2005-OP. Same for BP2000.

Knowing the number of neutrinos emitted by the Sun at 1 second we calculate the neutrino flux at 1 cm^2 at a distance of one astronomical unit. The results of the calculations are following:

- Gauss potential, BS2005-OP:

$\Phi_{pp} = 6.196\times10^{10}\ cm^{-2}s^{-1}$; $\Phi_{pep} = 2.042\times10^{8}\ cm^{-2}s^{-1}$;
$\Phi_{pp} / \Phi_{pep} = 303.4$

- Yukawa potential, BS2005-OP :

$\Phi_{pp} = 6.053\times10^{10}\ cm^{-2}s^{-1}$; $\Phi_{pep} = 1.995\times10^{8}\ cm^{-2}s^{-1}$;
$\Phi_{pp} / \Phi_{pep} = 303.4$

- exponential potential, BS2005-OP :

$\Phi_{pp} = 6.134\times10^{10}\ cm^{-2}s^{-1}$; $\Phi_{pep} = 2.138\times10^{8}\ cm^{-2}s^{-1}$;
$\Phi_{pp} / \Phi_{pep} = 286.9$

- exponential potential, BP2000 :

$\Phi_{pp} = 6.015\times10^{10}\ cm^{-2}s^{-1}$; $\Phi_{pep} = 2.092\times10^{8}\ cm^{-2}s^{-1}$;
$\Phi_{pp} / \Phi_{pep} = 287.6$

- Malfliet-Tjon potential, BS2005-OP :

$\Phi_{pp} = 6.156\times10^{10}$ cm^{-2}s^{-1} ; $\Phi_{pep} = 1.817\times10^{8}$ cm^{-2}s^{-1};
$\Phi_{pp} / \Phi_{pep} = 338.5$

Survival probability of neutrino

The earlier measurement of boron ν_e flux by SNO, combined with Super-Kamiokande data, gives strong evidence for neutrino oscillations.
The survival probability ν_e in vacuum is

$$P(\nu_e) = 1 - \sin^2 2\theta \, \sin^2\left(\frac{1.27\Delta \mathrm{m}^2 \mathrm{L}}{E}\right), \qquad (30)$$

NOTE: L(m), E(MeV) , Δm^2 (eV^2), $\Delta m^2 = m^2_{\nu_\mu} - m^2_{\nu_e}$
For the MSW probability the mixture angle θ is replaced by θ_m , depending on the density of the medium.

$$P(\nu_e) = 1 - \sin^2 2\theta_m \, \sin^2\left(\frac{1.27\Delta \mathrm{m}^2 \mathrm{L}}{E}\right), \qquad (31)$$

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(\cos 2\theta - \frac{\xi}{\Delta m^2}\right)^2 + \sin^2 2\theta}},$$

$$\xi = 1.526 \times 10^{-7} \frac{Z}{A} \rho k \ \mathrm{eV}^2,$$

NOTE: density ρ(kg/cm^3), neutrino momentum k(MeV/c), Z is atomic number while A is atomic weight of the element of the medium.

In the MSW oscillation should be resonance transform ν_e -> ν_μ at defined value of the medium density. However, there is big problem to define the correct value of mixture angle θ and exact value of Δm. Therefore the different behavior of the survival probability is observed, which can be seen the pictures below.

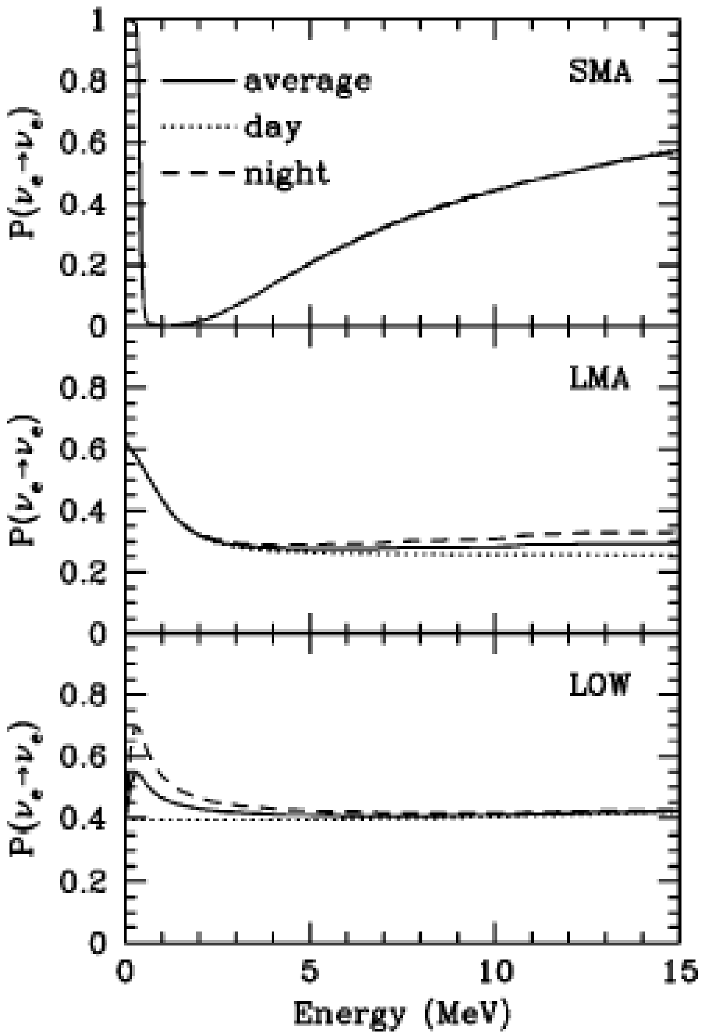


FIG. 6. Survival probabilities for MSW solutions. The figure presents the yearly-averaged survival probabilities for an electron neutrino that is created in the sun to remain an electron neutrino upon arrival at the SuperKamiokande detector. [J. N. Bahcall, P. I. Krastev, A. Yu. Smirnov, Phys. Rev. D58, 096016-1 (1998)]

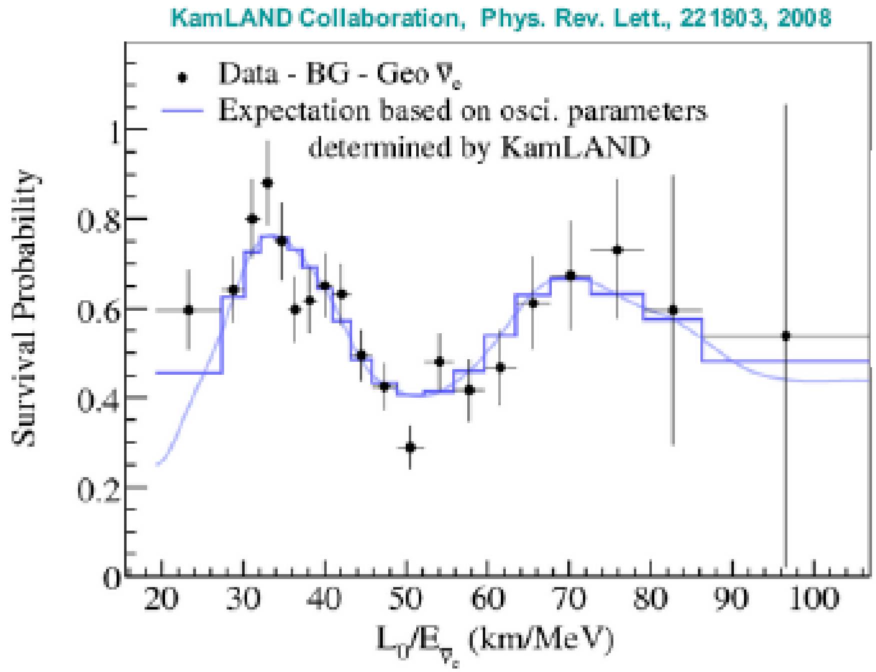


Fig. 7

KamLAND RESULTS:

Observe: 1609 events
Expect: 2179±89 events (if no oscillations)

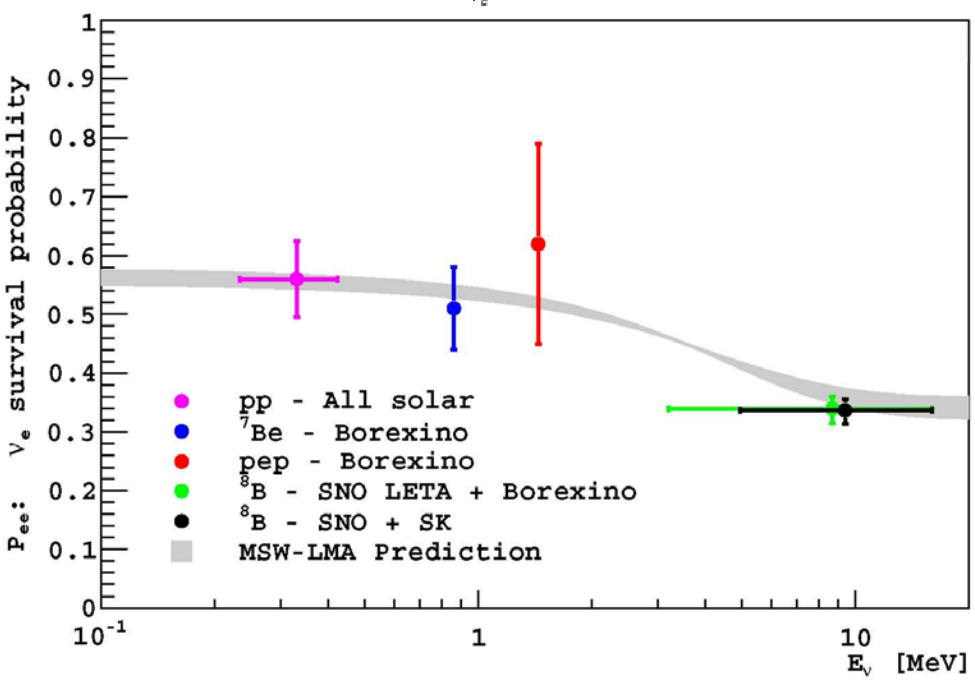


Fig. 8 Electron neutrino survival probability as a function of energy. The red line corresponds to the measurement by Borexino collaboration. [PRL, **108**, 051302 (2012)]

Taking into account that survival probability of neutrino (due to neutrino oscillation) in *pep* reaction predicted by Borexino collaboration equals to P=0.62 ±0.17 at 1.44 MeV we find the neutrino flux at 1 AU:

- Gauss potential, BS2005-OP:

Φ_{pep}= $(1.266 \pm 0.347)\times 10^{8}$ $cm^{-2}s^{-1}$.

- Yukawa potential, BS2005-OP:

Φ_{pep}= $(1.237 \pm 0.339)\times 10^{8}$ $cm^{-2}s^{-1}$.

- Exponential potential, BS2005-OP:

Φ_{pep}= $(1.326 \pm 0.363)\times 10^{8}$ $cm^{-2}s^{-1}$.

- Exponential potential, BP2000:

Φ_{pep}= $(1.297 \pm 0.356)\times 10^{8}$ $cm^{-2}s^{-1}$.

- Malfliet-Tjon potential, BS2005:

Φ_{pep}= $(1.130 \pm 0.310)\times 10^{8}$ $cm^{-2}s^{-1}$

All our calculated fluxes from the pep reactions with different types of the potential and SMM lie close to the Borexino results

.

Comparison with Borexino experimental data and Bahcall's results

- **Borexino Collaboration (assuming the MSW-LMA effect)** $\Phi = (1.6 \pm 0.3) \times 10^8\ cm^{-2}s^{-1}$

Table 3. Predicted fluxes (without survival probability), in units of 10^{10}(pp), 10^8(pep) $cm^{-2}s^{-1}$. Data are taken from Ref.: **John N. Bahcall, Aldo M. Serenelli, Sarbani Basu,** arXiv:astro-ph/0412440v3.

Model	pp	pep	Φ_{pp} / Φ_{pep}
BP04(Yale)	5.94	1.40	424.2
BP04(Garching)	5.94	1.41	421.3
BS04	5.94	1.40	424.2
BS05(14N)	5.99	1.42	421.3
BS05(OP)	5.99	1.42	421.3
BS05(AGS,OP)	6.06	1.45	417.9
BS05(AGS,OPAL)	6.05	1.45	417.2

Conclusion

- Three used NN potentials lead to the almost same value of the astrophysical S-factors for **pp** reaction obtained by Bahcall *et al.*.
- The method of hyperharmonic function gives possibility to get with enough accuracy **pep** three-body wave function in the initial state if the energy corresponds to the Sun interior condition.
- In the framework of the three-body approach the probability of the process $\mathbf{ppe^{-} \rightarrow d+\nu_e}$ at the conditions in the center of Sun, has been found.
- We have suggested the definition of the astrophysical S factor for **pep** reaction.
- The value of neutrino flux from **pep** reaction obtained by us on ~40% is larger than it calculated by Bahcall *et al.* It can be explained by increasing probability of reaction due to screening the charges of the proton-proton system by electron change. If neutrino oscillation is taken into account our calculated flux would close to the Borexino experimental data.


Thanks