Nuclear Polarizability and the Lamb Shift in Muonic Atoms

Chen Ji[1], **Nir Nevo Dinur**[2], **Sonia Bacca**[1], **Nir Barnea**[2]

[1]**TRIUMF, Vancouver, Canada**
[2]**The Hebrew University, Jerusalem, Israel**

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האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

How large is the proton?

- **electron-proton**
  1. e-p scattering: $r_p = 0.875(10)$ fm
  2. eH atomic spectroscopy: $r_p = 0.8768(69)$ fm
  3. CODATA-2010: $r_p = 0.8775(51)$ fm

     Mohr *et al.*, Rev. Mod. Phys. (2012)

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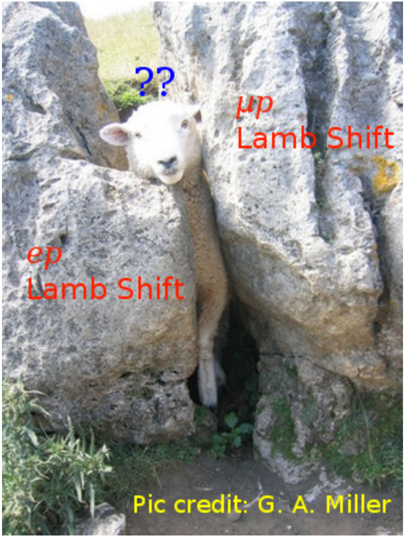
- **muonic hydrogen Lamb shift (2S-2P)**
  1. μH $2S_{1/2}^{F=1}$-$2P_{3/2}^{F=2}$: $r_p = 0.84184(67)$ fm (5σ)

     Pohl *et al.*, Nature (2010)
  2. Combine μH $2S_{1/2}^{F=0}$-$2P_{3/2}^{F=1}$: $r_p = 0.84087(39)$ fm (7σ)

     Antognini *et al.*, Science (2013)

Origins of the discrepancy?

Discrepancy of r_p between ep and μp measurements



Origins of the discrepancy?

- **study r_p's discrepancy between μp and ep experiments**
  - systematic errors in atomic Lamb shift and ep scattering
  - new physics that distinguishes μp and ep interactions
  - high-precision measurements $\Longleftrightarrow$ accurate theoretical inputs

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- **new experiments at PSI**
  - Lamb shift in μD

    CREMA collaboration, finishing
  - Lamb shift in muonic helium

    CREMA collaboration, planned in 2013
  - μp scattering experiment

    MUSE collaboration, in development

Extract nuclear charge radius

- $\langle r^2 \rangle$ from Lamb shift

$$\Delta E_{LS} = \delta_{QED} + \delta_{pol} + \frac{m_r^3}{12}(Z\alpha)^4 \langle r^2 \rangle - \frac{m_r^4}{24}(Z\alpha)^5 \langle r^3 \rangle_{(2)}$$

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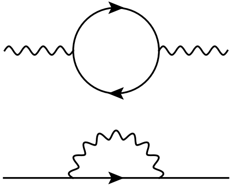
- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects



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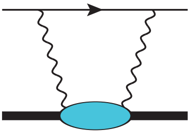
- Nuclear polarization corrections (inelastic):
  - exchange of two virtual photons
  - dominant contribution $\sim (Z\alpha)^5$



- Intrinsic inelastic nucleon polarization corrections

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- Nuclear finite-size corrections (elastic):
  - leading term $\sim (Z\alpha)^4$: $\frac{m_r^3}{12}(Z\alpha)^4 \langle r^2 \rangle$
  - Zemach moment $\sim (Z\alpha)^5$: $-\frac{m_r^4}{24}(Z\alpha)^5 \langle r^3 \rangle_{(2)} \quad \propto \langle r^2 \rangle^{3/2}$

Uncertainty in nuclear polarization

$$\Delta E_{LS} = \delta_{QED} + \boldsymbol{\delta_{pol}} + \frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle - \frac{m_r^4}{24}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

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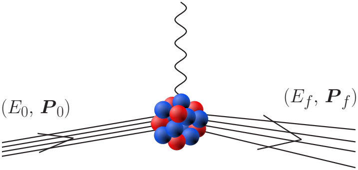
- Nuclear polarization $\Longrightarrow$ inputs from nuclear response function

$$S_O(\omega) = \sum\!\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2\delta(E_f - E_0 - \omega)$$



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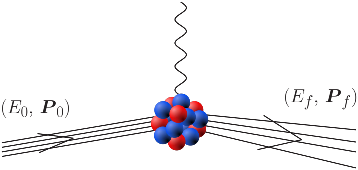

- Early calculations of δ_{pol} in muonic atoms:
  $\Rightarrow S_O(\omega)$ inputs were not accurate enough

Calculations of $S_O(\omega)$ & δ_{pol}

- **Toy models**
  - $\mu\,^{12}$C (square-well) Rosenfelder '83
  - μD (Yamaguchi) Lu & Rosenfelder '93

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- $S_O(\omega)$ **from photoabsorption cross sections**
  - $\mu\,^{4}\mathrm{He}^{+}$: Bernabeu & Jarlskog '74; Rinker '76; Friar '77
  - $\delta_{pol} = -3.1$ meV $\pm 20\%$
  - c.f. experimental requirement $\sim \pm 5\%$

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- **state of the art**
  - μD: AV14 (Leidemann & Rosenfelder, '95); AV18 (Pachucki, '11)
  - μD: Effective range expansion (Friar, '13)

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- **δ_{pol} in other light muonic atoms (*e.g.*, $\mu\,^3$He$^+$, $\mu\,^4$He$^+$, $\cdots$)**
  - experimental input for S_O is either too scattered or inexistent
  - need to calculate S_O using **modern potentials and *ab-initio* methods**

Calculations of δ_{pol} in $\mu\,^4$He$^+$

We perform the first *ab-initio* calculation of nuclear polarization in $\mu\,^4$He$^+$ with state-of-the-art potentials

Ji, Nevo Dinur, Bacca & Barnea, arXiv:1307.6577 (2013)

- Effective Interaction Hyperspherical Harmonics method **EIHH**
- Lorentz Integral transform method **LIT**
- Nuclear Hamiltonian: AV18/UIX and χEFT
  $\Longrightarrow$ response functions
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  $\Longrightarrow$ nuclear polarization in $\mu\,^4$He$^+$

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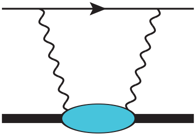
- **Final Goal:**

  provide δ_{pol} with an accuracy comparable to the $\pm 5\%$ experimental needs

- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb

$$\Delta H = \alpha \sum_i^Z \Delta V(\boldsymbol{r}, \boldsymbol{R}_i) \equiv \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- Evaluate ΔH's inelastic effects to the muonic atom spectrum in 2nd-order perturbation theory

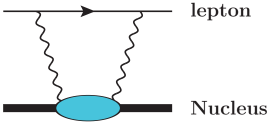
$$\delta_{pol} = \sum_{N \neq N_0, \mu} \langle N_0 \mu_0 | \Delta H | N \mu \rangle \frac{1}{E_{N_0} - E_N + \epsilon_{\mu_0} - \epsilon_\mu} \langle N \mu | \Delta H | \mu_0 N_0 \rangle$$

$|\mu_0\rangle$: muon wave function for $2S/2P$ state

Systematic contributions to nuclear polarization

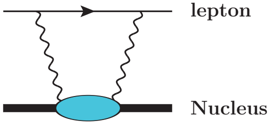
- non-relativistic limit δ_{NR}
- longitudinal and transverse relativistic polarizations $\delta_L + \delta_T$
- Coulomb distortions δ_C
- corrections from finite nucleon size δ_{NS}

- Neglect Coulomb interactions in the intermediate state



Non-relativistic limit

- **Neglect Coulomb interactions in the intermediate state**
- **Expand muon matrix element in powers** $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$



$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

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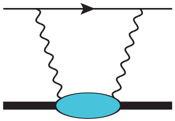

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- $|\boldsymbol{R}-\boldsymbol{R}'| \Longrightarrow$ "virtual" distance a proton travels in 2γ exchange
- uncertainty principal $|\boldsymbol{R}-\boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
- $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} \approx 0.17$ for $\mu\,^4He^+$

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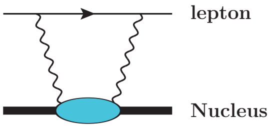

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- $\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)} \Longrightarrow$ LO + NLO + N^{2}LO

NR limit at LO: $\delta_{NR}^{(0)}$

$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$

- $\delta_{NR}^{(0)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

$$\delta_{D1}^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

- $S_{D_1}(\omega) \Longrightarrow$ electric dipole response function
- $\delta_{D1}^{(0)}$ is the dominant contribution to δ_{pol}

NR limit at NLO: $\delta_{NR}^{(1)}$

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$

$$\boxed{\delta_{NR}^{(1)} = \delta_{R3pp}^{(1)} + \delta_{Z3}^{(1)}}$$

$$\delta_{R3pp}^{(1)} = -\frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R}d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \langle N_0|\hat{\rho}^\dagger(\boldsymbol{R})\hat{\rho}(\boldsymbol{R}')|N_0\rangle$$

$$\delta_{Z3}^{(1)} = \frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R}d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0(\boldsymbol{R})\rho_0(\boldsymbol{R}')$$

- $\delta_{R3pp}^{(1)} \Longrightarrow$ 3rd-order proton charge correlation
- $\delta_{Z3}^{(1)} \Longrightarrow$ 3rd-order Zemach moment
  cancels Zemach moment in finite-size corrections
  c.f. Pachucki '11 & Friar '13 (μ D)

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(2)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$

$$\delta_{NR}^{(2)} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S_{R^2}(\omega) + \frac{16\pi}{25} S_Q(\omega) + \frac{16\pi}{5} \mathcal{S}_{D_1 D_3}(\omega) \right]$$

- $S_{R^2}(\omega) \Longrightarrow$ monopole response function
- $S_Q(\omega) \Longrightarrow$ quadrupole response function
- $S_{D_1 D_3}(\omega) \Longrightarrow$ interference between D_1 and D_3 $[\ \hat{D}_3 = R^3 Y_1(\hat{R})\ ]$

- **Test Run:**
  electric-dipole polarization effects in μ D
  - μ-D's nuclear polarization (AV18): Pachucki, '11
  - we calculate contributions from dipole excitation (AV18)
    [$S_{D_1}(\omega)$ from Bampa, Leidemann & Arenhövel '11]

$\delta^{(0)}$ [meV]		Pachucki, '11	Our work
non-rel dipole	$\delta_{D1}^{(0)}$	-1.910	-1.907
relativistic	$\delta_{L}^{(0)}$	0.035	0.029
	$\delta_{T}^{(0)}$	–	-0.012
Coulomb	$\delta_{C}^{(0)}$	0.261	0.259

  - The difference in $\delta_{L}^{(0)}$ is due to small energy expansion used in Pachucki, '11

nuclear polarization in $\mu\,^4$He$^+$

[meV]		AV18/UIX	χEFT★
$\delta^{(0)}$	$\delta^{(0)}_{D1}$	-4.418	-4.701
	$\delta^{(0)}_{L}$	0.289	0.308
	$\delta^{(0)}_{T}$	-0.126	-0.134
	$\delta^{(0)}_{C}$	0.512	0.546

★ NN(N^{3}LO)/$3N$(N^{2}LO)

c_D=1, c_E=-0.029

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$\delta^{(1)}$	$\delta^{(1)}_{R3pp}$	-3.442	-3.717
	$\delta^{(1)}_{Z3}$	4.183	4.526

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$\delta^{(2)}$	$\delta^{(2)}_{R2}$	0.259	0.324
	$\delta^{(2)}_{Q}$	0.484	0.561
	$\delta^{(2)}_{D1D3}$	-0.666	-0.784

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δ_{NS}	$\delta^{(1)}_{R1pp}$	-1.036	-1.071
	$\delta^{(1)}_{Z1}$	1.753	1.811
	$\delta^{(2)}_{NS}$	-0.200	-0.210

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	$\delta^{(2)}_{NS}$	-0.200	-0.210
δ_{pol}		-2.408	-2.542

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nuclear polarization in $\mu\,{}^4\text{He}^+$

[meV]	AV18/UIX	χEFT★
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{pol}	-2.408	-2.542

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$
  in a systematic expansion of
  $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{m_r/M_N} \approx 0.17$

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  $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{m_r/M_N} \approx 0.17$
- δ_{pol} with AV18/UIX & χEFT differ:
  $\sim 5.5\%$ (0.134 meV)

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c_D=1, c_E=-0.029

The work is not completed yet ...

Nuclear physics uncertainty

^{4}He		AV18/UIX	χEFT	Difference
$\mu\,^4$He$^+$ nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

Nuclear physics uncertainty

^{4}He		AV18/UIX	χEFT	Difference
binding energy	B_0 [MeV]	28.422	28.343	0.28%
point-proton radius	R_{pp} [fm]	1.432	1.475	3.0%
electric-dipole polarizability	α_E [fm^3]	0.0651	0.0694	6.4%
$\mu\,^{4}$He^{+} nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09

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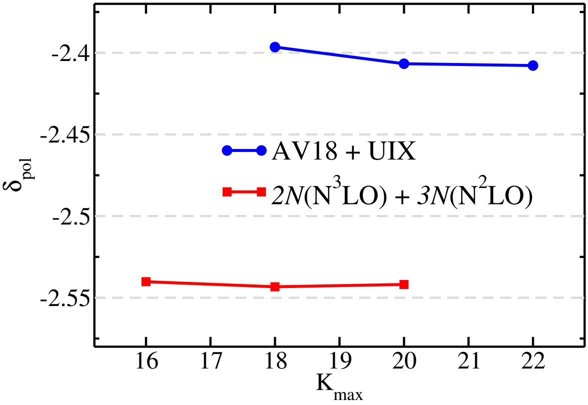
binding energy	B_0 [MeV]	28.422	28.343	0.28%
point-proton radius	R_{pp} [fm]	1.432	1.475	3.0%
electric-dipole polarizability	α_E [fm^3]	0.0651	0.0694	6.4%
$\mu\,^4$He$^+$ nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09
- systematic uncertainty in δ_{pol} from nuclear physics:
  $\Longrightarrow \frac{5.5\%}{\sqrt{2}} \rightarrow \pm 4\%$

- Convergence with the largest model space K_{max}
- Difference btw K_{max} & $K_{max} - 4$
  - AV18/UIX $\sim 0.4\%$
  - EFT $\sim 0.2\%$



Atomic physics uncertainty

- $(Z\alpha)^6$ effects (beyond 2nd-order perturbation theory)
- relativistic & Coulomb corrections to other multipoles (other than dipole)
- higher-order nucleon-size corrections

- combine these corrections $\Longrightarrow$ an additional few percent error

δ_{pol} in $\mu\,^4\text{He}^+$

- combine all errors in a quadratic sum
- our prediction: $\delta_{pol} = -2.47$ meV $\pm 6\%$
- more accurate than early calculations: $\delta_{pol} = -3.1$ meV $\pm 20\%$

  Bernabeu & Jarlskog '74; Rinker '76; Friar '77
- our accuracy is comparable to the 5% requirement for the future $\mu\,^4\text{He}^+$ Lamb shift measurement

  Antognini *et al.* '11

Summary

- Lamb shifts in muonic atoms
  - raise interesting questions about lepton universality
  - high precision nuclear charge radius measurement

- We perform the first *ab-initio* calculation for $\mu\,^4\mathrm{He}^+$ polarization corrections
  - utilize the EIHH and the LIT methods
  - use modern phenomenological & chiral potentials

- We obtain $\delta_{pol} = -2.47$ meV $\pm 6\%$
  - more accurate than early calculations
  - will significantly improve the precision of $\langle r^2 \rangle$ extracted from future $\mu\,^4\mathrm{He}^+$ Lamb shift measurement (2013)

Outlook

- Study higher-order atomic-physics corrections
- Narrow uncertainty in nuclear physics
  - understand the discrepancy btw AV18/UIX & EFT results
  - explore other choices for potential parameterizations
  - include higher-order χEFT forces
- Investigate nuclear polarization in *e.g.* $\mu\,{}^3\mathrm{He}^+$, $\cdots$

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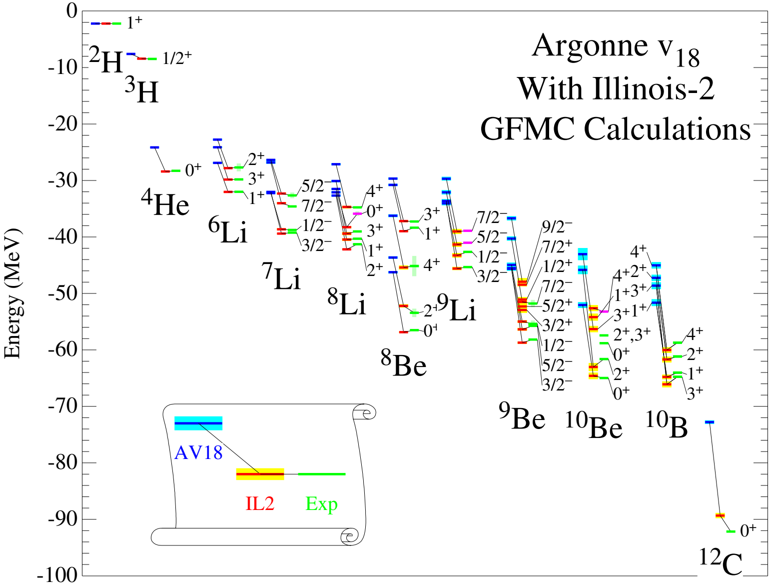
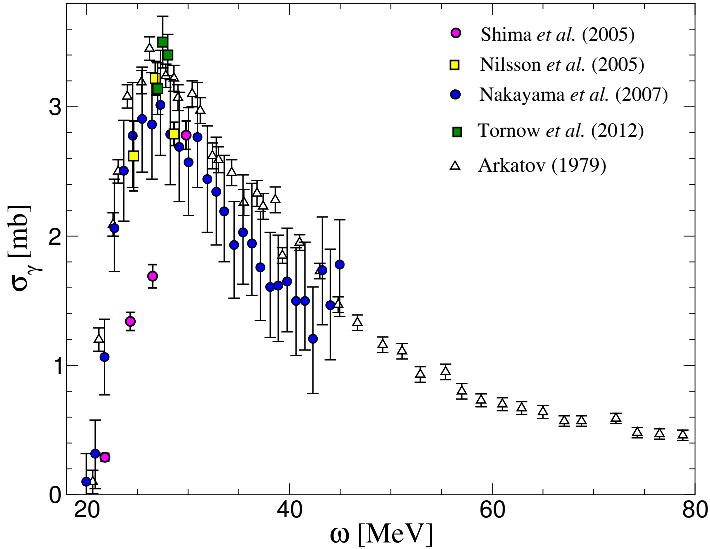
BACK UP

Phenomenological potentials



^{4}He photoabsorption cross sections

electric-dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega S_{D1}(\omega)$



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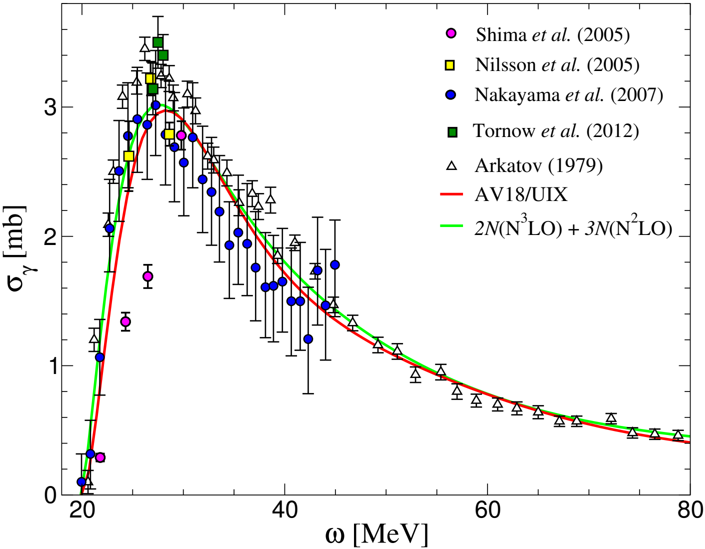

Non-Relativistic Approximation

- Coulomb interactions in the (virtual) intermediate state are neglected
- Results are expressed in an expansion of $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

$$\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$$

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1. $\sim R^2$ **term:**

- $\Delta E_{NR}^{(2)}$ **is the dominant polarizability contribution**

$$\Delta E_{NR}^{(2)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S^{D_1}(\omega)$$

- $S^{D_1}(\omega) = \frac{1}{2J_0+1} \sum_{N\neq N_0, J} |\langle N_0 J_0 || \hat{D}_1 || NJ \rangle|^2 \delta(\omega - E_N + E_{N_0})$
- $\hat{D}_1 = \frac{1}{Z}\sum_i^Z R_i Y_1(\hat{R}_i)$

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$$\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$$

2. $\sim R^3$ **term:**

$$\Delta E_{NR}^{(3)} = -\frac{m_r^4}{24}(Z\alpha)^5 \left[\iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \langle N_0|\hat{\rho}^\dagger(\boldsymbol{R})\hat{\rho}(\boldsymbol{R}')|N_0\rangle - \langle r^3\rangle_{(2)} \right]$$

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  vanishes in point-nucleon limit

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- 1st term: charge correlation function
  vanishes in point-nucleon limit

- 2nd term: Zemach moment
  $\langle r^3\rangle_{(2)} = \iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \rho_0(\boldsymbol{R})\,\rho_0(\boldsymbol{R}')$
  $\rho_0(\boldsymbol{R}) = \langle N_0|\hat{\rho}(\boldsymbol{R})|N_0\rangle$

  cancels exactly the **Zemach term** in (elastic) finite-size corrections
  c.f. Pachucki PRL 2011 (μD)

3. $\sim R^4$ **term:**

- $\Delta E_{NR}^{(4)}$ **corresponds to higher-multipole corrections**

$$\Delta E_{NR}^{(4)} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S^{Q_0}(\omega) + \frac{16\pi}{25} S^{Q_2}(\omega) + \frac{16\pi}{5} S^{D_{13}}(\omega) \right]$$

- $S^{R^2}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{R}^2 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$

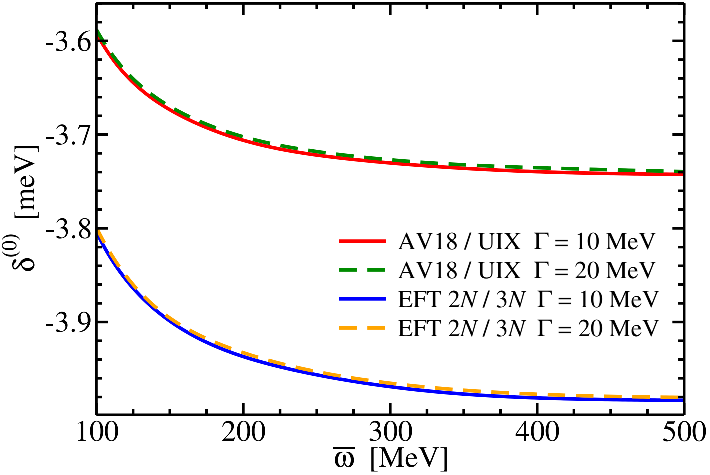
$$S^{Q_2}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{Q}_2 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$$

$$S^{D_{13}}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} (-1)^{J_0 - J} \times \mathrm{Re}\left(\langle N_0 J_0 || \hat{D}_3 || N J \rangle \langle N J || \hat{D}_1 || N_0 J_0 \rangle \right) \delta(\omega - E_N + E_{N_0})$$

- $\hat{R}^2 = \frac{1}{Z} \sum_i^Z R_i^2$ $\quad$ $\hat{D}_1 = \frac{1}{Z} \sum_i^Z R_i Y_1(\hat{R}_i)$

$$\hat{Q}_2 = \frac{1}{Z} \sum_i^Z R_i^2 Y_2(\hat{R}_i) \qquad \hat{D}_3 = \frac{1}{Z} \sum_i^Z R^3 Y_1(\hat{R}_i)$$

Convergence of $\bar{\omega}$



Convergence of Ab-initio calculations

$\delta^{(0)}$ convergence with the largest model space K_{max}